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3D image formation of the earth surface relief in the aperture synthesis mode when rotating the receiving antenna phase center and the transceiver module diversion

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Abstract: **The features of 3D radar image formation during the synthesis of an artificial aperture due to rotation of the phase center of the receiving antenna and stationary diversed transceiver module have been considered during the interferometric interpretation of incoming data. The main mathematical relations, associated with determination of the relief height are given, the algorithm for processing the trajectory signal based on the interferometric approach, as well as the results of evaluating the efficiency of the proposed algorithm, obtained by computer simulation, are presented.**

Keywords: **radar system, antenna aperture synthesis, antenna phase center, interferometer base, interferometric processing, radar image, point target, antenna phase center rotation, transceiving module, receiving module, distributed radar system**

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Contents

1. INTRODUCTION

The development of theoretical foundations and technical capabilities for the creation of synthetic aperture radar systems (SAR) makes it possible to receive radar images (RI) of observed objects with a resolution of the order of meters or less [1-4]. At the same time when solving a number of practical problems airborne radars require great information capabilities, which can only be provided by obtaining 3D RIs. In particular, obtaining 3D RIs of the observed surface makes it possible to reduce errors in determining coordinates of the observed ground objects due to their height, and consequently, to increase the accuracy of determining the location of the radar carrier relative to ground radar-contrast landmarks when using image data. Besides, the formation of 3D RIs allows to obtain information about the relief of the observed area in any weather conditions and time of day, which allows the radar carrier to solve the problems of following the terrain in the conditions of poor optical visibility. The issues of the formation of 3D RIs were previously considered in the scientific and technical literature [5-6]. So, the formation of 3D RIs during the translational movement of the carrier (the antenna phase center (APC) of the radar) is considered in [5], and a possibility of forming 3D RIs during the rotation of the APC of the radar transceiving module is shown in [6].

The purpose of the article is to consider the features of forming 3D RIs in a distributed radar system, consisting of a receiving module (Rx), the antenna phase center of which rotates around a circle, and a stationary receiving-transmitting (Rx-Tx) module.

2. PROBLEM STATEMENT

Let us assume that there is a distributed radar in which the phase centers of the Rx and Rx-Tx modules are separated in space. The antenna phase center of Rx-Tx radar module is located in point A (**Fig. 1**) and it has the following

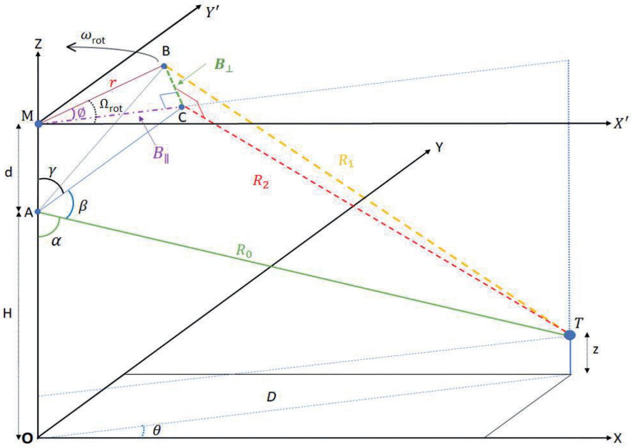


Fig. 1. *The geometry of location of the APC and the target in 3D space.*

coordinates: $x_{pc} = 0$, $y_{pc} = 0$, $z_{pc} = H$. But the APC of the receiving module is located in point B, it rotates strictly around the circle with a radius r with a constant angular velocity ω_{rot} in a plane (M, X', Y') relative to the center of rotation (p. M). The plane of rotation (M, X', Y') is parallel to the horizontal plane (O, X, Y). As a result the APC coordinates of the Rx module can be represented as

$$x_{rrm}(t) = r\cos(\omega_r t + \varphi_0), \quad (1)$$

$$y_{rrm}(t) = r\sin(\omega_r t + \varphi_0), \quad (2)$$

$$z_{rrm} = H + d, \quad (3)$$

where φ_0 – the initial phase of APC rotation of the receiving antenna, $\Omega_r(t) = \omega_r t + \varphi_0$ – a current angle of APC rotation relative to axis X', d – a distance between the APC of the receiving-transmitting module and the center of the APC rotation of the receiving module in the vertical plane of the module (Fig. 1).

The study of the probing signal is carried out by the Tx module (p. A) and the reception is carried out by both Rx-Tx module (p. A) and Rx module (p. B). The processing of the trajectory signal is carried out jointly in p. A and B, realizing an interferometric processing system. Under these conditions the interferometer base $\left\|\overline{AB}\right\|$, which we denote as I, does not change with the angle change $\Omega_r(t)$ and can be presented as

$$I(t) = \sqrt{\left(x_{pc}(t) - x_{rrm}(t)\right)^2 + \left(y_{pc}(t) - y_{rrm}(t)\right) + d^2}, \quad (4)$$

$$I = \sqrt{\left(r\cos(\Omega_r(t))\right)^2 + \left(r\sin(\Omega_r(t))\right)^2 + d^2} = \sqrt{r^2 + d^2}. \quad (5)$$

It is known [6], that with interferometric processing of signals in the process of aperture synthesis due to APC rotation, it is possible to form 3D RIs. Based on the described situation we will consider the process of forming 3D RIs during the rotation of APC of the receiving module and separated in space but stationary located APC of the Rx-Tx module. First, let's consider the basic mathematical relationships that determine the possibility of indicating the height of the observed object using the example of a point target (Fig. 1).

