DOI: 10.17725/j.rensit.2023.16.249

Intraband absorption of electromagnetic radiation by electrons with optical phonon participation in quantum dot superlattices

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Received December 08, 2023, peer-reviewed December 15, 2024, accepted December 22, 2024, published April 25, 2024

Abstract: **In this paper, we investigate the absorption of electromagnetic radiation by a free electron gas interacting with lattice vibrations in a quantum dot superlattice. It is assumed that the electron gas in the quantum dot superlattice is limited by the anisotropic parabolic potential. The absorption of light by free carriers with the participation of phonons is calculated in the second order of the perturbation theory. When calculating the absorption coefficient, the matrix elements of the electron-photon and electron-phonon interactions (with polar and nonpolar optical phonons) are used. We can expect three possible transitions in the absorption coefficient for electron- polar phonon scattering: (1) a transition due to the size subband levels for only the x direction, (2) a transition due to the size subband levels for only the y direction and (3) a transition due to the size subband levels for both the x direction and the y direction. For electron- nonpolar phonon scattering we can expect only one possible transitions in the, the absorption coefficient: a transition due to the size subband levels for both the x direction and the y direction.**

Keywords: **quantum dot, superlattice, miniband, intraband absorption**

UDC: 621.315.592

For citation: Guseyn B. Ibragimov, Raida Z. Ibayeva. Intraband absorption of electromagnetic radiation by electrons with optical phonon participation in quantum dot superlattices. *RENSIT: Radioelectronics. Nanosystems. Information Technologies,* 2024, 16(1):249-255e. DOI: 10.17725/j.rensit. 2024.16.249.

Contents

1. INTRODUCTION

Advances in methods for growing crystals with sizes close to the interatomic distance, such as molecular beam epitaxy and metal-organic vapor deposition, have made it possible to obtain such low-dimensional systems as quantum wells, superlattices, quantum dots, etc. Since the beginning of the pioneering work of Esaki and Tsu [1], a large amount of research has been devoted to semiconductor superlattices due to their transport properties and their application in electronic devices such as generators, tunnel diodes, hot electron transistors, and optoelectronic devices [2,3]. Modern nanotechnology makes it possible to manufacture quantum dots of various shapes, which are successfully used in various devices, quantum cascade lasers based on quantum dot superlattices [1–3]. Such systems have a significantly higher performance compared to existing quantum cascade lasers based on quantum well superlattices [4]. Superlattices of quantum dots attract the attention of researchers also due to their unusual optical and optoelectronic properties, which can be used to develop new optoelectronic devices [5-13].

It is well known that the electron-phonon interaction is an important factor affecting the physical properties of polar crystals, such as the binding energy of impurities, charge transfer, linear and nonlinear optical properties, especially in low-dimensional quantum systems [14,15].

In this paper, we develop a theory of light absorption by free carriers in a superlattice of quantum dots with an anisotropic parabolic potential upon scattering of carriers by polar and nonpolar optical phonons.

In bulk semiconductors, a quantum mechanical theory of light absorption by free carriers using the second-order perturbation theory was given by Dumke [16]. Later, it was studied in detail for acoustic phonons (through the deformation potential and piezoelectric coupling), non-polar and polar optical phonons, and impurities in bulk semiconductors [17]. These studies have shown a different power-law dependence of the absorption coefficient on wavelength to radiation for different scattering mechanisms. Moreover, these power laws differ from the classical dependence of the absorption coefficient. The absorption of light by free carriers in the scattering of electrons by phonons has been theoretically investigated in semiconductor quantum wells and wires [18-28] and in quantum dots [29,30].

