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Localization of the geological layer boundaries using the reverse time migration method

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Abstract: **The article is devoted to solving an important practical problem - determining the structure of the subsurface space of the geological environment based on surface seismic data. Thanks to the registration of seismic waves reflected from the boundaries of geological layers, it is possible to delineate hydrocarbon deposits, which makes it possible to effectively plan a field development scheme. Optimizing the production process makes it possible to make it profitable, including when developing unconventional hydrocarbon deposits. The paper discusses the technology of constructing a migration image using the reverse time migration method. In the general case, an analytical derivation of the calculation formulas was carried out. For the practically significant case of an acoustic environment, simplified calculation formulas are explicitly written out and implemented in the form of a software algorithm. The issue of improving the quality of the migration image without significantly increasing the computational complexity of the problem is discussed separately. The authors demonstrated the performance of this approach using the widely used Marmousi geological model.**

Keywords: **mathematical modeling, seismic survey, reverse-time migration**

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Contents

1. INTRODUCTION

Seismic exploration is the main method for detailed study of the geological medium structure. Most of the work is carried out using the reflected wave method in the modification of the common depth point

(CDP). The seismic cross-sections obtained are interpreted by geologists. Based on them, they construct geological models and assess the resource base of minerals.

CDP seismic cross-sections have several disadvantages: diffracted waves associated with faults and small objects are displayed as diffraction hyperbolas, inclined boundaries are not displayed in their true position, synclinal structures are displayed as loops. In cases where there are velocity anomalies in the geological model, the underlying reflecting horizons are less well focused, and their relief is distorted. The seismic data migration procedure is aimed at minimizing these shortcomings and constructing a reliable seismic image.

One of the advanced migration methods is the reverse-time migration method (RTM). The foundations of this method were laid in fundamental works [1-3]. The method was subsequently developed and generalized to more complex media models using various parameterizations, integral formulas, and post-processing techniques by various researchers [4-8].

This article presents successful experience in using the RTM method to construct a migration image of a complex geological medium in a two-dimensional setting. In the general case, a detailed derivation of the fundamental formulas is presented, as well as a method for reducing them to the special case of an acoustic model. The results of the work contain a description of the computer experiments performed and the constructed migration images.

2. MATERIALS AND METHODS

2.1. Basic idea of RTM

Let the data $D = \vec{D}(t)$ be recorded as a result of seismic exploration: a time-dependent signal at the receivers at points $\vec{x}_i$, Having denoted the wave field (generally speaking, unknown) in the entire geological massif by u, we can write that $D = Ru$, where R is the restriction operator. Moreover, $u \in U$, where U is the space of admissible functions of coordinates and time. Let the physical and mathematical model of the medium be described by the differential equation

$$F[m]u = f, \tag{1}$$

where f is the known right-hand side, $m \in M$ are the parameters of the medium from a certain space of admissible parameters M, $F[m]$ is a differential operator acting on functions from the space U. The differential problem based on equation (1) is well-posed when specifying suitable initial and boundary conditions. We can assume that these conditions are included in the definition of the space U.

Then we can write the following residual function:

$$\begin{aligned} \Psi(m) &= \frac{1}{2}\left\|(Ru)(t) - \vec{D}(t)\right\|^2_{R^n(t)} = \\ &= \frac{1}{2}\int_0^T \left\|\left(\overrightarrow{Ru}\right)(t) - \vec{D}(t)\right\|^2_{R^n} dt = \\ &= \frac{1}{2}\sum_{i=0}^{n}\int_0^T \left(\left(\overrightarrow{Ru}\right)(t) - \vec{D}(t)\right)_i^2 dt. \end{aligned} \tag{2}$$

In this formula, the wave field u is the only solution to equation (1), therefore, $u = u[m]$ and $\Psi = \Psi[m]$.

Let us write down the definitions of several concepts used later in the article.

