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Application of Chimeric Meshes for Explicit Accounting for Inhomogeneities in Modeling the Propagation of Elastic Waves

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Abstract: **The method of chimeric meshes is applied to simulate the propagation of elastic perturbations in media containing porous and fractured inclusions. A model of a linearly elastic isotropic medium is considered, which describes the state of a geological rock. The grid-characteristic method with the third-order accurate Rusanov scheme is used for numerical modeling of the dynamic propagation of elastic disturbances. Special attention is paid to the presence of separate inclusions of pores or fractures, which introduce heterogeneity into the medium and can substantially influence the response of elastic disturbances. The use of the chimera grid method allows for both the position and shape of such inclusions to be described explicitly, taking into account their influence on the propagation of elastic disturbances. As a result of the conducted investigation, a methodology for numerical modeling of the propagation of elastic disturbances in media with porous and fractured inclusions was developed, which can be used to assess the influence of such inclusions on the dynamic response of elastic disturbances. The presented results can be applied in geophysical and seismic research related to modeling the dynamics of various processes in soils and rocks.**

Keywords: **grid-characteristic method, chimera grids, elastic wave, pores, cracks**

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Contents

1. INTRODUCTION

Seismic exploration is one of the oldest and most widespread methods used to search for and explore hydrocarbon deposits. Its main purpose is to determine the structure of the subsurface based on ground and well observation data. However, no less important element of this method is the description of the process of propagation of seismic waves from the source into the geological environment. This allows tasks such as refining hydrocarbon reserves in areas with a high well activity, generating synthetic seismograms for regional studies, and refining the presence and characteristics of hydrocarbons in known geological structures to be solved. Significant progress in this area is achieved by using numerical modeling of the propagation of seismic waves in reliable geological models, as it allows for the creation of arbitrary internal structures of the environment and the estimation of the synthetic response signal.

In this paper, we consider a method for studying spatial dynamic processes occurring in geological environments with porous and fractured inclusions in the process of seismic exploration. The grid-characteristic method is used for numerical integration of emerging systems of partial differential equations [1,2].

Geological rocks containing porous and fractured inclusions are one of the main sources of hydrocarbons, but understanding their structure and properties remains a difficult task for geologists. This is due to the fact that such structures have a complex geometry and their properties can vary depending on many factors, such as a combination of porosity, permeability and pore density. One of the tasks that can be solved by studying dynamic processes in geological environments with porous and fractured inclusions is to determine the optimal places for hydrocarbon production. Developed network of microcracks and relatively high porosity play an important role in the extraction of natural gas in dense gas-bearing sandstone reservoirs. These structures can be used for controlled gas storage and migration due to their low porosity and permeability [3].

At the same time, the development of natural microcracks contributes to the formation of a network of pore-cracks during hydraulic fracturing, which is also an important factor in the production of hydrocarbons. Wave scattering on porous inhomogeneities makes significant changes in the nature of elastic wave propagation, which was shown in [4].

Classical works, in the construction of which it is assumed that the pores are equally distributed in volume, such as the Gassman or Bio models [5,6,7], do not allow taking into account all the features of the distribution of porous inclusions. This necessitates the creation of models that allow us to describe the physical properties of such structures with maximum accuracy, describing their shape [8,9] and including the seismic characteristics of pores and microcracks. Solving the inverse problem based on seismic data is one of the ways to create such models. This approach allows you to create a model of the geological structure under the surface and determine the optimal places for drilling wells. The use of models that take into account various physical properties of pores and microcracks makes it possible to predict and detect manifestations associated with the extraction of hydrocarbons, and to control such processes.

2. MATERIALS AND METHODS

The concept of chimeric (overset) grids arose from the need to model multicomponent systems in which each component requires an optimal grid adapted to its shape. By implementing an arbitrary overlap between adjacent grids in an overlapping system, each grid can be generated independently. The emphasis when creating a grid can be focused on maintaining high quality cells, such as orthogonality and cell size. The flexibility of this approach usually leads to significantly better grid quality compared to hierarchical grids, where grid points on the borders of neighboring zones must completely coincide. In addition, the overset grid approach allows you to identify local geometry changes, such as adding or removing components, without the need for a complete restructuring of the grid system.

