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Extended orthogonal feedback precoding for spatial multiplexing systems

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Abstract: **Various methods of generating of space-time codes and algebraic structures for increasing noise immunity of MIMO communication systems with a large number of transmitting and receiving antennas are considered. The possibility of generation of an orthogonal precoding matrix in MIMO systems depending on changing reception conditions in a radio channel using feedback from a transmitter to a receiver is shown. It is obtained that to transmit such a precoding matrix in MIMO systems with 4×4, 8×8, 16×16 antenna configurations, feedback is required with the ability to transmit only 2 bits on the reverse link at an energy gain of 1.5 dB... 2.5 dB compared to an open loop system. The precoding matrices obtained in the article and their formation algorithms can be used in the development of new MIMO mobile communication systems.**

Keywords: **MIMO, spatial multiplexing, algebraic codes, precoding, Golden code, maximum likelihood, ML, MMSE**

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Contents

1. INTRODUCTION

Shannon–Hartley theorem modification for the channel capacity defined that MIMO channel is more effective than SISO due to independent communication channels between transmitters and receivers [1]. The number of communication channels is $\min(N_{tx}, N_{rx})$ [1,2], where N_{tx} – is the number of transmitting antennas and N_{rx} – is the number of receiving antennas.

Diversity on both sides of the system and coherent signal processing are the special features of MIMO systems. Those features together with increasing of number of independent communication channels allows to improve noise immunity [1,3,4].

The fact that the number of independent communication channels for MIMO systems is $N_{tx} \cdot N_{rx}$ but the real number of communication channels which used for one symbol transmission is N_{rx}. That is the reason why different symbols received with different quality [1,5].

Spatial multiplexing is used in MIMO systems when one symbol transmits via one antenna per one timeslot. Improving noise immunity of the system we may combine symbols in special space algebraic codes which are presented in matrix format [6,7]. One of these codes used for 2×2 MIMO systems is so called Golden code for two transmitting and two receiving antennas [8,9]. Golden code performance gain is 1.5-2 dB [8,10,11]. The breaking factor for widespread of algebraic codes for massive MIMO systems is high computational complexity of optimal demodulation [10,12]. For example, computational complexity of Maximum Likelihood demodulator in algebraic codes is growing in line with $2^{k_b N_{tx}^2}$, where k_b – is the number of bits per modulated symbol.

In those papers [13,14,15] offered extended orthogonal precoding method for MIMO systems with full diversity reception using special orthogonal matrices with dimensions $\left(N_{tx}^2 \times N_{tx}^2\right)$. The method demonstrates that resulted orthogonal matrices give us the minimum of maximum variance of *QAM* symbols for linear demodulation method [16].

The other way to impact on noise immunity of the system to get the information about the channel to have the chance of choosing the space time code. There is a popular way to have the control closed loop to transmit the information [1,17,18] within the system. The key factor for the efficiency of the method is precise information about the channel and the capacity of reverse closed loop [17,19,20].

This paper offers the combination of orthogonal precoding and channel state information for choosing appropriate precoding matrices. The necessary minimum value of reverse channel information is only few bits. As shown below for MIMO 8×8 it's only 3 bits are enough.

2. MIMO SYSTEM MODEL

MIMO spatial multiplexing system model describes the connections between the transmitter and receiver and might be expressed as [1,2]:

$$\mathbf{y}_n = \mathbf{H}\mathbf{x}_n + \boldsymbol{\eta}_n, \tag{1}$$

where $\mathbf{y}_n = \left[y_n^{(1)} \quad y_n^{(2)} \quad \cdots \quad y_n^{(N_{rx})} \right]^T$ – (N_{rx}×1)-dimensioned vector of received signal; $\mathbf{H}$ – (N_{rx}×N_{tx})-dimensioned MIMO channel matrix which consist of scalar channel parameters (complex transmission coefficients) $h^{(i,j)}$ which are non-correlated zero mean Gaussian with the variances $E\left\{\left|h^{(i,j)}\right|^2\right\} = \frac{1}{N_{tx}}$, which means that the channel is independent Rayleigh fading channel; $\boldsymbol{\eta}_n$ – (N_{rx}×1)-dimensioned Gaussian noise vector with covariance matrix $\mathbf{R}_\eta = E\left\{\boldsymbol{\eta}_n \boldsymbol{\eta}_n^H\right\}$, which are mostly diagonal [1].