The Frechet derivative of a functional A acting from a normed space E into R^1 ($A{:}E \to R^1$) at the point $x_0 \in E$ is a linear functional

$$\left.\frac{\partial A}{\partial x}\right|_{x_0} : E \to R^1 \left(\left.\frac{\partial A}{\partial x}\right|_{x_0} (\tilde{x}) = y,\ \tilde{x} \in E, y \in R \right)$$

such that

$$\begin{gathered} A(x) - A(x_0) = \left.\frac{\partial A}{\partial x}\right|_{x_0} (x - x_0) + \alpha(x)\left\|x - x_0\right\|, \\ \lim_{x \to x_0} \alpha(x) = 0. \end{gathered} \tag{3}$$

Also, we will use the definition of the adjoint operator A^* to the operator A for spaces with the scalar product $\langle\cdot,\cdot\rangle$:

$$\langle Ax, y\rangle = \langle x, A^* y\rangle. \tag{4}$$

The distribution of medium parameters $m_* = \operatorname{argmin}\Psi[m]$ can be considered a solution to the inverse problem of finding medium parameters m using the known equation (1), the known right-hand side f and the known recorded data D. To find m_*, we can use various optimization methods, for example, gradient methods using derivatives in the Frechet sense (3) are often used. It was noted that if the initial approximate distribution of parameters is a smoothed version of the true one (this is often the case due to the estimate of the average value of the velocity), then the desired increment of medium parameters $m_* - m_{\text{initial}}$ corresponds to the migration image of the medium. Based on this observation, the RTM method was proposed, which consists of finding the gradient $\partial\Psi/\partial m$ and then processing it to obtain a migration image.

2.2. RTM in the operator form

In this subsection, we consider a method for finding the derivative $\partial\Psi/\partial m$. Direct differentiation of the residual functional (2) with respect to the parameters of the medium m will require finding the derivatives $\partial(Ru)/\partial m$, which in the discrete case corresponds to the Jacobian matrix of the first derivatives $\left\|\frac{\partial(\mathrm{Ru})_i}{\partial m_j}\right\|$, the calculation of which is extremely expensive. Therefore, to calculate the derivative $\partial\Psi/\partial m$, the technique of solving the adjoint equation is used, which does not require explicit calculation of $\partial(Ru)/\partial$m. The corresponding formulas can be derived in several ways [5]. We chose the Lagrange multiplier method for the continuous optimization problem with constraints, presented below.

To optimize the residual functional $\frac{1}{2}\left\|(\mathrm{Ru})(\mathrm{t}) - \vec{\mathrm{D}}(\mathrm{t})\right\|^2_{\mathrm{R}^n(\mathrm{t})}$ under the additional constraint $F[m]u = f$, consider the Lagrange functional

$$L\left[m,\tilde{u},\tilde{\lambda}\right] = \frac{1}{2}\left\|(R\tilde{u})(t) - \vec{D}(t)\right\|^2_{R^n(t)} + \\ + \left\langle \tilde{\lambda}, F[m]\tilde{u} - f\right\rangle \tag{5}$$

with arbitrary $m \in \mathcal{M}$, $\tilde{u} \in U$, $\tilde{\lambda} \in U^*$, where the space U^* defines functions of the same smoothness as in the space U, with the same boundary conditions, but with conditions at the final time instead of initial conditions. From optimization theory it is known that the minimum of the functional is achieved at the point where the derivatives of the Lagrangian are equal to zero [9]. First, we write down the Frechet differentials of the functional L with respect to $\tilde{\mathrm{u}}$ and $\tilde{\lambda}$, using the definition of the adjoint operator (4):

$$\left.\frac{\partial L}{\partial\tilde{\lambda}}\right|_{m,\,\tilde{\mathrm{u}},\tilde{\lambda}}\left(\tilde{\lambda}\right) = 0 + \left\langle\tilde{\lambda}, F[m]u - f\right\rangle, \tag{6}$$

$$\left.\frac{\partial L}{\partial\tilde{u}}\right|_{m,\tilde{u},\tilde{\lambda}}(\tilde{u}) = \left\langle R\tilde{u} + D, R\hat{u}\right\rangle + \left\langle F[m]^*\lambda, \hat{u}\right\rangle = \\ = \left\langle R^*(R\tilde{u}) + D\right\rangle + F[m^*\lambda, \hat{u}]. \tag{7}$$