3. DETERMINING THE OBJECT HEIGHT BY RADAR OBSERVATION

The registration will be done relative to the point where APC of Rx-Tx module is located (p. A). Then the height of the target Z_t can be presented as

$$Z_t = H - R_0 cos(\alpha), \tag{6}$$

where H – the distance between the horizontal plane OXY and the point of APC of Rx-Tx module location (height); R_0 – distance from APC of Rx-Tx module to the point target (point T); α – antenna look angle.

Consider the MOT plane in Fig. 1, which includes the OZ axis and the target location point (p. T). Taking into account this consideration, the expression (6) can be written as

$$Z_t = H + R_0 cos(\beta + \gamma), \tag{7}$$

where β – the angle between R_0 and vector $\overrightarrow{AC}$, which determines the point of APC of Rx module location relative to R_0 direction, and γ – the angle between $\overrightarrow{AC}$ vector and OZ axis. In the general case, p. C determines the position of APC Rx module projection on the MOT plane. Both angles $\beta(t)$, $\gamma(t)$ are functions of time, since the APC location of Rx module changes in time and accordingly its projection on the (MOT) plane changes. But at the same time, the sum of these angles does not depend on time and, accordingly, angle α – antenna look angle – does not change when observing a single point target (PT).

To define the PT height Z_t, it is necessary to know the following information: the value of the signal phase at the output of interferometer $\psi_z(t)$, formed by Rx-Tx and Rx modules; the slant distance to the target R_0; the radius rotation value r of APC Rx module; distance d between the APC of Rx-Tx module and antenna rotation plane of Rx module; the antenna look angle θ in the azimuth plane; the angular position Ω of the APC Rx relative to the X axis.

Based on geometry in Fig. 1, the phase $\psi_z(t)$ of the signal at the output of interferometer can be presented as:

$$\psi_z(t) = \frac{2\pi}{\lambda}\left(R_1(\mathrm{t}) - R_0\right), \tag{8}$$

where $R_1(t)$ – distance from the APC of Rx module to PT, λ – probing oscillation wavelength, R_0 – distance from PT to APC of Rx-Tx antenna.

Basing on ΔTAC (see Fig. 1), one can write down:

$$\left(R_2(t)\right)^2 = R_0{}^2 + \left(\|AC\|(t)\right)^2 - 2R_0\|AC\|(t)cos\left(\beta(t)\right). \tag{9}$$

From equation (9), it follows that angle β can be presented as

$$\beta(t) = arccos\left(\frac{R_0{}^2 + \|AC\|^2 - R_2(t)^2}{2R_0\|AC\|(t)}\right). \tag{10}$$

Then from the right triangle ΔBCT, we get the following:

$$\begin{gathered} R_2(t)^2 = R_1(t)^2 - B_\perp(t)^2 = \\ = \left(\frac{\lambda\psi_z(t)}{2\pi} + R_0\right)^2 - B_\perp(t)^2, \end{gathered} \tag{11}$$

where $B_\perp = \left\|\overrightarrow{CB}\right\|$ – distance from the APC of Rx module to MOT' plane. As $\|AC\|^2 = d^2 + B_\|^2$, $I^2 = B_\perp(t)^2 + \|AC\|(t)^2$, we can write:

$$\beta(t) = arccos\left(\frac{I^2 - \left(\left(\frac{\lambda\psi_z(t)}{2\pi}\right)^2 + \frac{\lambda\psi_z(t)}{\pi}R_0\right)}{2R_0\sqrt{d^2 + B_\|(t)^2}}\right), \tag{12}$$

where $B_\| = \left\|\overrightarrow{MC}\right\|$ – projection of APC of Rx module location point on MOT plane equal to:

$$B_\|(t) = r\cos(\phi(t)), \tag{13}$$

$\phi(t)$ – angle between the directions $\overrightarrow{MB}$ and $\overrightarrow{MC}$, which can be presented as:

$$\phi(\mathrm{t}) = \omega_r t - \theta. \tag{14}$$

Then, taking into account expressions (14) and (13), we can write

$$B_\| = r\cos(\omega_r t - \theta). \tag{15}$$

From expression (15) it follows that the value of angle β can be defined as

$$\beta(t) = arccos\left(\frac{I^2 - \left(\left(\frac{\lambda\psi_z(t)}{2\pi}\right)^2 + \frac{\lambda\psi_z(t)}{\pi}R_0\right)}{2R_0\sqrt{d^2 + \left(r\,cos\left(\omega_r t - \theta\right)\right)^2}}\right). \tag{16}$$