In such MIMO system where N_{tx} – is the number of transmitting antennas and N_{rx} – is the number of receiving antennas the input modulated symbols stream divided into number of (N_{tx}×1)-dimensioned vectors $\mathbf{x}_n = \left[x_n^{(1)} \quad x_n^{(2)} \quad \cdots \quad x_n^{(N_{tx})} \right]^T$, where every m-element of n-vector is $x_n^{(m)} = s_{(n-1)N_{tx}+m}$, $m = \overline{1, N_{tx}}$, $n = 1,2,...$ Every modulated symbol is zero mean $E\{s_i\}$ = 0, and unit power $E\left\{\left|s_i\right|^2\right\} = 1$.

Let's take the same approach as it was in [13,14,15], when using the transmission of extended vector of modulated symbols $\tilde{\mathbf{x}} \triangleq \left[\mathbf{x}_1^T \quad \mathbf{x}_2^T \quad \cdots \quad \mathbf{x}_L^T \right]^T$, which is one structure of L – vectors and size of this vector is (LN_{tx}×1). Extended channel model for this vector is

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\boldsymbol{\eta}}, \qquad (2)$$

where

$$\tilde{\mathbf{y}} \triangleq \begin{bmatrix} \mathbf{y}_1^T & \mathbf{y}_2^T & \cdots & \mathbf{y}_L^T \end{bmatrix}^T,$$

$$\tilde{\boldsymbol{\eta}} \triangleq \begin{bmatrix} \boldsymbol{\eta}_1^T & \boldsymbol{\eta}_2^T & \cdots & \boldsymbol{\eta}_L^T \end{bmatrix}^T,$$

$$\tilde{\mathbf{H}} \triangleq \begin{bmatrix} \mathbf{H} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \mathbf{H} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{H} \end{bmatrix}$$

– is ($LN_{rx} \times LN_{tx}$)-dimensioned block diagonal extended channel matrix.

Let's have new matrix $\tilde{\mathbf{F}}$, which is ($LN_{rx} \times LN_{tx}$)-dimensioned and use it for precoding our extended vector $\tilde{\mathbf{x}}$. We may get new vector $\mathbf{z}$ which is $\mathbf{z} = \tilde{\mathbf{F}}\tilde{\mathbf{x}}$. New channel model has the form:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{F}}\tilde{\mathbf{x}} + \tilde{\boldsymbol{\eta}}. \qquad (3)$$

MMSE-based demodulated vector and covariance matrix for this vector are given below:

$$\tilde{\mathbf{x}} = \left(\tilde{\mathbf{F}}'\tilde{\mathbf{H}}'\tilde{\mathbf{H}}\tilde{\mathbf{F}} + \frac{1}{\rho}\mathbf{I}_{LN_{tx}} \right)^{-1} \tilde{\mathbf{F}}'\tilde{\mathbf{H}}'\tilde{\mathbf{y}}$$
$$\tilde{\mathbf{V}} = \left(\rho\tilde{\mathbf{F}}'\tilde{\mathbf{H}}'\tilde{\mathbf{H}}\tilde{\mathbf{F}} + \mathbf{I}_{LN_{tx}} \right)^{-1}. \qquad (4)$$

Than we have

$$\tilde{\mathbf{x}} = \left(\tilde{\mathbf{F}}'\tilde{\mathbf{H}}'\tilde{\mathbf{H}}\tilde{\mathbf{F}} + \frac{1}{\rho}\mathbf{I}_{LN_{tx}} \right)^{-1} \tilde{\mathbf{F}}'\tilde{\mathbf{H}}'\tilde{\mathbf{y}} =$$
$$= \tilde{\mathbf{F}}' \left(\tilde{\mathbf{H}}'\tilde{\mathbf{H}} + \frac{1}{\rho}\mathbf{I}_{LN_{tx}} \right)^{-1} \tilde{\mathbf{H}}'\tilde{\mathbf{y}} = \tilde{\mathbf{F}}'\hat{\mathbf{z}}, \qquad (5)$$
$$\tilde{\mathbf{V}} = \tilde{\mathbf{F}}' \left(\rho\tilde{\mathbf{H}}'\tilde{\mathbf{H}} + \mathbf{I}_{LN_{tx}} \right)^{-1} \tilde{\mathbf{F}} = \tilde{\mathbf{F}}'\tilde{\mathbf{V}}_z\tilde{\mathbf{F}},$$

where $\hat{\mathbf{z}} = \left(\tilde{\mathbf{H}}'\tilde{\mathbf{H}} + \frac{1}{\rho}\mathbf{I}_{LN_{tx}} \right)^{-1} \tilde{\mathbf{H}}'\tilde{\mathbf{y}}$ – is MMSE-based vector for $\mathbf{z}$, $\tilde{\mathbf{V}}_z = \left(\rho\tilde{\mathbf{H}}'\tilde{\mathbf{H}} + \mathbf{I}_{LN_{tx}} \right)^{-1}$ – covariance matrix for $\mathbf{z}$ vector.