2. THEORETICAL FRAMEWORK

It is known that the motion of an electron in a superlattice is limited and its energy spectrum is quantized. It is assumed that the quantization takes place in the z-direction. The electron-phonon interaction in a quantum dot superlattice with a periodic potential $U(z)$ of period d along the z-direction of the form [31] is considered:

$$\mathrm{H} = \mathrm{G}_{\mathrm{GAPL}}(\mathrm{v}') = \mathrm{Concat}_{\mathrm{j}=1}^{\mathrm{O}'}\left(\frac{1}{\mathrm{B}}\sum_{\mathrm{a}=1}^{\mathrm{B}}\mathrm{v}_{\mathrm{a}}^{\mathrm{j}}\right) \in \mathbb{R}^{1\times \mathrm{O}'}. \tag{1}$$

where m^* is the effective mass and ω_x, ω_y are the confinement frequencies in the x- and y-directions, respectively. In the strong-coupling approximation, the Hamiltonian for charge carriers in a quantum dot superlattice can be written as [8,9]:

$$H = \frac{(p_x^2 + p_y^2)}{2m^*} + \frac{m^*}{2}(\omega_x^2 x^2 + \omega_y^2 y^2) + \frac{\Delta}{2}(1 - \cos\frac{p_z d}{\hbar}), \tag{2}$$

where Δ is the miniband width. The normalized eigenwave functions of the electron $\Psi_{n,e,k_z}(r)$ and the eigenvalues of the energy $E_{n,l}(k_z)$ in the conduction band are set, respectively, in the form [10]:

$$\Psi_{n,l,k_z}(r) = \frac{1}{\sqrt{L_z}}\Psi_n(x)\Psi_l(y)\xi_{k_z}(z), \tag{3}$$

$$E_{n,l}(k_z) = (n+\frac{1}{2})\hbar\omega_x + (l+\frac{1}{2})\hbar\omega_y + \\ + \frac{\Delta}{2}(1-\cos k_z d) = \varepsilon_{n,l} + \varepsilon(k_z), \tag{4}$$

where n = 0, 1, 2, ... and l = 0, 1, 2, ... are the level indices of the electronic subbands, k_z is the component of the wave vector in the z-direction, $\Psi_n(x) = \sqrt{1/\sqrt{\pi}l_x 2^n n!}\exp\left[-x^2/2l_x^2\right]H_n(x/l_x)$, $\Psi_l(y) = \sqrt{1/\sqrt{\pi}l_y 2^l l!}\exp\left[-y^2/\left(2l_-^2\right)\right]H_l(y/l_-)$ are the eigenwave functions of the harmonic oscillator, $H_n(x)$ is the Hermite polynomial, $l_x = \sqrt{\hbar/\left(m^*\omega_x\right)}, l_y = \sqrt{\hbar/\left(m^*\omega_y\right)}, \xi_{k_z}(z)$ is the Bloch function in the approximation of strong coupling in the z-direction, and L_z is the normalized length in the direction.

The absorption of light by free carriers with the participation of phonons is calculated in the second order of the perturbation theory. The absorption coefficient is determined by the well-known formula [20]:

$$\alpha = \frac{\varepsilon^{1/2}}{n_0 c}\sum_i W_i f_i, \tag{5}$$

where ε is the dielectric constant, n_0 is the number of photons in the radiation field, f_i is the distribution function of free current carriers, W_i is the transition probability, determined by the following expression:

$$W_i = \frac{2\pi}{\hbar}\sum_{fq}\left[\left|\langle f|M_+|i\rangle\right|^2\delta\left(E_f - E_i - \hbar\Omega - \hbar\omega_q\right) + \right. \\ \left. + \left|\langle f|M_-|i\rangle\right|^2\delta\left(E_f - E_i - \hbar\Omega + \hbar\omega_q\right)\right], \tag{6}$$

where E_i and E_f are the energies of the initial and final states of electrons, respectively, $\hbar\Omega$ is the photon energy, $\hbar\omega_q$ is the phonon energy, and $\langle f|M_\pm|i\rangle$ are the elements of the matrix of transition from the initial state to the final state for the interaction between electrons, phonons, and photons. Elements of the transition matrix can be represented as:

$$\langle f|M_\pm|i\rangle = \sum_\alpha\left(\frac{\langle f|H_R|\alpha\rangle\langle\alpha|V_s|i\rangle}{E_i - E_\alpha \mp \hbar\omega_q} + \frac{\langle f|V_s|\alpha\rangle\langle\alpha|H_R|i\rangle}{E_i - E_\alpha - \hbar\Omega}\right), \tag{7}$$

where i, α, f indices denote the initial, intermediate and final states of an electron with quantum numbers k, n, l, H_R is the operator of electron-photon interaction, V_s is the operator of electron-phonon interaction.