In its turn it follows from Fig. 1 (see ΔMAC) that angle γ is equal to the following

$$\gamma(t) = arctan\left(\frac{B_{||}(t)}{d}\right) = arctan\left(\frac{r\cos(\omega_r t - \theta)}{d}\right). \qquad (17)$$

Basing on (16) and (17), the expression (7), determining the point target height Z_t, can be presented as:

$$Z_t = H + R_0 \cos\left(arccos\left(\frac{I^2 - \left(\left(\frac{\lambda\psi_z(t)}{2\pi}\right)^2 + \frac{\lambda\psi_z(t)}{\pi}R_0\right)}{2R_0\sqrt{d^2 + \left(r\cos(\omega_r t - \theta)\right)^2}}\right) + arctan\left(\frac{r\cos(\omega_r t - \theta)}{d}\right)\right). \qquad (18)$$

The expression (18) makes it possible to determine the PT height at its various positions, defined by angle *θ*.

Fig. 2 shows the results of simulating the nature of the change in angles α, *β* and *γ*. The simulation was conducted under the following conditions: $d = 3$ m, $r = 8$ m, $\lambda = 3$ cm, $D = 10$ km, $H = 500$ m, $\omega_r = 10\pi$ rad/s, $\varphi_0 = 0$, $Z_t = 10$ m, $\theta = 0°$ (Fig. 2*a*); θ = 45°(Fig. 2*b*).

The presented results show that angles *β* and *γ* change with time when the PC of a real Rx antenna rotates. At the same time the antenna look angle of the point target α, equal to $[\pi - (\beta + \gamma)]$, remains unchanged in time. The results also show, that the nature of the change in angles *β*, *γ* does not depend on the location of a PT and *d*, *r*, *z*, *H* and R_0 parameters.

4. CONDITIONS FOR UNAMBIGUOUS DETERMINATION OF THE OBSERVATION OBJECT HEIGHT

When using interferometric processing to determine the height of the relief an ambiguity problem arises, which is due to the fact that the phase of the output signal of the phase detector, which compares the phases of the received signals, varies in the range from $-\pi$ to π: $\psi(t) \in [-\pi, \pi]$. This fact must be taken into account when determining the target height, basing on the phase change of the interferometer output signal.

Let us determine the conditions under which the unambiguous measurements of the PT height will be ensured. To do this, let's consider how the phase $\psi(t)$ changes at the maximum given value of the height $\psi_h(t)$ and at the height corresponding to the zero level $\psi_0(t)$, and find their difference.

$$\psi_h(t) - \psi_0(t) = \frac{2\pi}{\lambda}\left(\Delta R_h(t) - \Delta R_0(t)\right), \qquad (19)$$

where $\Delta R_0(t) = R_{1,0} - R_{2,0}(t)$; $\Delta R_h(t) = R_{1,h_{uniq}} - R_{2,h_{uniq}}(t)$; $R_{1,0}$ – the distance from the PC of the receiving-transmitting antenna to PT_0; PT_0 is a point target, the height of which is considered to be zero;