Equating the derivatives above to zero, we get a direct problem with respect to u and an adjoint problem with respect to λ:

$$\frac{\partial L}{\partial\tilde{\lambda}} = 0 \Leftrightarrow F[m]u - f = 0, \tag{8}$$

$$\frac{\partial L}{\partial\tilde{u}} = 0 \Leftrightarrow R^*\left(R\tilde{u} + D\right) + F[m]^*\lambda = 0. \tag{9}$$

Equation (8) is a direct problem that can be solved by a numerical method. Many different approaches have been developed for seismic problems, for example [10-12]. Equation (9) is an adjoint problem with respect to λ, which can also be solved numerically. In the next section it will be shown that for the acoustic model the operator F and its conjugate F^* are such that to solve the conjugate problem it is possible to use the same solver as for the direct problem.

Now consider the Frechet differential L with respect to the parameters of the medium m under the assumption that the terms of the operator F are linear with respect to m ($\tilde{u}$, $\tilde{\lambda}$, do not depend on m):

$$\left.\frac{\partial L}{\partial m}\right|_{m,\tilde{u},\tilde{\lambda}}(\hat{m}) = \frac{\partial}{\partial m}\left.\left\langle \tilde{\lambda}, F[m]u \right\rangle\right|_{m,\tilde{u},\tilde{\lambda}}(\hat{m}) = = \left\langle \tilde{\lambda}, F[m+\hat{m}] - F[m](u) \right\rangle_U = = \left\langle I(\vec{x}), \hat{m} \right\rangle_M . \quad (10)$$

Thus, the derivative of the functional with respect to the parameters m is an integral operator on the space $\mathcal{M}$ with kernel I:

$$\left\langle I(\vec{x}), \hat{m} \right\rangle_M = \int_\Omega \int_0^T \lambda \cdot \left(F[m+\hat{m}] - F[m] \right) u \, dt d\vec{x}. \quad (11)$$

The kernel $I = I(\vec{x})$ defines the integral operator defining the Frechet derivative, so I is usually called the gradient $\partial\Psi/\partial m$. This kernel is the desired derivative.

So, the algorithm for calculating $\partial\Psi/\partial m$ is as follows:

1. Solve direct problem (8) – find u.
2. Solve the conjugate problem (9) – find λ.
3. Calculate the gradient $I = I(\vec{x})$ using the integral formula (11) based on the calculated u and λ.

2.3. Acoustic model

In this work, acoustic equations were used to describe the seismic wave propagation in the geological media. This model correctly describes the propagation of P-waves, their multiple passage and reflection at the interfaces between layers. Mathematically, the model can be written as a scalar wave equation

$$u_{tt} = c^2 \Delta u + f. \quad (12)$$

In this equation, $u = u(\vec{x}, t)$ is the deviation of pressure in the medium from equilibrium state; $c = c(\vec{x}) > 0$ is the propagation velocity of longitudinal waves, Δ is the Laplace operator. Thus, the operator F can be written as

$$F[m] = \frac{\partial^2}{\partial t^2} - c^2 \sum_{i=1}^{d} \frac{\partial^2}{\partial x_i^2}. \quad (13)$$

The problem is considered with zero initial conditions, since before the start of seismic exploration the medium was at rest. On the only physical boundary – the free surface – a zero boundary condition is set. Thus, we can define the spaces $\mathcal{U}, \mathcal{U}^*$:

$$\mathcal{U} = \left\{ f \in C^2 : f\big|_{t=0} = 0, f_{t|_{t=n}} = 0, f\big|_{\vec{x}\in\partial\Omega} = 0 \right\}, \quad (14)$$

$$\mathcal{U}^* = \left\{ f \in C^2 : f\big|_{t=T} = 0, f_{t|_{t=T}} = 0, f\big|_{\vec{x}\in\partial\Omega} = 0 \right\}, \quad (15)$$

