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Frequency-polarization processing for multipath mitigation in local positioning systems

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Abstract: **The properties of a local positioning system operating in the 2.4 GHz ISM band, which is capable of supplementing the constellation with local system signals to achieve high positioning accuracy in a complex interference environment, are investigated. In this case, global navigation satellite systems (GNSS) signal receivers often also implement other methods for estimating the position or parameters associated with it: inertial systems, ultra-wideband systems, radars, etc. In this paper, the study of the properties of a local positioning system operating in the 2.4 GHz ISM band , capable of supplementing the constellation with local system signals to achieve high positioning accuracy in difficult conditions. The emphasis here is on indoor operation of the system when GNSS signals are unavailable. In addition, there are a large number of reflected signals in the room. The paper presents a method for describing reflected signals using the Jones vector, as well as the synthesis of optimal algorithms for estimating signal parameters under complex conditions for simplified scenarios. With the help of modeling, it is shown that the optimal methods are poorly applicable to real situations, however, they allow a better understanding of the physics of processes. Based on this understanding, a signal shaping technique is proposed to create a large redundancy of measurements in the receiver, as well as an empirical method for rejecting false measurements. With the help of this method, it has been experimentally shown that indoors in difficult conditions and in the presence of multipath, it is possible to achieve the RMS positioning error of less than 10 centimeters.**

Keywords: **local positioning, navigation, multipath, phase measurements, polarization processing, OFDM**

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1. INTRODUCTION

The construction of a combined system [1] for determining the location and transmitting information has features inherent in the systems from which it is built. The first such feature is the restriction on the geometric arrangement of base and mobile stations. Depending on their relative position (geometric factor), the accuracy of estimating the coordinates of the mobile station may deteriorate. In addition, when receiving signals from multiple base stations, there may be a situation where the mobile station is close to one base station and far from another. Then the receiving device may not have enough dynamic range to receive both signals, or the level of cross-correlation of the signals will be too high. From the foregoing, it follows that for the correct operation of such a system, the organization of time or frequency division of channels for transmitting stations is required. Another feature inherent in navigation systems is the need to ensure the synchronous operation of the system elements. To solve a navigation problem using the difference-range method, it is necessary that the base station system be tightly synchronized and have a single time scale. Thus, the base station system must have synchronization mechanisms on its signals to control the time scales. A well-known method for implementing this task is the classification of base stations into master and slave. In this case, each slave base station monitors the signal of the master base station, according to which it synchronizes its time scale.

An analysis of the functional features of the IEEE 802.11 protocol showed that the main problem in the implementation of location determination is the use of multiple access protocol with carrier sense and collision avoidance. When implementing a positioning system by known methods within a network with a similar access method, it is not possible to ensure the strict frequency of transmission of navigation sequences necessary for the implementation of high-precision positioning. However, the protocol devices have a procedure for checking the busyness of the transmission medium, called the Clear Channel Assessment (CCA). As part of this procedure, the devices scan the air for the presence of protocol signals. When there is a Wi-Fi signal on the air, devices do not initiate data transfer, waiting for the medium to be free.

Given the above, we can conclude that the output system contains (**Fig. 1**):

- a modified access point of the IEEE 802.11 protocol (Master Station (Access Point) – MSTA/MAP), acting as a master base station,

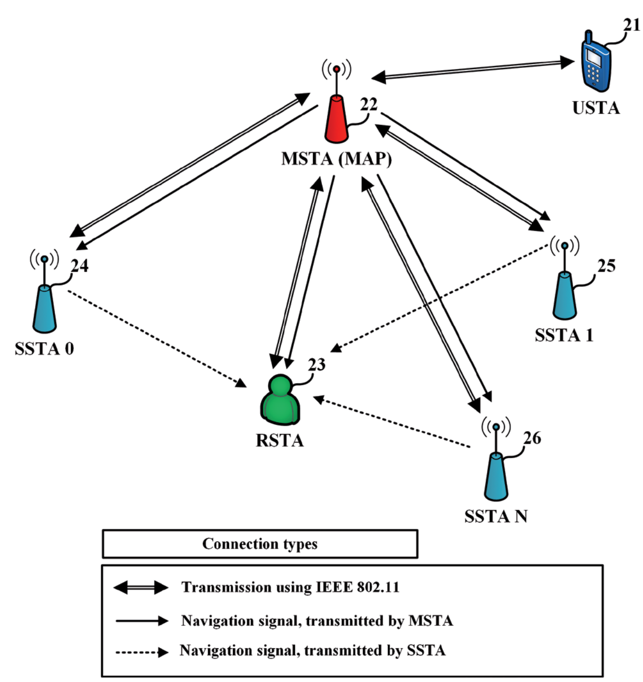

Fig. 1. *Combined system diagram and types of emitted signals.*

and implementing, in addition to standard media access mechanisms, the proposed algorithms for combining systems. Device 22 in Fig. 1;