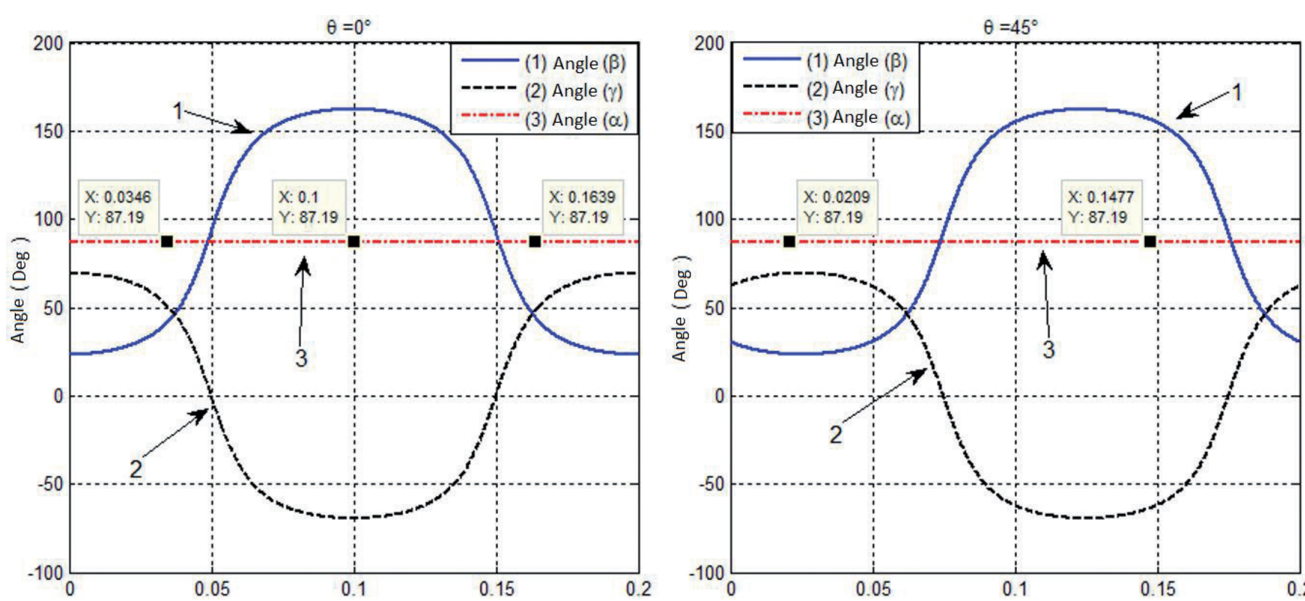


Fig. 2. *The nature of the change in angles β (curve 1), γ (curve 2) and a (curve 3) when rotating APC Rx module.*

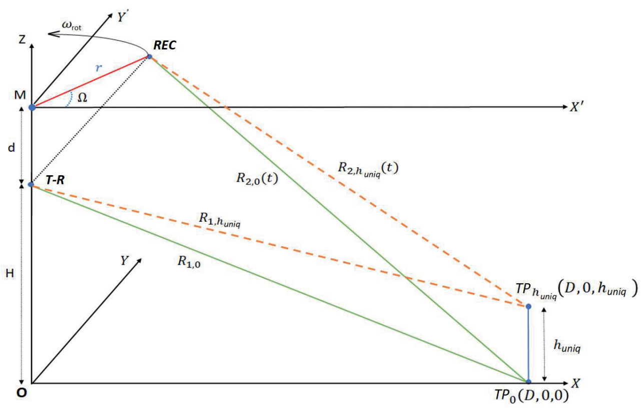


Fig. 3. *Geometric positions of APC and PT with zero height and the height corresponding to the maximum value within the unambiguous measurement.*

$R_{2,0}$ – the distance from the PC of the receiving antenna to PT_0; $R_{1,h_{uniq}}$ – the distance from the PC of the receiving-transmitting antenna to $PT_{h_{uniq}}$, $PT_{h_{uniq}}$ – a point target, the height of which is set within the range of unambiguous measurement; $R_{2,h_{uniq}}$ – the distance from the PC of the receiving antenna to $PC_{h_{unit}}$ (see **Fig. 3**). The expression (19) can be rewritten as follows

$$\Delta\psi_h(t)-\Delta\psi_0(t)= \\ =\frac{2\pi}{\lambda}\left[\left(R_{1,h_{uniq}}-R_{1,0}\right)+\left(R_{2,0}(t)-R_{2,h_{uniq}}(t)\right)\right]. \qquad (20)$$

Taking into account the expansion $\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{8}x^2$... and the fact that $H, d, r << D$; $\varphi_0 = 0°$, and $R_{1,0}=\sqrt{D^2+H^2}$, we get

$$R_{1,h_{uniq}}-R_{1,0}\cong\frac{h_{uniq}^2}{2R_{1,0}}-\frac{Hh_{uniq}}{R_{1,0}}-\frac{(h_{uniq}^2-2Hh_{uniq})^2}{8R_{1,0}^3}, \qquad (21)$$

$$R_{2,0}(t)-R_{2,h_{uniq}}(t)\cong\frac{h_{uniq}^2-2(H+d)h_{uniq}}{2R_{1,0}}- \\ -\frac{r\cos(\omega_r t)[h_{uniq}^2-2(H+d)h_{uniq}]\sqrt{R_{1,0}^2-H^2}}{2R_{1,0}^3}. \qquad (22)$$

At the same time taking into account that $h_{uniq}^4 \ll 8R_{1,0}^3$, and relying on (20)-(22), the expression (19) can be represented as

$$\psi_h(t)-\psi_0(t)= \\ =\frac{2\pi}{\lambda}\left[\frac{dh_{uniq}}{R_{1,0}}+\frac{(H+d)\left(\sqrt{R_{1,0}^2-H^2}\right)r\cos(\omega_r t)h_{uniq}}{R_{1,0}^3}\right]. \qquad (23)$$

To ensure unambiguous height measurement the phase difference of the output signal of the interferometric processing system must satisfy the condition [$\psi_h(t) - \psi_0(t) \le \pi$]. When solving (23) we obtain the measured value of the unambiguous height of the observed target. It follows from the expression (23) that the unambiguous measured height is related to the rotation of the PC of the receiving antenna. In this case the unambiguous height depends on variables λ, r, H, d, ω_r and $R_{1,0}$ as follows

$$h_{uniq}(t)\cong\frac{\lambda R_{1,0}^3}{2\left[dR_{1,0}^2+r(H+d)\cos(\omega_r t)\sqrt{R_{1,0}^2-H^2}\right]}. \qquad (24)$$

The nature of the change $h_{uniq}(t)$ when varying variables d, r, H is shown in **Fig. 4-6**.