In that transformation the resulted covariance matrix is block diagonal matrix, as given below:

$$\tilde{\mathbf{V}}_z = \begin{bmatrix} \mathbf{V}_{MMSE} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \mathbf{V}_{MMSE} & \cdots & \mathbf{O} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{V}_{MMSE} \end{bmatrix}, \qquad (6)$$

$$\mathbf{V}_{MMSE} = \left(\rho\mathbf{H}'\mathbf{H} + \mathbf{I}_{N_{tx}} \right)^{-1}, \qquad (7)$$

where $\mathbf{V}_{MMSE}$ is ($N_{rx} \times N_{tx}$)-dimensioned MMSE-based covariance matrix for symbol vector in the system with simple spatial multiplexing, for channel model (1).

3. ORTHOGONAL PRECODING USING INCOMPLETE EXTENDED VECTOR

As we may see in [13], orthogonal precoding of transmitted symbols does not affect the trace of covariance matrix of estimation errors. It means that average values of SNR after linear demodulation have not been changed but in same time the probability distribution is changed, which means that noise immunity of the system is getting better due to the minimum of maximum variance criteria [16]. Next important point in [13] is that we may choose types of orthogonal matrices which give us the way to reduce the variety of minimum and maximum variance in covariance matrix and reduce the mean error probability. When we apply our extended vector with $L = N_{tx}$ size there are no variety between minimum and maximum variance and all variances are equal to the mean. The other side of the approach is negative because we must operate with large size vectors and matrices. It would be good to have the approach which operates with smaller vectors and matrices. For example, two vectors with ($N_{tx} \times 1$) size give us one extended vector with ($2N_{tx} \times 1$) size and orthogonal precoding matrix will be ($2N_{tx} \times 2N_{tx}$) size.

Let's take system with N_{tx} and $L = 2$, choose the precoding matrix as following:

$$\tilde{\mathbf{F}} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_4 & e^{-j\varphi}\mathbf{P}_4' \\ -e^{j\varphi}\mathbf{P}_4 & \mathbf{I}_4 \end{bmatrix}, \tag{8}$$

where φ – is rotation angle for symbol constellation; $\mathbf{P}_4$ – is (4×4)-dimension permutation matrix where every element is 0 except one which is equal to 1 and it means that $\tilde{\mathbf{F}}$ is orthogonal matrix.

For the maximum diversity gain, we should have 0 at main diagonal for matrix $\mathbf{P}_4$. This is the situation when every symbol transmits via two different antennas.

Considering new covariance matrix of estimation errors according to (5) - (7)

$$\tilde{\mathbf{V}} = \tilde{\mathbf{F}}'\tilde{\mathbf{V}}_z\tilde{\mathbf{F}} = \\ = \frac{1}{2}\begin{bmatrix} \mathbf{V}_{MMSE} + \mathbf{P}_4'\mathbf{V}_{MMSE}\mathbf{P}_4 & \ddots \\ \ddots & \mathbf{V}_{MMSE} + \mathbf{P}_4'\mathbf{V}_{MMSE}\mathbf{P}_4 \end{bmatrix}. \tag{9}$$

Since we are interested in variances of error of estimation only which are located at main diagonal and then we do not have any concerns about the rest of block matrices inside. Having our permutation matrix $\mathbf{P}_4$ we may get new vector of diagonal elements of matrix $\tilde{\mathbf{V}}$:

$$\tilde{\mathbf{v}} \triangleq \mathrm{diag}\left(\tilde{\mathbf{V}}\right) = \frac{1}{2}\begin{bmatrix} \mathbf{v}_{MMSE} + \mathbf{P}_4\mathbf{v}_{MMSE} \\ \mathbf{v}_{MMSE} + \mathbf{P}_4\mathbf{v}_{MMSE} \end{bmatrix}, \tag{10}$$

where diag(**A**) is the operator which creates vector from diagonal elements of **A** matrix, $\mathbf{v}_{MMSE} \triangleq \mathrm{diag}\left(\mathbf{V}_{MMSE}\right)$ – N_{tx}-dimensioned vector of diagonal elements of MMSE-based covariance matrix.