From this we can obtain the self-adjoint condition $F = F^*$:

$$\forall u \in \mathcal{U} \circ \forall \lambda \in \mathcal{U}^* \to \left\langle F[m]u, \lambda \right\rangle_{L_2} = = \left\langle u, F[m]\lambda \right\rangle_{L_2} . \quad (16)$$

The restriction operator R represents taking values at points $\vec{x}_i$, , and its conjugate R^* is an operator that reduces the time signal at a set of points to the sum of delta functions over a coordinate at that set of points. Thus, the right side of the conjugate equation (9) $-R^*(Ru - D)$ is the sum of the point sources located at the data receiver locations D.

To calculate I using formula (11), it remains to express in explicit form the expression $\left(F[m+\hat{m}] - F[m] \right) u$. In this case, $\hat{m}$ will not be included in the final formula, being transferred to another factor of the scalar product according to formula (10). Considering the parameter m to be the square of the velocity, from (13) we obtain

$$\left(F[m+\hat{m}] - F[m] \right) u = \hat{c}^2 \Delta u, \quad (17)$$

$$I(\vec{x}) = \int_0^T \lambda \cdot \Delta u \, dt. \quad (18)$$

However, it should be noted that for other parameterizations of the model under consideration the result will be different. For example, considering the slowness $\beta = 1/c^2$ as a medium parameter, we can write $F_1 = \beta u_{tt} - \Delta u$, from which it follows $\left(F_1\left[m+\hat{m}\right]-F_1\left[m\right]\right)u = \beta u_{tt}$ and formula

$$I_1\left(\vec{x}\right)=\int_0^T \lambda\cdot u_{tt}\,dt. \quad (19)$$

The last formula, taking into account the zero initial conditions on u and the final conditions on λ, according to the rule of integration by parts, can easily be rewritten in the form

$$I_1\left(\vec{x}\right)=\int_0^T \lambda\cdot u_{tt}\,dt=-\int_0^T \lambda_t\cdot u_t\,dt=\int_0^T \lambda_{tt}\cdot u\,dt. \quad (20)$$

In this work, we used the simplified formula widely used in practice

$$I\left(\vec{x}\right)=\int_0^T \lambda\cdot u\,dt. \quad (21)$$

2.4. Gradient post-processing for the migration image calculation

For some real problems, the resulting gradient $\partial\Psi/\partial m$ is quite far from the desired migration image. There are several approaches to improve the result. For example, Least-Squares RTM performs a gradient descent procedure to achieve $dm \approx m_* - m_{\text{initial}}$, but this method is computationally expensive. In this work, we used several simpler techniques that allow us to significantly improve the final migration image in a short computational time.

The first such method is the seismic signal attenuation compensation. It is known that the amplitude of waves decreases due to geometric divergence even in a model of the geological media without dissipation. Consequently, the amplitude of the gradient obtained using integral (21) decays with depth, because the sources of both the direct and conjugate problems are located near the surface. To compensate for this effect, it was proposed to replace the integral formula (21) with the following:

$$\text{Image}\left(\vec{\text{x}}\right)=\frac{\sum_{\text{s:sources}}\int_0^T u_s\lambda_s\,dt}{\sum_{\text{s:sources}}\int_0^T u_s^2\,dt+\delta}. \quad (22)$$

The non-negative denominator is separated from zero using the small constant δ. We also explicitly added summation over all sources to this formula: for each source there is its own set of data D, and its own calculation of the direct and conjugate problems is carried out.

A standard Laplace filter is used to suppress low-frequency noise components in the resulting image.

Since the amplitudes of $I(\vec{\text{x}})$ in the immediate vicinity of sources and receivers are extremely large, but are not of significant interest, when displaying I for visual analysis, values near the surface (down to depths of the order of 100 m) are set to zero.