From the results, presented in Fig. 4-6 one can see that the unambiguous measured height changes according to the periodic law. Moreover, the interferometer base, i.e., the vertical distance

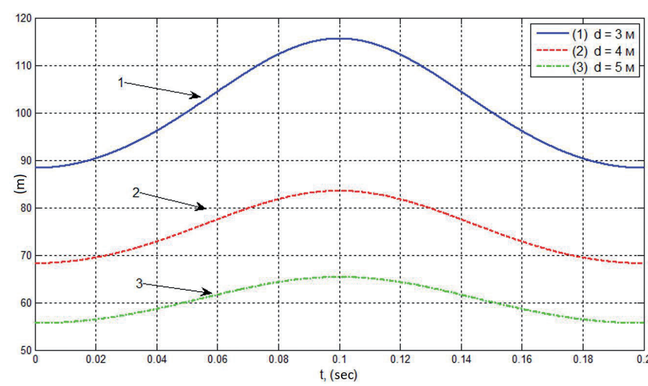


Fig. 4. *The change in the unambiguously measured height in the process of rotation at H = 500 m, r = 8 m, D = 10 km and varying the vertical distance between the PCs of the real antennas: d = 3, 4, 5 m.*

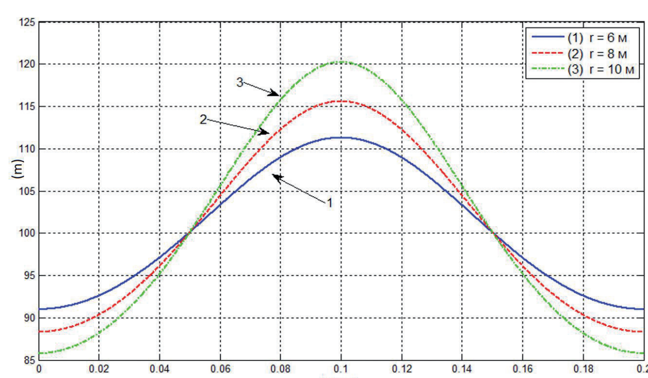


Fig. 5. *The change in the unambiguously measured height in the process of rotation at H = 500 m, d = 3 m, D = 10 km and varying the PC rotation radius value of the receiving antenna.*

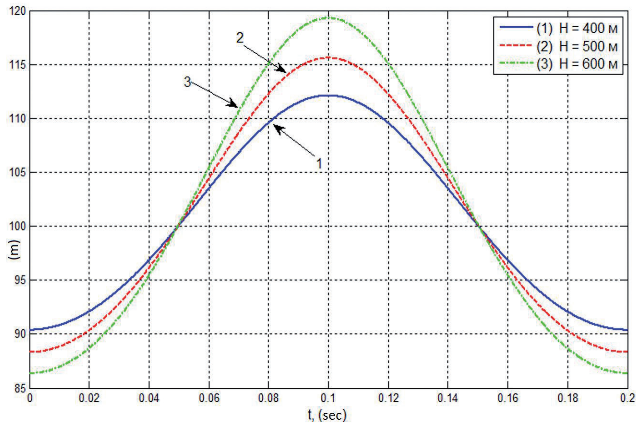


Fig. 6. *The change in the unambiguously measured height in the process of rotation at d = 3 m, r = 8 m, D = 10 km and variation of the radar carrier height: H = 400, 500, 600 m.*

between the PCs of real Rx and Rx-Tx antennas has the greatest influence on the result of height evaluation. In this case the maximum value of $h_{uniq}(t)$ is reached at $t = 0.5\ T$, where T – is the time corresponding to one rotation turn of the APC of Rx module, at which the angle between the direction to the PT and the position of the receiving antenna is π rad. In this situation the distance from the PT to the PC of the Rx antenna will be the maximum possible and, accordingly, the difference $[\psi_h(t) - \psi_0(t)]$ – the minimum.

One can also see from **Fig. 5** and **6**, that the maximum value of $h_{uniq}(t)$ is reached at $t = 0.5\ T$, when $\omega_r t = \pi$. At the values of $\omega_r t = \pi/2$ и $\omega_r t = 3\pi/2$ (i.e., when the angle between the PT and Rx antenna is $\pi/2$ and $(-\pi)/2$ accordingly), the height of the unambiguous measurement is defined by one and the same value, which makes it possible to state: the radius of rotation r and the height H of the radar carrier do not influence $h_{uniq}(t)$ at $t = 0.25\ T$ and $0.75\ T$.