Substituting expressions for wave functions (3) into expressions (7) for matrix elements of electron-photon interaction, we obtain:

$$\left\langle n'l'k_z'\left|H_R\right|nlk_z\right\rangle=\frac{e\Delta d\sin\left(k_zd\right)}{2\hbar}\left(\frac{2\pi\hbar n_0}{V\Omega\in}\right)^{1/2}\delta_{nn'}\delta_{ll'}\delta_{kk_z}, \quad (8)$$

where V is the volume of the crystal, the radiation field is polarized along the z-direction, ε – is the vector of polarization.

The electron distribution function for a nondegenerate electron gas has the form:

$$f_0\left(E_{nlk_z}\right)=\frac{2n_{1d}de^{\frac{\Delta}{k_BT}}\sinh\left(\frac{\hbar\omega_x}{2k_BT}\right)\sinh\left(\frac{\hbar\omega_y}{2k_BT}\right)}{M\left(\frac{1}{2},1,\frac{\Delta}{k_BT}\right)}\exp\left(-\frac{E_{nlk_z}}{k_BT}\right), \quad (9)$$

where n_{1D} is the number of electrons per unit length and $M(a, c, x)$ is the confluent hypergeometric function [31]

$$M(a,c;x)=\sum_{n=0}^{\infty}\frac{(a)_n x_n}{(c)_n n!}. \quad (10)$$

The matrix element of the electron-phonon interaction has the following form:

$$\left|\left\langle k_z'n'l'\left|V_s\right|k_znl\right\rangle\right|^2=C_j'^2J_{nn'}\left(x\right)J_{ll'}\left(y\right)I\left(q_z\right), \quad (11)$$

where V_s is the operator of the interaction energy of an electron with a phonon, C_j is a function that characterizes the interaction between electrons and phonons,

$$\begin{aligned}J_{nn'}\left(q_x\right)&=\int_{-\infty}^{\infty}e^{iq_xx}dx\Psi_n\left(x\right)\Psi_{n'}\left(x\right),\\ J_{ll'}\left(q_y\right)&=\int_{-\infty}^{\infty}e^{iq_yy}dy\Psi_l\left(y\right)\Psi_{l'}\left(y\right),\\ I\left(q_z\right)&=\int_0^d\xi_{k_z}\left(z\right)\xi_{k_k'}\left(z\right)e^{iq_zz}dz,\end{aligned} \quad (12)$$

For the interaction of an electron with polar optical phonons, we have:

$$\begin{aligned}\tilde{N}_{POL}^2&=2\pi e^2\hbar\omega_0\left\{\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right\},\\ F_{POL}&=\frac{N_0^{\pm}}{q^2},\ \ N_0=\left[\exp\left(\frac{\hbar\omega_0}{K_BT}\right)-1\right]^{-1},\end{aligned} \quad (13)$$

where $N_0^-=N_0,\ \ N_0^+=N_0+1.$

Here ε_∞ and ε_0 are the high-frequency and static dielectric constants of the semiconductor, respectively. The phonon energy is taken $\hbar\omega_q=\hbar\omega_0=\text{const}$

$$N_0=\left[\exp\left(\frac{\hbar\omega_0}{K_BT}\right)-1\right]^{-1},\ N_0^-=N_0,\ N_0^+=N_0+1, \quad (14)$$

where $N_0^-\left(N_0^+\right)$ describes the annihilation and creation of a phonon.