3. RESULTS

The computational algorithm described in the work was implemented by the authors as a computer program in Python. To numerically solve the governing equations of the direct and adjoint problems, the open-source solver SpecFem2D, based on the spectral element method, was used [13]. Fourth order elements were used as a basis for expanding the solution. Near the lateral and lower boundaries, absorbing PML layers were additionally used [14]. During the modeling process, the calculated pressure fields were saved to the hard disk, after which they were used to calculate the integral using formula (22).

A two-dimensional formulation of the problem with the widely used test geological model Marmousi [15] in a monoparametric acoustic formulation (13) was considered. **Fig. 1** shows the *P*-wave velocity distribution in the true media model. To build an initial guess model, this distribution was smoothed using the gaussian_filter function of the

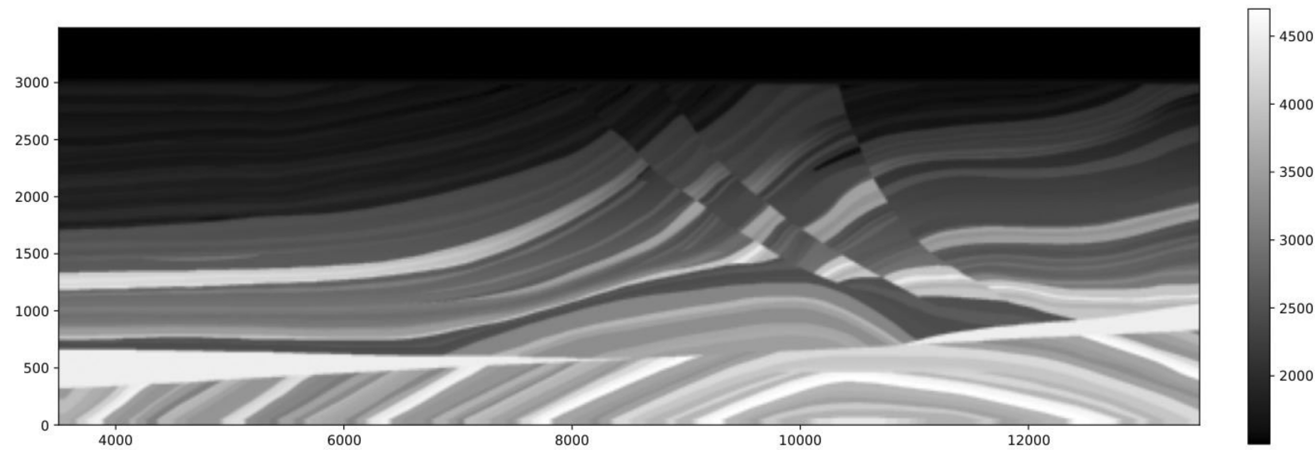

Fig. 1. *Spatial distribution of the P-wave velocity in the Marmousi model.*

SciPy.ndimage library. For clarity, the difference between the initial guess and the true distribution $m_* - m_{\text{initial}}$ is presented in **Fig. 2**.

When carrying out computer calculations, a square computational grid with a step of 20 m was used, covering a geological model with dimensions of 10×3.5 km. The total time of the computer experiment was 3.5 *s*, the time step was chosen equal to $3.5 \cdot 10^{-4}$ s. The source function was a Ricker pulse with a peak frequency of 25 Hz. On the day surface, data were recorded by 491 receivers, uniformly located with a step of 20 m at a depth of 10 m. To construct a migration image, 61 sources were used with a step of 150 m at a depth of 10 m. **Fig. 3** shows the result of the RTM method.

4. CONCLUSIONS

The work considers the seismic exploration inverse problem – determination of the boundary positions between the geological layers. The derivation of a calculation algorithm for the reverse time migration of seismic data in the general case is presented. Simplified calculation formulas for the acoustic model of the geological media were obtained. The computational algorithm was implemented as a computer program. It was successfully used to solve the migration problem for the Marmousi test model. Analysis of the resulting migration image confirms the possibility of localizing reflective horizons and suppressing noise.