From the obtained results we can conclude that in order to eliminate the phase shift, caused by the rotation of the APC Rx it is necessary to choose $t = 0.25\ T$. In this case the expression (24) can be presented as

$$h_{uniq} \cong \frac{\lambda}{2d} R_{1,0}. \qquad (25)$$

The expression (25) shows, that the height of an unambiguous measurement is inversely proportional to the value d – the vertical distance between the PCs of real antennas and is directly proportional to the wavelength of the probing signal λ and distance to the target $R_{1,0}$. In particular, at $\lambda = 3$ cm, $d = 3$ m, $D = 10$ km, $H = 500$ m we get $h_{uniq}(t)$ 50 m.

In future we will assume that the height report is carried out at $t = 0.25\ T$ or $\omega_r t =$ rad. In this case the expression (18) can be written as

$$Z_c = H - \frac{\left(\left(\frac{\lambda \psi_z(T_1)}{2\pi}\right)^2 + \frac{\lambda \psi_z(T_1)}{\pi} R_0\right) - I^2}{2d}, \qquad (26)$$

where $T_1 = 0.25\ T$. Expression (26) directly implies the following: what information it is necessary to have in order to be able to determine the height of the surface object of observation.

5. ALGORITHM FOR DETERMINING THE HEIGHT OF THE OBSERVED EARTH'S SURFACE RELIEF

It follows from the obtained results that in order to determine the height H of the observed surface relief it is necessary to do the following steps:

1. Determine the phase difference $\psi_h(m,n)$ of the received signals from the observed object (relief) at the output of the processing system (at the output of phase detector PD) in each resolution element in azimuth and range, where m,n are the current numbers of resolution elements in range and azimuth, respectively. As $\psi_h(m,n) = \varphi \pm 2\pi N$ is periodical, it is necessary to determine the number of the whole periods of N phase reversal.
2. Determine the phase difference $\psi_0(m,n)$ of the output signals corresponding to the zero height for each resolution cell

$$\psi_0(m,n) = \frac{2\pi}{\lambda}\left(R_{1,0}(m,n) - R_{2,0}(m,n)\right).$$

In view of the fact that the distance $R_{1,0}(m,n)$ is unknown in practice, we calculate it using $R_{1,h}(m,n)$, and get the following in the result

$$\psi_0(m,n) \cong \frac{\pi}{\lambda}\left(\frac{I^2 + 2dH}{R_{1,h}(m,n)}\right), \qquad (27)$$

where I – interferometer base. This phase is also a periodic function, so it's necessary to estimate the number of the whole periods of phase reversal.

3. Determine the relief height $h(m,n)$, based on (26) in the assumption that $\omega_r t = \pi/2$ and $\theta = 0$ degrees, we get

$$h(\mathrm{m,n}) = \mathrm{z}_h(\mathrm{m,n}) - \mathrm{z}_0(\mathrm{m,n}), \tag{28}$$

$$h(m,n) \cong \frac{\lambda^2}{8\pi^2 d}\left(\psi_0^2(m,n) - \psi_h^2(m,n)\right) + \\ + \frac{\lambda}{2\pi d}\left(\psi_0(m,n) - \psi_h(m,n)\right) R_{1,h}(m,n), \tag{29}$$

$$h(m,n) \cong \frac{\lambda}{2\pi d}\left(\psi_0(m,n) - \psi_h(m,n)\right) R_{1,h}(m,n). \tag{30}$$

Expression (30) actually contains the whole information necessary to determine the relief height during the PC rotation of the receiving antenna and the antenna diversity of Rx-Tx module, namely: the distance to the target $R_{1,h}(m,n)$, phase difference $\psi_h(m,n)$ of the received signals at the output of the processing system and parameters r, d, H, λ, that define the observation conditions.

The determination of an unambiguous phase (disclosure of the phase difference) is performed using the algorithm presented in [8], the essence of which is to integrate the phase difference between two adjacent resolution elements provided that this difference does not exceed the value of π, and there are no sharp changes in height relief. Otherwise a large number of phase turns occurs. More details about this algorithm can be found in [9].