When using the method of chimeric grids, which can include various numerical methods, including the grid-characteristic method, to solve problems related to the calculation of physical parameters describing the behavior of the medium at subsequent points in time, it is possible to perform calculations independently in the main computational grid and in chimeric grids at each time step separately. After completing the simulation of the propagation of disturbances at this time step in all chimeric and basic grids, it is necessary to interpolate the values of the physical state of the medium from the chimeric grids into the main grids that intersect with them. This is necessary to be able to take into account the influence of inhomogeneities described by these chimeric grids when calculating at the next time step [10]. Further, the next time step begins with interpolation from the main grid to the boundary nodes of the chimeric grids that intersect with the main grid. Thus, the simulation of the interaction of various regions described by chimeric grids with other parts of the simulated area is achieved. This approach is designed to achieve greater accuracy when performing calculations, especially in cases where the calculation area contains heterogeneous areas located in different places of the simulated area.

3. CRACKS

Regular rectangular grids were used to represent the fractured medium. The continuous medium was described by a single rectangular grid, and the cracks were defined using overset rectangular grids, coaxial to the described cracks. The method used for calculating a straight crack on a regular rectangular grid and its implementation in relation to the grid-characteristic method was described in the article [11]. The adjusted regular rectangular grid was necessary to account for the crack of the coaxial rotated grid. Thus, in the proposed method, it is necessary to use the number of overset grids equal to the number of cracks taken into account.

However, there is another way to define a rotated crack that does not require the use of overset grids. This method works by replacing one crack with many small cracks tied to the nodes of the main grid, and was also described in [12,13]. Despite the fact that both methods provide sufficient accuracy of calculations, the use of many small cracks can lead to an increase in the number of computational operations, and in some cases, may introduce an additional error associated with the "ladder structure" of the crack described by such a method. Thus, the choice between these methods may depend on the specific conditions and the required accuracy of calculations.

To verify this method, a number of test calculations were performed to calculate the propagation of wave disturbances originating from a point source with a frequency of $f = 15$ Hz for various angles of rotation of the crack model. The angle of rotation of the crack is the angle between the axes *OX* of the overset and the main grid. The disturbances created by the source are described by the Ricker wavelet.

During the calculations, a regular rectangular grid of 700×700 nodes, each measuring 2 m, was used to describe a continuous medium. To describe rotated cracks, rotated regular rectangular grids of 32×13 nodes were used, consisting of cells of 2 m. The characteristics of the continuous medium were as follows: density ρ = 400 kg/m^3; longitudinal velocity of the elastic wave C_p = 2850 m/s; lateral velocity C_s = 1650 m/s.

In this work, the propagation of dynamic wave disturbances was calculated during a time step of dt = 0.3 ms for 0.75 seconds. The scheme of setting up in a series of test calculations for various angles of rotation

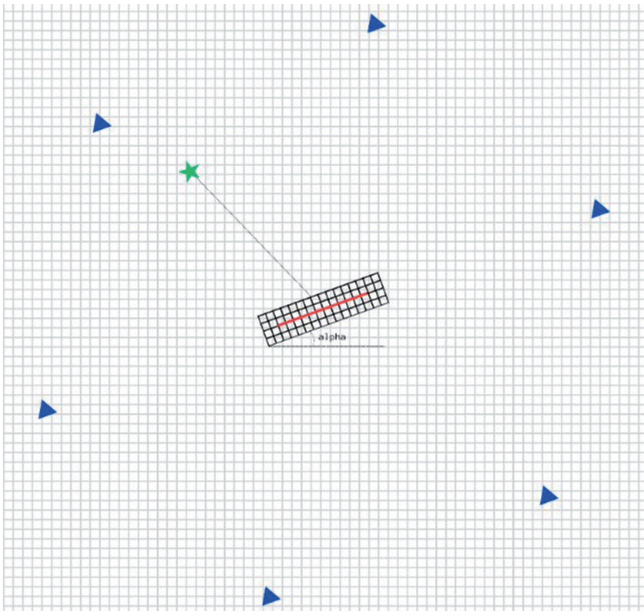

Fig. 1. *Gray lines indicate the edges of the main grid, black edges of the overset one. The blue triangles indicate the positions of the receivers, the green star indicates the position of the source of elastic waves.*