As a further continuation of the research, the generalization of the considered approach

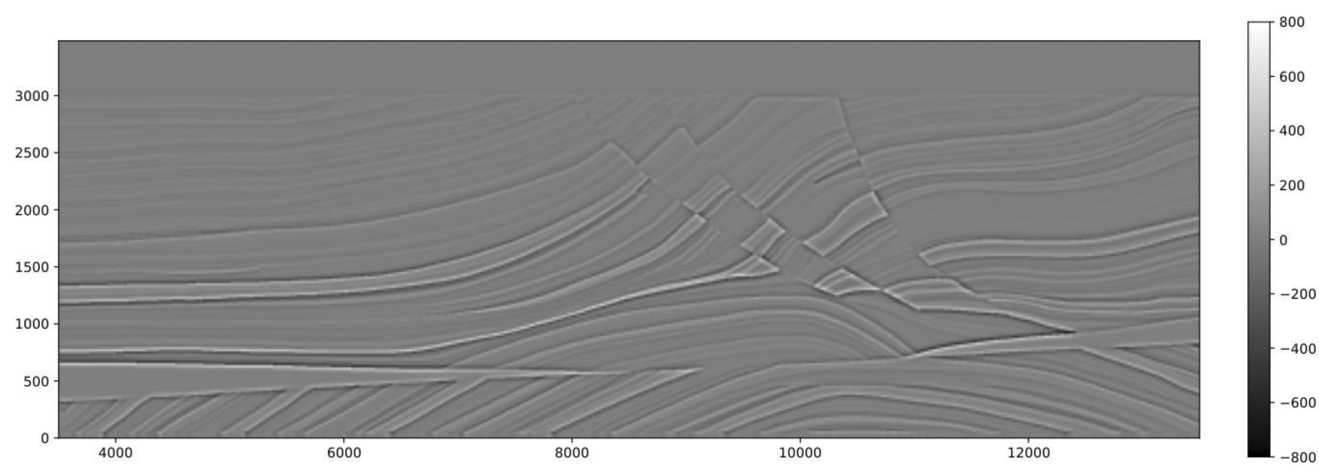

Fig. 2. *The difference between true model and used initial guess model.*

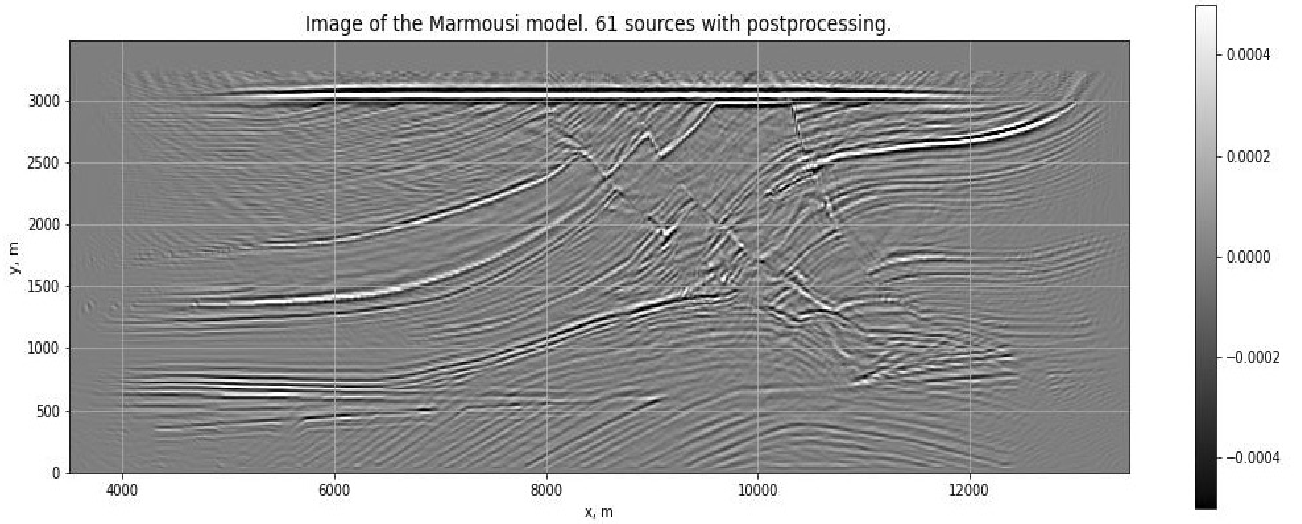

Fig. 3. *Obtained migration image.*

to more complex models of geological media, for example, isotropic and anisotropic linear elastic models is prominent.

REFERENCES

1. Lailly P. The Seismic Inverse Problem as a Sequence of before Stack Migrations. In: Bednar JB, Robinson E, Weglein A. (eds.). *Conference on Inverse Scattering – Theory and Application*, SIAM, Philadelphia, 1983, pp. 206-220.
2. Tarantola A. Inversion of seismic reflection data in the acoustic approximation. *Geophysics*, 1984, 49:1259-1266.
3. Baysal E, Kosloff DD, Sherwood JWC. Reverse time migration. *Geophysics*, 1983, 48:1514-1524.
4. Plessix RE, Mulder WA. Frequency-domain finite-difference amplitude-preserving migration. *Geophys. J. Int.*, 2004, 157:975-987.
5. Plessix RE. A review of the adjoint-state method for computing the gradient of a functional with geophysical applications. *Geophys. J. Int.*, 2006, 167(2):495-502.
6. Yang J, Liu Y, Dong L. Least-squares reverse time migration in the presence of density variations. *Geophysics*, 2016, 81(6):497-509.
7. Sun M, Dong L, Yang J, Huang C, Liu Y. Elastic least-squares reverse-time migration with density variations. *Geophysics*, 2018, 83(6):1-62.
8. Malovichko MS, Shevchenko AV, Golubev VI. On imaging conditions for elastic reverse-time migration. *Conference Quasilinear Equations, Inverse Problems and Their Applications (QIPA)*, 2022.