- modified stations of the IEEE 802.11 protocol (Slave (fixed) Station - SSTA), acting as slave base stations, also implementing the proposed algorithms for combining systems. Devices 24,25,26 in Fig. 1;
- modified stations of the IEEE 802.11 protocol (Rover (mobile) Station - RSTA), acting as mobile subscribers (rovers), determining their location by the system signals using the difference-range method, as well as participating in the exchange of information with the access point and other stations. Device 23 in Fig. 1.
- standard IEEE 802.11 protocol devices (User Station - USTA). Device 21 in Fig. 1.

The basic mechanisms of signal formation and the technique for constructing tracking loops and estimating signal parameters in the receiver are described in more detail in [1]. This article focuses on the features of the operation of this system in difficult conditions in the presence of a large number of reflected signals.

2. TECHNIQUE FOR DESCRIBING POLARIZATION OF RE-REFLECTED ELECTROMAGNETIC WAVES

Reflection of waves, taking into account polarization distortions, is conveniently considered by representing a plane homogeneous electromagnetic wave by the Jones vector [2]. Let's introduce this concept.

2.1. Representation of electromagnetic waves by the Jones vector

Let there be a plane homogeneous TE wave (Transverse Electric) whose electric intensity vector for the case of an arbitrary elliptical polarization can be written in the right Cartesian coordinate system as

$$\bar{E}(z,t) = \left[E_x \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_x) \right] \bar{x} + \left[E_y \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_y) \right] \bar{y}. \quad (1)$$

Here $\bar{x}$ and $\bar{y}$ – unit vectors that determine the orientation of the electric vectors of linearly polarized waves

$$E_x \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_x) \text{ and } E_y \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_y).$$

The superposition of these waves, which have different phases and amplitudes, leads to an elliptical polarization of the wave. When analyzing its transformations in a radio channel, there is no need to use the full expression (1). First of all, it is not necessary to keep the spelling of the unit vectors $\bar{x}$ and $\bar{y}$ to take into account the orientation of the components E_x and E_y, but it is advisable to take this orientation into account by using the column vector

$$\bar{E}(z,t) = \begin{bmatrix} E_x \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_x) \\ E_y \cos(\omega t - \frac{2\pi z}{\lambda} + \varphi_y) \end{bmatrix}. \quad (2)$$

Here and below, it is assumed that the upper line corresponds to the projection of the vector of the analyzed field on the OX axis, and the lower one, to the OY axis.

Both the harmonic time dependence and the constant phase shift, which is the same for both components, do not carry information about the state of wave polarization. In this regard, these values can be excluded from consideration. Given the above, expression (2) can be represented in the following form:

$$\dot{\bar{\boldsymbol{E}}} = \begin{bmatrix} E_x \exp(j\varphi_x) \\ E_y \exp(j\varphi_y) \end{bmatrix} = \begin{bmatrix} \dot{E}_x \\ \dot{E}_y \end{bmatrix}. \quad (3)$$

This representation of an electromagnetic wave in the form (3) is called the Jones vector. The total intensity of the wave represented by the Jones vector is equal to the sum of the intensity of the projections of the electrical wave vector onto the orthogonal axes OX, OY:

$$J = \dot{\bar{\boldsymbol{E}}}^{\dagger} \dot{\bar{\boldsymbol{E}}} = E_x^2 + E_y^2,$$

where † means Hermitian conjugation.

The total intensity of the wave does not determine its polarization properties, therefore, for the convenience of analysis, we normalize to the total intensity. The Jones vector in this case satisfies the condition $\dot{\bar{E}}^{\dagger}\dot{\bar{E}}=1$ and is called the normalized Jones vector.

Since the complex elements of the Jones vector can independently take on any possible values, this makes it possible to obtain all polarization states of an electromagnetic wave. For example, linearly polarized poles whose electric vectors are oriented either along the OX axis or along the OY axis can be represented by the Jones vectors

$$\dot{\bar{E}}_{OX}=\begin{bmatrix}1\\0\end{bmatrix},\dot{\bar{E}}_{OY}=\begin{bmatrix}0\\1\end{bmatrix}$$

and form a pair of orthogonal linear polarized waves.