Considering the example of following permutation matrix:

$$\mathbf{P}_4^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

In that case for the first half of diagonal vector $\tilde{v}$ we can write the following expressions:

$$\begin{aligned} \tilde{v}_1 &= \frac{1}{2}\left(v_{MMSE,1} + v_{MMSE,4}\right) \\ \tilde{v}_2 &= \frac{1}{2}\left(v_{MMSE,2} + v_{MMSE,3}\right) \\ \tilde{v}_3 &= \frac{1}{2}\left(v_{MMSE,3} + v_{MMSE,2}\right) \\ \tilde{v}_4 &= \frac{1}{2}\left(v_{MMSE,4} + v_{MMSE,1}\right). \end{aligned} \tag{12}$$

The second half of diagonal vector $\tilde{v}$ is the same as the first one.

As the result of orthogonal precoding (12) there are vector with two variances instead of four different variances for MMSE-based algorithm and maximum of those new variances guaranteed less than original ones. We should note that chosen permutation matrix does not realize that maximum of new variances would be minimal because there no all combinations located at main diagonal.

We may choose other permutation matrices with different locations of "1", for example:

$$\mathbf{P}_4^{(2)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{P}_4^{(3)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{13}$$

For these matrices, we obtain the following variance values of estimation errors using orthogonal precoding:

$$\begin{aligned} \tilde{v}_1^{(2)} &= \frac{1}{2}\left(v_{MMSE,1} + v_{MMSE,3}\right) & \tilde{v}_1^{(3)} &= \frac{1}{2}\left(v_{MMSE,1} + v_{MMSE,2}\right) \\ \tilde{v}_2^{(2)} &= \frac{1}{2}\left(v_{MMSE,2} + v_{MMSE,4}\right) & \tilde{v}_2^{(3)} &= \frac{1}{2}\left(v_{MMSE,2} + v_{MMSE,1}\right) \\ \tilde{v}_3^{(2)} &= \frac{1}{2}\left(v_{MMSE,3} + v_{MMSE,1}\right), & \tilde{v}_3^{(3)} &= \frac{1}{2}\left(v_{MMSE,3} + v_{MMSE,4}\right) \\ \tilde{v}_4^{(2)} &= \frac{1}{2}\left(v_{MMSE,4} + v_{MMSE,2}\right) & \tilde{v}_4^{(3)} &= \frac{1}{2}\left(v_{MMSE,4} + v_{MMSE,3}\right). \end{aligned} \tag{14}$$

Using three variants of permutation matrices $\mathbf{P}_4^{(1)}, \mathbf{P}_4^{(2)}, \mathbf{P}_4^{(3)}$ we may search the whole set of combinations for diagonal vector. We may transmit the information selecting the right combination for transmission with minimum of maximum variance. For our system with $N_{tx} = 4$ it is only 2 bits needed in the feedback loop to transmit the information about the view of precoding matrix.

Large scale configurations for MIMO systems may use the same approach (see (11), (13)) and the number of those matrices to search the whole set of combinations is $(N_{tx} - 1)$ when $N_{tx}=2^m$. The number of bits is needed to transmit the information is *m*.

Those kinds of systems when we may select the precoding matrix and transmit that information via reverse link, are the systems with the control closed loop.

4. SIMULATION RESULTS

The section offers simulation results for proposed orthogonal precoding algorithm with matrix selection for various MIMO configurations. The simulation has been done with following conditions:

- MIMO channel with independent Rayleigh fading;
- QPSK modulation;
- MMSE-based demodulation.

In **Fig. 1** we show bit-error rates (BER) for 4×4nMIMO configuration for the following modes:

- SMx – is ordinary spatial multiplexing;
- SMx + OrtPr – is spatial multiplexing for two time slots;
- SMx + CL-OrtPr – is spatial multiplexing with proposed orthogonal precoding for time slots and close loop (Close Loop Orthogonal Precoding).

We may see that the orthogonal precoding with close loop increases noise immunity in 2-3 dB for BER range 0.01-0.001 with comparison to ordinary spatial multiplexing system. Compared to a MIMO system using orthogonal open-loop (uncontrolled) precoding, the proposed method provides a gain of ~ 1 dB.

For the configuration MIMO 8×8 see **Fig. 2**, the gain compared to conventional multiplexing is 1.6-4.1 dB, and compared to orthogonal open-loop precoding, the gain is 0.5-1.5 dB.