9. Ciarlet PG. *Introduction to Numerical Linear Algebra and Optimization*. Cambridge University Press, New York, 1989.
10. Virieux J, Calandra H, Plessix RE. A review of the spectral, pseudo-spectral, finite-difference and finite-element modelling techniques for geophysical imaging. *Geophysical Prospect*ing, 2011, 59(5):794-813.
11. Golubev V, Shevchenko AV, Petrov I. Raising convergence order of grid-characteristic schemes for 2D linear elasticity problems using operator splitting. *Computer Research and Modeling*, 2022, 14(4):899-910.
12. Golubev V, Shevchenko A, Khokhlov N, Petrov I, Malovichko M. Compact Grid-Characteristic Scheme for the Acoustic System with the Piece-Wise Constant Coefficients. *International Journal of Applied Mechanics*, 2022, 14(02). DOI: 10.1142/S1758825122500028.
13. Peter D, Komatitsch D, Luo Y, Martin R et al. Forward and Adjoint Simulations

of Seismic Wave Propagation on Fully Unstructured Hexahedral Meshes. *Geophys. J. Int.*, 2011, 186(2):721-39. DOI: 10.1111/J.1365-246X.2011.05044.X.

14. Zhinan X, Komatitsch D, Martin R, et al. Improved Forward Wave Propagation and Adjoint-Based Sensitivity Kernel Calculations Using a Numerically Stable Finite-Element PML. *Geophys. J. Int.*, 2014, 198(3):1714-1747.
15. Brougois A, Bourget M, Lailly P, Poulet M, Ricarte P, Versteeg R. Marmousi, model and data. *Conference: EAGE Workshop - Practical Aspects of Seismic Data Inversion,* 1990. DOI: 10.3997/2214-4609.201411190.