Waves of right and left circular polarization can be represented using the Jones vector as follows:

$$\dot{\bar{E}}_{R}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\j\end{bmatrix},\dot{\bar{E}}_{L}=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-j\end{bmatrix}.$$

Let us now consider in the general case the technique for obtaining the Jones vector of the sum wave in the basis of the receiving antenna, taking into account the multipath signal propagation. First, let us turn to the wave received upon reflection from some stationary object. The consideration will be carried out assuming that the reflecting object does not introduce changes into the spatial spectrum of the wave, i.e. does not give a diffraction image [2]. Let there be three bases: a transmitting antenna XOY, a reflecting object XOY_{mp} and a receiving antenna XOY_{rx}. In the basis of the transmitting antenna, the initial Jones vector $\dot{\bar{E}}$ is set, characterized by a certain (fixed) ratio of the amplitudes of its projections and some phase difference between them. These values determine the parameters of the polarization ellipse of the output wave of the transmitting antenna, i.e. of the input wave of the reflecting object, assuming that no distortion occurs in the propagation space of the wave. Since the orientation of the transmitting antenna's basis relative to the object's own basis is generally arbitrary, we translate the original vector $\dot{\bar{E}}$ into the object's own basis using the rotation operator $\left\|\boldsymbol{R}(\theta)\right\|$:

$$\dot{\bar{E}}_{mp}^{IN}=\boldsymbol{R}(\theta)\dot{\bar{E}}=\begin{bmatrix}C_{\theta} & S_{\theta}\\-S_{\theta} & C_{\theta}\end{bmatrix}\times\begin{bmatrix}\dot{E}_1\\\dot{E}_2\end{bmatrix}=\begin{bmatrix}\dot{E}_{mp1}^{IN}\\\dot{E}_{mp2}^{IN}\end{bmatrix},$$

where $C_{\theta}=\cos(\theta),S_{\theta}=\sin(\theta)$, θ – angle between the positive directions of the semiaxes OX, OX_{mp}, let's call it the angle of transition between the bases.

Since only the amplitude and absolute phase of the polarization ellipse (which in this case is degenerate into a line) change for each of the orts of the eigencoordinate system, this means that the mathematical description of the transformation of the Jones vector from the input to the output of the object, when considered in its own basis, can be introduced on the basis of using a diagonal matrix

$$\dot{\bar{E}}_{mp}^{OUT}=\begin{bmatrix}\dot{k}_1 & 0\\0 & \dot{k}_2\end{bmatrix}\times\begin{bmatrix}\dot{E}_{mp1}^{IN}\\\dot{E}_{mp2}^{IN}\end{bmatrix}=\begin{bmatrix}\dot{k}_1\dot{E}_{mp1}^{IN}\\\dot{k}_2\dot{E}_{mp2}^{IN}\end{bmatrix},$$

where $\dot{k}_1$ and $\dot{k}_2$ – complex transfer coefficients corresponding to the orts of the object's own basis.

Completing the transformation procedure, it is necessary to make a transition from the object's own basis to the receiving antenna basis using the rotation operator $\left\|\boldsymbol{R}(\theta)\right\|$, where φ – angle between the positive directions of the semiaxes OX_{mp}, OX_{rx}. Then the Jones vector for the receiving antenna for non-direct propagation ray (Non-Line-of-Sight – NLOS) can be written as:

$$\dot{\bar{E}}_{rx\_mp}^{IN}=\boldsymbol{R}(\varphi)\times\begin{bmatrix}\dot{k}_1 & 0\\0 & \dot{k}_2\end{bmatrix}\times\boldsymbol{R}(\theta)\times\dot{\bar{E}}. \qquad (4)$$

For a direct propagation ray (Line-of-Sight – LOS), the Jones vector can be written using the rotation operator $\boldsymbol{R}(\alpha)$:

$$\dot{\bar{E}}_{rx\_LOS}^{IN}=\boldsymbol{R}(\alpha)\times\dot{\bar{E}},$$

where α – angle between the positive directions of the semiaxes OX, OX_{rx}.

Since in the proposed description the Jones vectors of the reflected and direct waves are presented in the basis of the receiving antenna, one can easily proceed to writing the Jones vector of the total wave in the presence of M singly reflected signals:

$$\dot{\vec{E}}^{IN}_{rx\_all} = \dot{\vec{E}}^{IN}_{rx\_LOS} + \sum_{j=1}^{M} \dot{\vec{E}}^{IN}_{rx\_mp\_j},$$

where

$$\dot{\vec{E}}^{IN}_{rx\_mp\_j} = \boldsymbol{R}(\varphi_j) \times \begin{bmatrix} \dot{k}_{1\_j} & 0 \\ 0 & \dot{k}_{2\_j} \end{bmatrix} \times \boldsymbol{R}(\theta_j) \times \dot{\vec{E}}.$$

In the considered case, the physical meaning of the reflection of the radiated wave is as follows: the Jones vector of the radiated wave, given in the basis of the transmitting antenna, is translated into the own bases of the reflecting objects. Orthogonal components, oriented along the orts of their own bases, acquire amplitude and phase changes. Then the Jones vectors of the received waves at the output of the objects are translated into their own basis of the receiving antenna and are added to each other, as well as with the Jones vector of the wave of the direct propagation beam, translated into the basis of the receiving antenna.