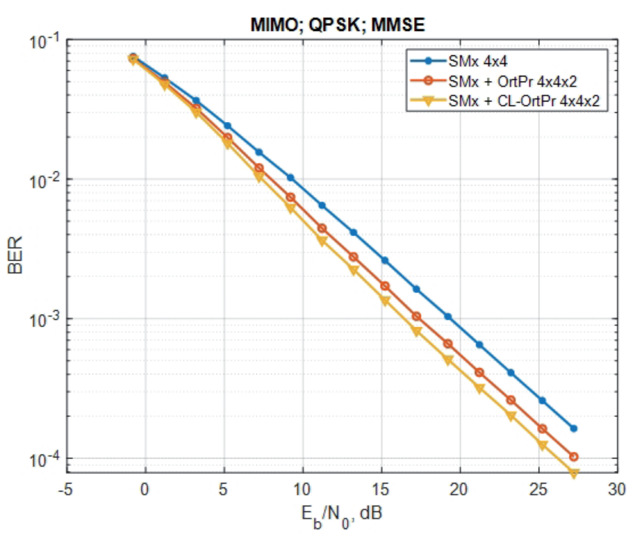

Fig. 1. *BER vs. SNR per bit for proposed system with orthogonal precoding and orthogonal precoding with closed loop for 4×4 MIMO system.*

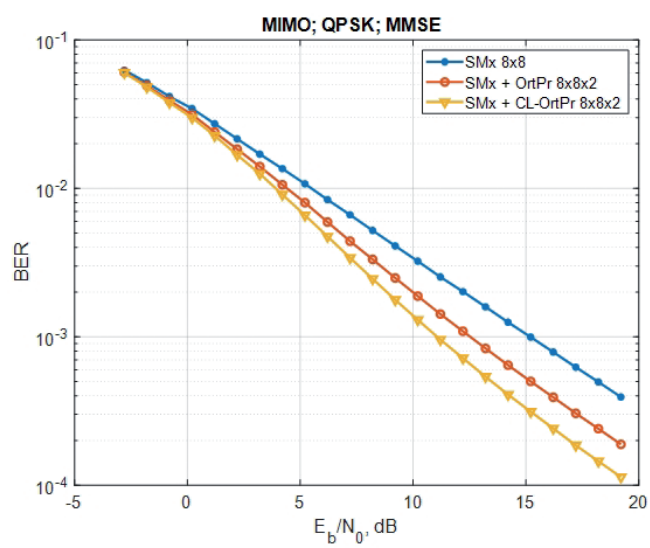

Fig. 2. *BER vs. SNR per bit for proposed system with orthogonal precoding and orthogonal precoding with closed loop for 8×8 MIMO system.*

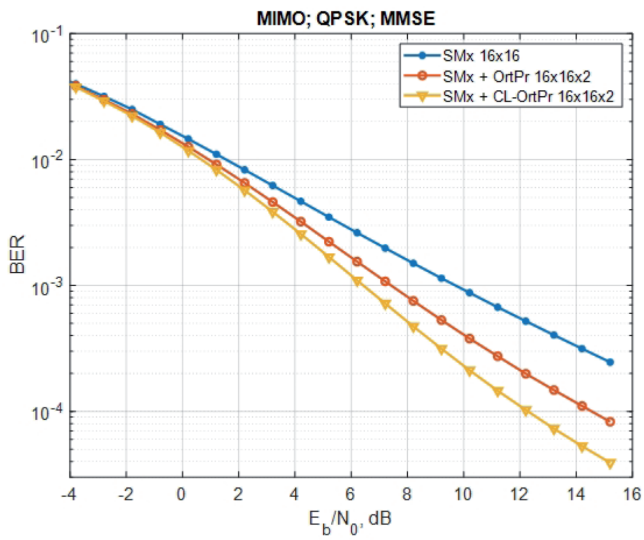

Fig. 3. *BER vs. SNR per bit for proposed system with orthogonal precoding and orthogonal precoding with closed loop for 16×16 MIMO system.*

Fig. 3 shows performance curves for is 16×16 MIMO configuration. Here, there are gains of 0.9-3.3 dB and 0.3-1.0 dB compared to systems with conventional spatial multiplexing and an orthogonal precoding system without control, respectively.

Note that for all configurations, the gain increases as the signal-to-noise ratio increases.

5. CONCLUSION

The orthogonal precoding method, which uses a closed reverse channel control loop to select a precoding matrix, improves the performance of a multi-antenna MIMO communication system by increasing the diversity order. Our precoding algorithm based on minimum of maximum variances optimal criteria which operate with elements of main diagonal of covariance matrix for MMSE-based demodulation algorithm.

Close loop utilizes for transmission the information about precoding matrices demonstrating the better diversity effect using part of extended vector with modulated symbols. The volume of the information needed for 4×4 MIMO system is only 2 bits. The combination of orthogonal precoding for strong sparse matrices and using only part of extended vector with modulated symbols do not increase computational complexity at transmitting and receiving sides of MIMO systems.

The simulation results for orthogonal precoding with close loop provide us with 1-4 dB gain which depends on dimensions of MIMO system and required BER interval.

As the signal-to-noise ratio increases, the gain from using the proposed orthogonal precoding method with a closed control loop increases.

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