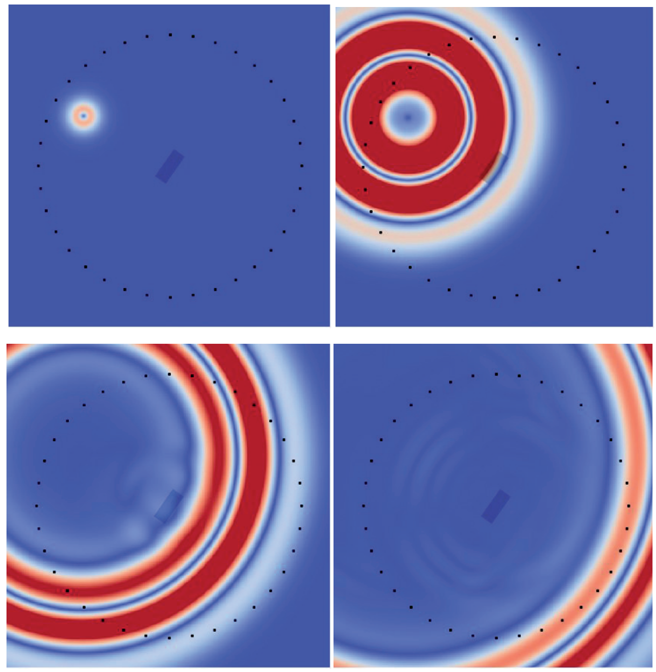

Fig. 2. *Wave patterns of the tension of a medium containing a rotated crack described by means of a overset grid in the presence of a point source of an elastic wave.*

of the crack relative to the axes of the main grid is shown in **Fig. 1**, in this series, the angle α is variable in Fig. 1, the angle between the normal to the crack and the segment from the center of the crack to the source remains unchanged and is 5°. At the same time, from calculation to calculation, the position of the source remains unchanged, and the overset grid-crack-receivers system rotates relative to the source. Visualization of the propagation of tension in the medium, under the conditions of the described calculation for α = 55° in the form of wave patterns is shown in **Fig. 2** visualization of the propagation of voltage in the medium, under the conditions of the described calculation for α = 55 ° in the form of wave patterns is shown in Fig. 2.

As a measure to assess the correctness of the proposed method of modeling a single crack at different angles of rotation, the average error according to the *L2* norm was used, calculated for all receivers throughout the entire modeling period. To compare the results, reference data obtained by turning the crack by an angle α = 0° were used. A graphical representation of the error dependence for various components of the medium tension tensor can be seen in **Fig. 3**.

The results showed that the method is quite accurate when modeling a crack at small angles of rotation of the crack relative to the axes of the main grid, but at steeper angles, additional research is required to refine the accuracy of the method. It should be emphasized that the maximum value of the error we obtained for both

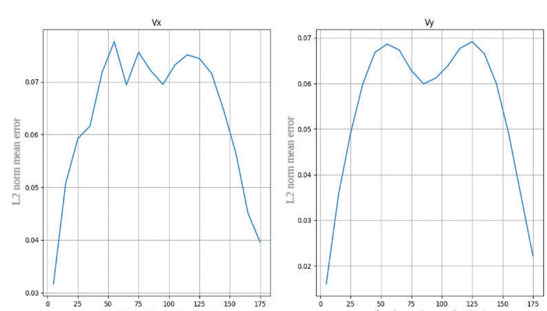

Fig. 3. *The average error according to the L2 norm for all receivers from the angle of rotation of the crack.*

components of the tension does not exceed 0.08, demonstrating the effectiveness of the method we propose.