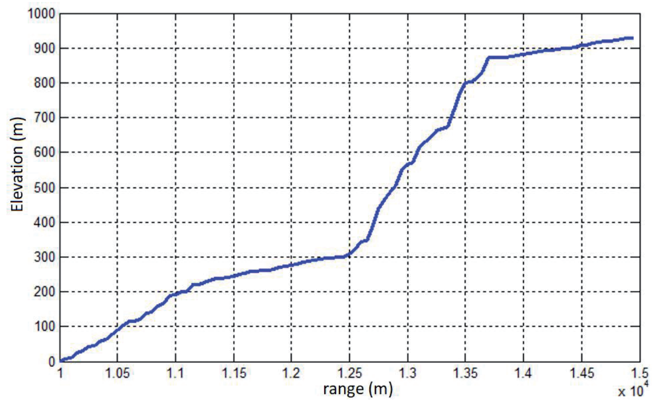
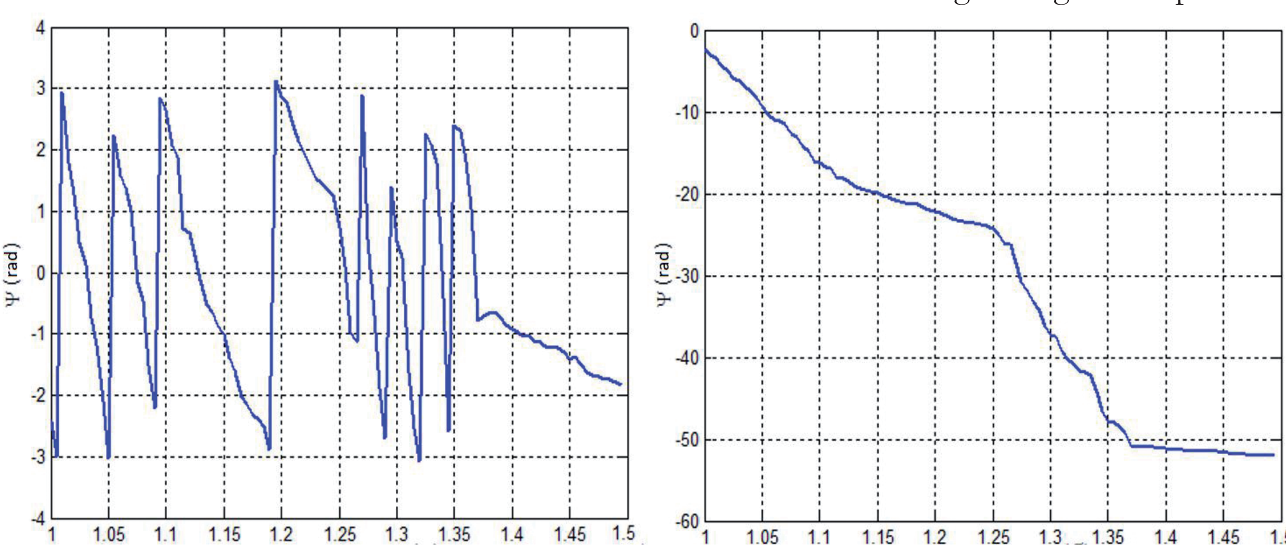


Fig. 7. *Modeled earth relief.*

The algorithm operability was tested by computer simulation. Initially the relief of the earth surface section was formed in the modeling environment, which is shown in **Fig. 7.** The relief has height jumps from 0 m up to 50 m within the range scale from 10 km to 15 km with a range step of 50 m. The height measurement range corresponds to half of its unambiguous measurement.

During modeling it was assumed that $d = 3$ m, $H = 500$, $r = 8$ m, $\lambda = 3$ cm, then in accordance with previously described algorithm the formation of the RI of the observed relief was carried out.

At the first step of the algorithm the phase difference $\psi_h(m,n)$ of the received signals was determined within each resolution element (**Fig. 8***a*), and then the unambiguous phase was determined using the algorithm presented



a *b*

Fig. 8. *Phase difference $\psi_h(m,n)$ of the received signals at the PD output before (a) and after (b) the processing.*

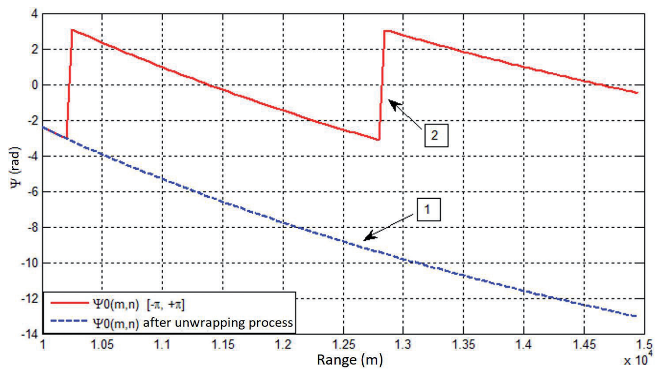


Fig. 9. *Phase difference $\psi_0(m,n)$ of the reflected signals, corresponding to the flat earth surface before (curve 1) and after the processing (curve 2).*

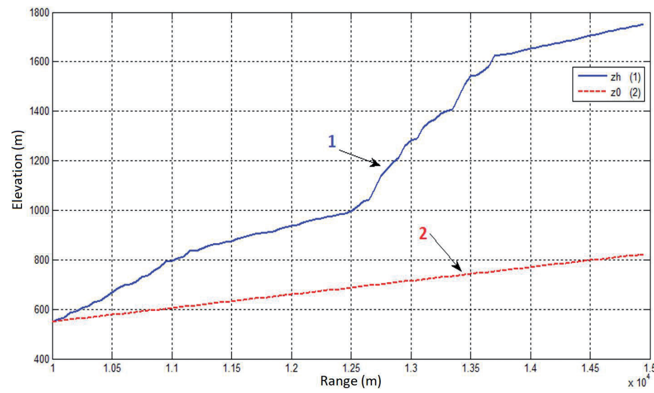


Fig. 10. *The relief of terrain $z_h(m,n)$ (1) and the component $z_0(m,n)$, corresponding to the zero height of the earth surface (2), along the range scale from 10 km to 15 km for one azimuth bin.*

in [8]. The results of processing are shown in Fig. 8*b*.