For the interaction of an electron with non-polar optical phonons

$$C_{np}^2=\frac{\hbar D}{2\rho\omega_0V},\ \ F_{np}\left(q\right)=N_0^{\pm}, \quad (15)$$

where D is the constant of the non-polar optical deformation potential.

Taking into account formulas (6) and (11) in (5) and summing over k_z', for the absorption coefficient we obtain:

$$\begin{aligned}\alpha=&\left(\frac{\pi e\Delta d}{\hbar^2}\right)^2\frac{1}{CV\Omega^3\in^{1/2}}\times\\ &\times\sum_{n'e'}\sum_{nek_z}\sum_{q}f_{nek_z}C_j^2\left|I_{nn'}\left(q_x\right)\right|^2\left|I_{ee'}\left(q_y\right)\right|^2\left(\sin\left(k_z+q_z\right)d-\sin k_zd\right)^2\times\\ &\times\left\{N_0\delta\left(\left(n'-n\right)\hbar\omega_x+\left(e'-e\right)\hbar\omega_y+\frac{\Delta}{2}\left(\cos k_zd-\cos\left(k_z+q_z\right)d-\hbar\Omega+\hbar\omega_q\right)\right)+\right.\\ &\left.+\left(N_0+1\right)\delta\left(\left(n'-n\right)\hbar\omega_x+\left(l'-l\right)\hbar\omega_y+\frac{\Delta}{2}\left(\cos k_zd-\cos\left(k_z+q_z\right)d\right)-\hbar\Omega-\hbar\omega_q\right)\right\}.\end{aligned} \quad (16)$$

In formula (16), passing from summation over k_z and q to integration over k_z and q using the expression:

$$\sum_q\frac{V}{\left(2\pi\right)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\pi/d}^{\pi/d}dq_xdq_ydq_z, \quad (17)$$

$$\sum_{k_z}\left(...\right)=\left(\frac{L_z}{2\pi}\right)\int_{-\pi/d}^{\pi/d}\left(...\right), \quad (18)$$

and using the properties of the Dirac δ-function:

$$\delta\left[f\left(x\right)\right]=\sum_i\frac{\delta\left(x-x_i\right)}{\left|f'\left(x_i\right)\right|}, \quad (19)$$

where x_i are simple zeros of the function $f(x)$,

$$\int_{-\pi/d}^{\pi/d}dq_zf\left(q_z\right)\delta\left(q_z-a\right)=f\left(a\right). \quad (20)$$

for the absorption coefficient for scattering by polar and non-polar optical phonons, we obtain:

$$\begin{aligned}\alpha_{pol}=&\frac{4\pi e^4\Delta d\omega_0L_z}{c\Omega^3\in^{1/2}\hbar^3}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right)\times\\ &\times\sum_{n'e'}\sum_{ne}\sum_{\pm}\int_{-\pi/d}^{\pi/d}dk_zf_{nek_z}\left(N_0+\frac{1}{2}\pm\frac{1}{2}\right)\frac{\sqrt{1-\frac{4}{\Delta^2}\theta_\pm^2}-\sin k_zd}{\sqrt{1-\frac{4}{\Delta^2}\theta_\pm^2}}\times\\ &\times\int_0^{\infty}\int_0^{\infty}\frac{\left|I_{nn'}\left(q_x\right)\right|^2\left|I_{ee'}\left(q_y\right)\right|^2}{\left(q_x^{\,2}+q_y^{\,2}\right)+a_\pm^{\,2}}dq_xdq_y,\end{aligned} \quad (21)$$

$$\alpha_{n.pol} = \frac{D^2 e^2 \Delta d}{\pi c \rho \omega_0 \Omega^3 L_x L_y \in^{1/2}} \times$$