The correctness of the method was checked when simulationg a single crack for different angles of rotation relative to the axes of the main grid. The results showed that the method gives fairly accurate results in this case. However, for convincing confirmation of the method's abilities in a wider range of conditions, additional research is needed. In particular, modeling of several cracks interacting with each other should be considered, as well as modeling of cracks at different angles between the crack normal and the direction to the source.

4. PORES

To accurately simulate the propagation of elastic waves with hollow holes, the method of overset grids was proposed [14]. It allows you to describe the free boundary conditions of a round or ellipse-shaped hole, forming an annular curved grid. This is achieved by a one-to-one transformation of a uniform regular grid into a curved one consisting of a ring of nodes. Periodic boundary conditions were applied at the grid nodes on the boundaries perpendicular to it. These borders, marked in red in **Fig. 4,** completely coincide and close the grid into a ring, which makes it possible to simulate the propagation of perturbation in nodes at these boundaries in any direction similar to the internal nodes of the grid.

Periodic boundary conditions are implemented in such a way that for nodes located close to one of the boundaries, the neighbor nodes are also located at the opposite boundary. The outer side of such a "looped" grid has no boundary conditions. The implementation of periodic boundary conditions makes it possible to describe hollow holes with an equilateral polygon, which is

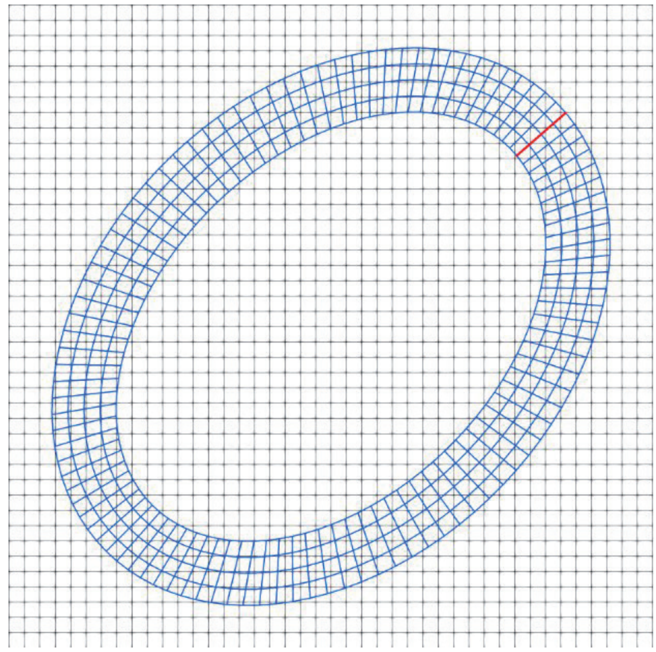

Fig. 4. *Black lines indicate the edges of the main grid, blue edges of the overset one. The red line indicates the position of the ends of the overset grid having periodic boundary conditions.*

formed on the inner side of the overset grid. Unlike a poly-line consisting of segments perpendicular to each other that coincide with the edges of the main regular grid, when using a "ladder" description of the boundary by a regular grid, this method is more accurate and correct.

The boundary of the overset grid, which is the inner boundary of the ring, is used to determine the boundary conditions of the free surface that form the wall of the hollow hole. Thus, it is possible to simulate the propagation of elastic waves in a medium in which there are hollow pore inclusions of various ellipse shapes, a special case of which are round pores. The wave patterns obtained as a result of modeling the interaction of an elastic wave with an ellipse-shaped hole implemented by the proposed method are shown in **Fig. 5**.