At the second step the phase difference was determined for the flat earth surface in each element of resolution at the given range on the basis of the expression (27) and the unambiguous phase $\psi_0(m,n)$ was determined too. The results of the algorithm work at the second step are given in **Fig. 9**.

At the third step, using the expression (18), the relief height was determined as the difference of heights $z_h(m,n)$ and $z_0(m,n)$ within the resolution elements in range and azimuth. In this case it was believed that the instant of time $t = 0.25\ T$ (i.e. $\omega_r t = \pi/2$). In this case the expression for $z_h(m,n)$ and $z_0(m,n)$ can be represented as

$$z_h(m,n) = \\ = \mathrm{H} - \left(\frac{\left(\left(\frac{\lambda\psi_h(m,n)}{2\pi} \right)^2 + \frac{\lambda\psi_h(m,n)}{\pi} R_{1,h}(m,n) \right) - I^2}{2d} \right), \quad (31)$$

$$z_0(m,n) = \\ = \mathrm{H} - \left(\frac{\left(\left(\frac{\lambda\psi_0(m,n)}{2\pi} \right)^2 + \frac{\lambda\psi_0(m,n)}{\pi} R_{1,h}(m,n) \right) - I^2}{2d} \right). \quad (32)$$

Fig. 10 shows the results of calculating $z_h(m,n)$ (curve 1) and $z_0(m,n)$ (curve 2) within the fixed element of resolution in azimuth.

Fig. 11 shows 3D image of the terrain relief, formed in the antenna aperture synthesis mode with rotation of the receiving antenna and the fixed position of the APC of Rx-Tx module when using the considered algorithm.

To assess the accuracy of the relief formation, the developed algorithm determined the error in calculating the height of the relief relative to the model used. The results of determining the error in the relief formation are presented in **Fig. 12**, where the nature of the change in this error on the range scale within the variation of the model relief height is shown.

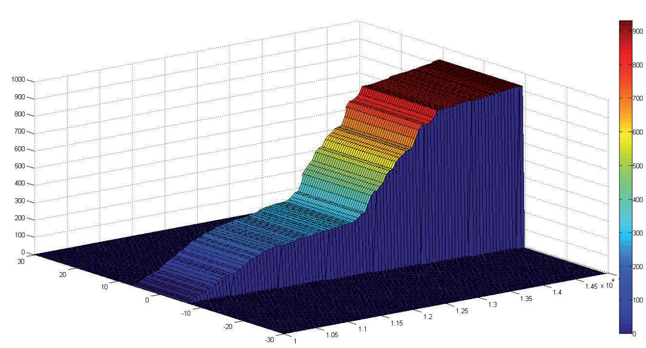


Fig. 11. *3D image of the simulated relief of terrain.*

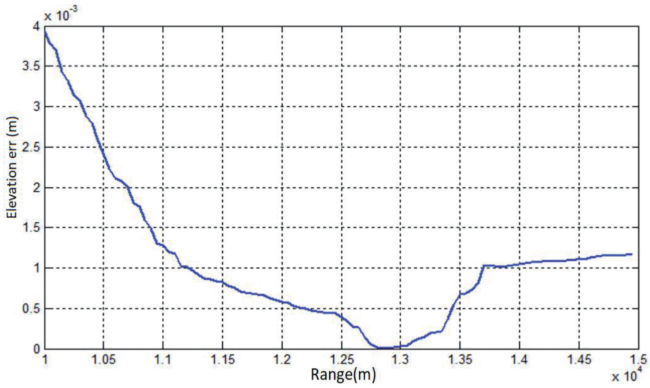


Fig. 12. *Changes in the error of relief formation along the range scale from 10 km to 15 km*.

Fig. 12 shows that the error in the restoring the relief height corresponds to the thousandths of a meter, which allows us to state that the developed algorithm adequately restores the relief of the observed surface. At the same time it should be noted that the obtained results correspond to a situation in which observation noise and possible errors in determining parameters *r,d* and *H* were not taken into account.

6. CONCLUSION

Thus, the results presented in the article, show that when synthesizing the antenna aperture due to rotation of the APC of the receiving module and the spaced but stationary position of antenna Rx-Tx module, it is possible to form 3*D* images of the observed earth surface. The formation of 3*D* images is provided by interferometric processing of the received signals in two reception points spaced relative to each other. In this case the errors in the restoration of the relief correspond to thousandths of a meter. The study of the influence of parameters *r, H* and *d* on unambiguity of the height measurement as well as the simulation results, showed that the vertical distance *d* between the PC of the receiving-transmitting antenna and the plane of rotating the PC of the receiving antenna has the greatest influence on the process of restoring the relief of the observed surface.

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