$$\times \sum_{n'e'} \sum_{ne} \sum_{\pm} \int_{-\pi/d}^{\pi/d} dk_z f_{nek_z} \left(N_0 + \frac{1}{2} \pm \frac{1}{2} \right) \frac{\sqrt{1 - \frac{4}{\Delta^2}\Theta_{\pm}^2} - \sin k_z d}{\sqrt{1 - \frac{4}{\Delta^2}\Theta_{\pm}^2}} \times \quad (22)$$

$$\times \frac{1}{n' - n} \cdot \frac{1}{l' - l},$$

where

$$\Theta_{\pm}(k_z d) = (n' - n)\hbar\omega_x + (l' - l)\hbar\omega_y \pm$$

$$\pm \hbar\omega_0 + \hbar\Omega + \frac{\Delta}{2}\cos k_z d, \quad (23)$$

$$a_{\pm}^2 = \frac{1}{d^2}\left(k_z d - \cos^{-1}\left(\frac{2}{\Delta}\Theta_{\pm}(k_z d) \right) \right)^2,$$

When deriving formula (22), it was taken into account that $\int_0^{\infty} \frac{|I_{nn'}(u)|^2 du}{u} = \frac{1}{n' - n}$.

3. RESULTS

From expression (22) it is seen that the absorption coefficient diverges whenever the condition $1 - 4\Theta_i^2/\Delta^2 = 0$ is satisfied. In addition, from the fact that $1 - 4\Theta_i^2/\Delta^2 = 0$ are real and positive, we can obtain an energy range in which the absorption coefficient are allowed.

As follows from equations (21-22), peaks are observed at certain frequencies of the incident photon. The resonance behavior of the absorption coefficient for electron-phonon scattering appears for frequencies satisfying the relation

$$N\hbar\omega_x + P\hbar\omega_y \pm \hbar\omega_0 = \hbar\Omega, \quad (24)$$

here $N = n' - n = 1, 2, 3, ...$, and $P = l' - l = 1, 2, 3, ...$

From Eq. (24), in the course of scattering events, the electrons in the subband levels specified by the level index $n(l)$ can make transitions to one of the subband levels $n'(l')$ by absorbing and/or emitting a photon of energy $\hbar\Omega$ during the absorption of a LO phonon of energy $\hbar\omega_0$.

Eq. (24) is the basic equation for the absorption coefficient spectral lineshape, which enable us to analyze resonant effects in semiconductors.

In **Fig. 1** α_+ dependence on $\hbar\Omega$ for $\hbar\omega_x = 0.03$eV, and $\hbar\omega_y = 0.02$eV, 0.06eV, 0.08eV is given. **Fig. 2** is presented α_{+max} dependence on ω_y for $\Delta = 0.0024$eV.

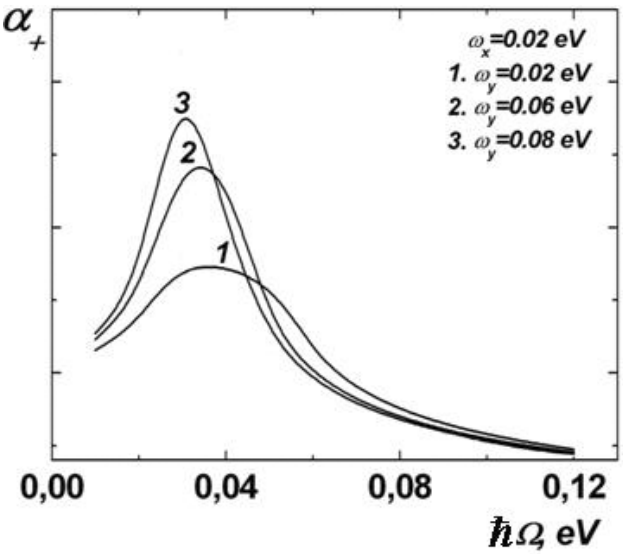

Fig. 1. *a_+ dependence on $\hbar\Omega$ at $\hbar\omega_x = 0.03eV$, and $\hbar\omega_y = 0.02$ eV, 0.06 eV, 0.08 eV.*