To verify the correctness of the use of our method in modeling the propagation of elastic waves in an inhomogeneous

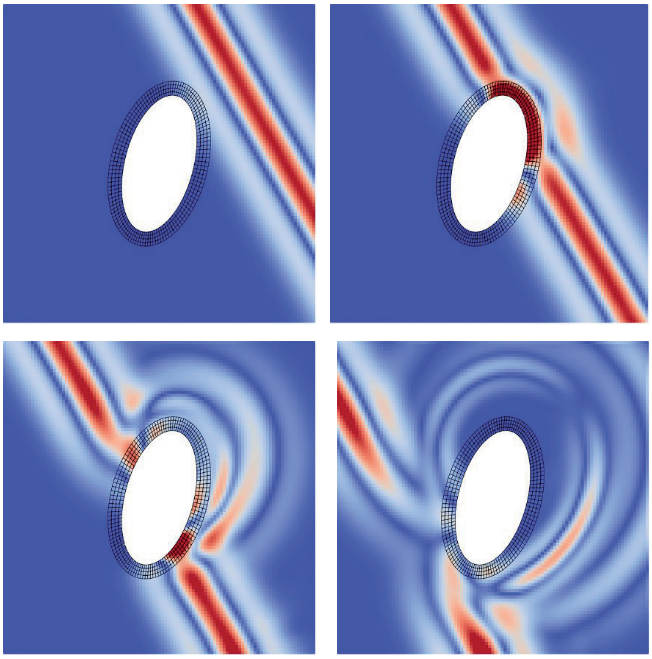

Fig. 5. *Wave patterns of the tension of a medium containing a hollow hole of elliptical shape during the passage of a plane elastic wave.*

medium through single holes, a series of calculations was carried out. They made it possible to check the symmetry of the implementation of the overset grid system and the main grid in the case of applying a periodic boundary condition on the closed boundaries of the overset grid forming a ring around the hole.

For numerical evaluation, we performed a series of calculations in a square area in the presence of a round hole. The characteristics of the continuous medium were as follows: density ρ = 2500 kg/m^3; longitudinal velocity of the elastic wave C_p = 3000 m/s; lateral velocity C_s = 1500 m/s. In the work, the propagation of dynamic wave disturbances was simulated during a time step of dt = 0.2 ms for 0.12 seconds.

The main grid is a square uniform grid of 450×450 nodes of 1.8 m each, the overset grid is also represented by a square uniform grid of 119×5 nodes, measuring 1.8 m. In addition, we have located 72 receivers at a

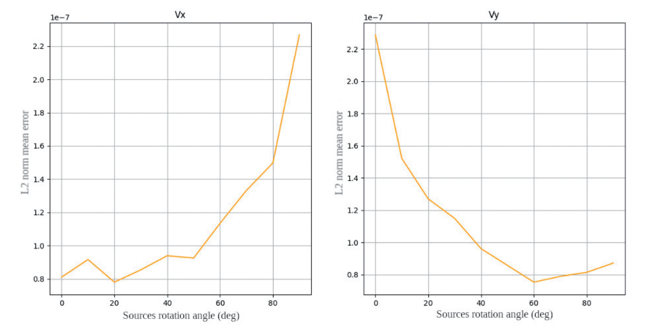

Fig. 6. *Graph of the dependence of the accumulated error according to the L2 norm on various angles between the incident wave front and the axis of the main grid.*

distance of 12 meters from the outer edge of the overset grid. Then we superimposed two plane waves with a frequency of 100 Hz on the described hole, symmetrically with respect to a straight line passing through the center of the hole at a variable angle. As a result of this calculation, the sum of errors of the signal received by the receivers according to the *L2* norm during the entire simulation period was calculated. The results of calculating this metric for various angles between a straight line parallel to the planes of incident waves and the Ox axis are shown in **Fig. 6.**

As can be seen from the graph shown in Fig. 6, the error for different components of the tension tensor increases at different angles. This dependence is explained by the features of the numerical method used in modeling the propagation of elastic perturbations along the longitudinal axes of the main computational grid. The calculations performed have shown the high accuracy of our method in modeling the propagation of elastic waves in a medium in the presence of a circular inhomogeneity. The data obtained demonstrate that our method can be used to solve complex problems related to modeling elastic waves in porous media.

5. CONCLUSION

The method of overset grids in combination with a grid-characteristic method for modeling the propagation of elastic waves in an inhomogeneous medium with various inhomogeneities, such as cracks and pores, is considered. In addition to describing the implementation of the proposed method, the ways of validating the applicability of the overset grid method are considered in detail. The results of validation of the proposed method are demonstrated, while the accuracy of the proposed method is estimated as a result of test calculations.

It is found that the proposed method provides high accuracy in modeling the propagation of elastic waves through an elastic medium having such inhomogeneities as cracks and pores. An important conclusion is that the presented method can be used in various fields related to the modeling of elastic waves in an inhomogeneous medium, and its potential can be revealed in future studies.

REFDRENCES

1. Favorskaya AV, Zhdanov MS, Khokhlov NI, Petrov IB. Modelling the wave phenomena in acoustic and elastic media with sharp variations of physical properties using the grid-characteristic method. *Geophysical Prospecting,* 2018, 66(8):1485-1502.
2. Petrov IB, Khokhlov NI. Modeling 3D seismic problems using high-performance computing systems. *Mathematical Models and Computer Simulations,* 2014, 8:342-350.
3. Qi Y, Chen X, Zhao Q, Luo X, Feng C. Seismic wave modeling of fluid-saturated fractured porous rock: Including fluid pressure diffusion effects of discrete distributed large-scale fractures. *EGUsphere*, 2023, 1-26 p. DOI:10.5194/egusphere-2022-1388.
4. Liu J, Wei XC, Ji YX, Chen TS, Liu CY, Zhang CT, Dai MG. An analysis of seismic scattering attenuation in a random elastic medium. *Applied Geophysics,* 2021, 8(4):344-354. DOI:10.1007/c11770-011-0296-y.
5. Biot MA. Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. *The Journal of the Acoustical Society of America*, 1956, 28(2):168-178.
6. Berryman JG. Origin of Gassmann's equations. *Geophysics,* 1999, 64(5):1627-1629. DOI: 10.1190/1.1444667.
7. Gassmann F. On elasticity of porous media. *Classics of Elastic Wave Theory*, 2007, pp. 389-408.
8. Li T, Wang Z, Yu N, Wang R, Wang Y. Numerical study of pore structure effects on acoustic logging data in the borehole environment. *Fractals,* 2020, 28(3):2050049-518. DOI: 10.1142/S0218348X20500498.
9. Cho Y, Gibson RL, Vasilyeva MV, Efendiev Y. Generalized multi-scale finite elements for simulation of elastic wave propagation in fractured media. *Geophysics*, 2017, 83(1):WA9-WA20. DOI: 10.1190/geo2017-0076.1.
10. Petrov IB, Khokhlov NI. Modeling 3D seismic problems using high-performance computing systems. *Mathematical Models and Computer Simulations,* 2014, 6(4):342-350. DOI: 10.1134/S2070048214040061.
11. Khokhlov N, Favorskaya A, Stetsyuk V, Mitskovets I. Grid-characteristic method using Chimera meshes for simulation of elastic waves scattering on geological fractured zones. *Journal of Computational Physics,* 2021, 446(1):110637. DOI: 10.1016/j.jcp.2021.110637.
12. Khokhlov N, Stognii P. Novel Approach to Modeling the Seismic Waves in the Areas with Complex Fractured Geological Structures. *Minerals*, 2020, 10(2):122. DOI: 10.3390/min10020122.

13. Stognii P, Khokhlov N, Zhdanov M. Novel approach to modelling the elastic waves in a cluster of subvertical fractures. *81st EAGE Conference and Exhibition,* 2019, 1:1-5. DOI: 10.3997/2214-4609.201901014.
14. Khokhlov NI, Stetsyuk VO, Mitskovets IA. Overset grids approach for topography modeling in elastic-wave modeling using the grid-characteristic method. *Computer Research and Modeling,* 2019, 11(6):1049-1059.