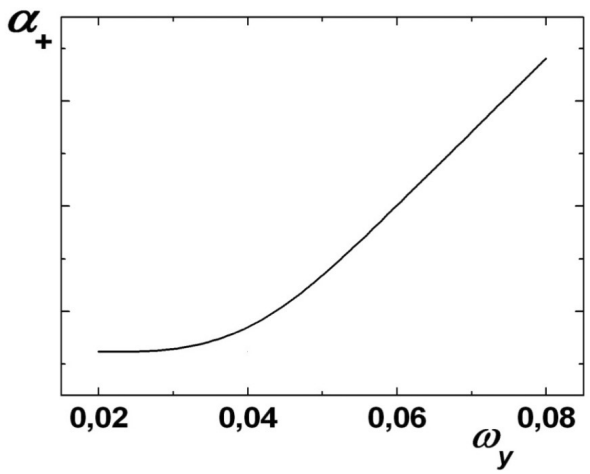

Fig. 2. *a_{+max} dependence on ω_y at $\Delta = 0.0024$ eV.*

As seen, with increasing ω_y the α_+ maximum grows. The ω_y values, affecting the change of α_{+max}, as well as the frequency at which it is achieved ($\hbar\Omega$), shift towards small frequencies.

4. CONCLUSION

In conclusion, we calculated free-carrier absorption of electromagnetic radiation with polar and non-polar optical phonons. It should be noted that the summations of Eq. (21) over the size subband levels contain three types of contributions: (i) $n' \neq n$, $l' = l$, (ii) $n' = n$, $l' \neq l$, and (iii) $n' \neq n$, $l' \neq l$. From the selection rules, we can expect three possible transitions in the absorption coefficient for electron-polar phonon scattering: (1) a transition due to the size subband levels for only the x-direction, (2) a transition due to the size subband levels for only the y-direction

and (3) a transition due to the size subband levels for both the *x*-direction and the *y*-direction.

From the Eq. (22) we can expect only one possible transitions in the, the absorption coefficient for electron-nonpolar phonon scattering: a transition due to the size subband levels for both the *x*-direction and the *y*-direction.

REFERENCES

1. Esaki L, Tsu R. Superlattice and negative differential conductivity in semiconductors. *IBM J. Res. Dev.,* 1970, 14:61-65.
2. Eaves L, Sheard FW, Toombs GAT. *Band Structure Engineering in Semiconductor Microstructures*. New York, Plenum, 1989, 400 p.
3. Mendez EE. Interfaces, Quantum Wells, and Superlattices. New York, Plenum, 1989, 403p.
4. Suris RA, Dmitriev IA. Bloch oscillations in quantum dot superlattices. *UFN*, 2003, 46(7):769.
5. Imran A, Jiang J, Eric D, Zahid MN, Yousaf M, Shah Z. Optical properties of InAs/GaAs quantum dot superlattice structures. *Results in Physics,* 2018, 9:297-302.
6. Hiura S, Itabashi K, Takeishi K, Takayama J, Kiba T, Murayama A. Quantum spin transport to semiconductor quantum dots through superlattice. *Applied Physics Letters,* 2019, 114:072406.
7. Eric D, Jiang J, Imran A, Zahid MN, Khan AA. Effects of quantum confinement on optical properties of InN/GaN quantum dots. *Conference: Applied Optics and Photonics China,* 2019, vol. 11336. Beijing, China.
8. Shu WM, Lei XL. Miniband transport in semiconductor superlattices in a quantized magnetic field. *Phys. Rev. B*, 1994, 50:17378.
9. Lee SC. Optically Detected Spin-Magnetophonon. *J. Korean Phys. Soc.,* 2007, 51:1973.
10. Lee SC. Electrophonon Resonance in Quantum-Dot Superlattices. *J. of Korean Phys. Soc*, 2008, 52:1081.
11. Kang DS, Lee SC, Kim SW. Magnetophonon Resonance by Quantum Size Effects in n-Ge-based and n-GaAs-based Quantum-dot Superlattices. *Journal of the Korean Physical Society,* 2010, 57:760.
12. Ermakov VA, Clemente da Silva Filho JM, Bonato LG, Vardhan Mogili NV, Montoro FE, Iikawa F, Nogueira AF, Cesar CL, Jiménez-Villar E, Chagas Marques F. Three-Dimensional Superlattice of PbS Quantum Dots in Flakes. *ACS Omega*, 2018, 3:2027.
13. Quang Bau N. Do Tuan Long Journal of Science: Impact of confined LO- phonons on the Hall effect in doped semiconductor superlattices. *Advanced Materials and Devices,* 2016, 1:209.
14. Hong-Jing X, Chuan-Yu Ch, Ben-Kun M. The bound polaron in a cylindrical quantum well wire with a finite confining potential. *J. Phys.: Condens. Matter*, 2000, 12:8623.
15. Hong-Jing X, Chuan-Yu Ch, Ben-Kun M. Bound polaron in a cylindrical quantum wire of a polar crystal. *Phys. Rev. B,* 2000, 61:4827.
16. Dumke WP. Quantum theory of free carrier absorption. *Phys. Rev.*, 1961, 124:1813.
17. Seeger K. *Semiconductor Physics: An Introduction, Springer Series in Solid State Sciences.* 9th ed. Springer-Verlag, Berlin Heidelberg, 2004, 538p.
18. Spector HN. Free-carrier absorption in quasi-two-dimensional semiconducting structures *Phys. Rev. B*, 1983, 28:971.
19. Adamska H, Spector HN. Free-carrier absorption from electrons in confined systems. *J. Appl. Phys.*, 1986, 59:619.
20. Wu CC, Lin CJ. Free-carrier absorption in n-type piezoelectric semiconductor films. *J. Phys.: Condens. Matter,* 1994, 6:10147.
21. Adamska H, Spector HN. Free carrier absorption in quantum well structures for polar optical phonon scattering. *J. Appl. Phys.,* 1984, 56:1123.
22. Kubakaddi SS, Mulimani BG. Free-carrier absorption in semiconducting quantum well wires for nonpolar optical phonon scattering. *J. Appl. Phys.*, 1988, 63:1799.
23. Kubakaddi SS, Mulimani BG. Free-carrier absorption in semiconducting quantum well wires. *J. Phys. C: Solid State. Phys.,* 1985, 18:6647.
24. Ibragimov GB. Intersubband optical absorption in parabolic quantum wires under a tilted magnetic field. *Phys. stat. sol. (b),* 2004, 241:1923.

25. Ibragimov GB. Free-carrier magnetoabsorption in quantum well wires. *J. Phys.: Condens. Matter*, 2003, 15:8949.
26. Ibragimov GB. Free-carrier magnetoabsorption in quantum well structures. *Ukr. J. Phys.*, 2003, 48:527.
27. Ibragimov GB, Huseyin D, Halil Y. Confined LO phonon limited free carrier absorption in quantum well wires. *Semiconductor Physics, Quantum Electronics & Optoelectronics*, 2006, 9:12.
28. Ibragimov GB, Ibaeva RZ. Theory of intraband absorption of electromagnetic radiation in cylindrical quantum wires. *Journal of Non-Oxide Glasses*, 2020, 12:4.
29. Shorokhov AV, Margulis VA. Intraband resonance scattering of electromagnetic radiation in anisotropic quantum dots. *Nanosystems: Physics, Chemistry, Mathematics*, 2010, 1:178.
30. Khoa DQ, Hieu NN, Bich TN, Phuong Le TT, Hoi BD, Tran PTL, Quach KQ, Nguyen CV, Phuc HV. Magneto-optical absorption in quantum dot via two-photon absorption process. *Optik*, 2018, 173:263.
31. George B. Arfken, Hans J. Weber. *Mathematical Methods for Physicists.* Academic Press, New York, 2001, 1182 p.