If there is a multiple reflection of the radiated wave, the final Jones vector can be obtained by sequentially multiplying the Jones vector of the radiated wave with matrices of weight coefficients and transition operators between the basis of individual objects. For example, in the case of double reflection, the Jones vector of the wave in the basis of the receiving antenna can be represented as follows [2]:

$$\dot{\vec{E}}^{IN}_{rx\_mp} = \boldsymbol{R}(\varphi) \times \begin{bmatrix} \dot{k}_{12} & 0 \\ 0 & \dot{k}_{22} \end{bmatrix} \times \\ \times \boldsymbol{R}(\upsilon) \times \begin{bmatrix} \dot{k}_{11} & 0 \\ 0 & \dot{k}_{21} \end{bmatrix} \times \boldsymbol{R}(\theta) \times \dot{\vec{E}}, \quad (5)$$

where υ – the angle between the positive directions of the semi-axes OX of the reflecting objects.

Applying the associativity property of matrix multiplication, we can transform expressions (4) and (5):

$$\dot{\vec{E}}^{IN}_{rx\_mp} = \boldsymbol{R}(\varphi) \times \boldsymbol{R}(\theta) \times \begin{bmatrix} \dot{k}_{1} & 0 \\ 0 & \dot{k}_{2} \end{bmatrix} \times \dot{\vec{E}} = \\ = \boldsymbol{R}(\varphi + \theta) \times \begin{bmatrix} \dot{k}_{1} & 0 \\ 0 & \dot{k}_{2} \end{bmatrix} \times \dot{\vec{E}}, \quad (6)$$

$$\dot{\vec{E}}^{IN}_{rx\_mp} = \boldsymbol{R}(\varphi) \times \boldsymbol{R}(\upsilon) \times \\ \times \boldsymbol{R}(\theta) \times \begin{bmatrix} \dot{k}_{12} & 0 \\ 0 & \dot{k}_{22} \end{bmatrix} \times \begin{bmatrix} \dot{k}_{11} & 0 \\ 0 & \dot{k}_{21} \end{bmatrix} \times \dot{\vec{E}} = \\ = \boldsymbol{R}(\varphi + \upsilon + \theta) \times \begin{bmatrix} \dot{k}_{12}\dot{k}_{11} & 0 \\ 0 & \dot{k}_{22}\dot{k}_{21} \end{bmatrix} \times \dot{\vec{E}}. \quad (7)$$

Based on expression (7), in this case it is possible to write the Jones vector for the case of N wave re-reflections:

$$\dot{\vec{E}}^{IN}_{rx\_mp} = \boldsymbol{R}\left(\sum_{l=1}^{N+1} \xi_l\right) \times \begin{bmatrix} \prod_{l=1}^{N} \dot{k}_{1\_l} & 0 \\ 0 & \prod_{l=1}^{N} \dot{k}_{2\_l} \end{bmatrix} \times \dot{\vec{E}}, \quad (8)$$

where ξ_l – angle of transition between the bases of two elements through which the wave passes successively. Thus, ξ_1 – transition angle between the transmitting antenna basis and the basis of the first reflecting object, ξ_{N+1} – angle of transition between the basis of the last reflecting object and the basis of the receiving antenna, other ξ_l – transition angles between the bases of reflecting objects through which the wave passes successively.

Thus, taking into account the above relations, we can write the general Jones vector for the sum wave in the basis of the receiving antenna:

$$\dot{\vec{E}}^{IN}_{rx\_all} = \dot{\vec{E}}^{IN}_{rx\_LOS} + \sum_{j=1}^{M} \dot{\vec{E}}^{IN}_{rx\_mp\_j},$$

where

$$\dot{\vec{E}}^{IN}_{rx\_mp\_j} = \boldsymbol{R}\left(\sum_{l=1}^{N_j+1} \xi_{j\_l}\right) \times \begin{bmatrix} \prod_{l=1}^{N_j} \dot{k}_{1\_j\_l} & 0 \\ 0 & \prod_{l=1}^{N_j} \dot{k}_{2\_j\_l} \end{bmatrix} \times \dot{\vec{E}},$$

and N_j can take any value from 1 to ∞.

It should be noted that the coefficients $\dot{k}_{1\_j\_l}$ and $\dot{k}_{2\_j\_l}$ are constant for given objects lying on the *j*-th path of wave propagation, parameter l in this case defines only the serial number of the reflecting object in the *j*-th path. In other words, the same objects in the general case can repeatedly re-reflect the signal between themselves. Angles ξ_{j_l} are determined based on the directions of wave propagation, and for each path can be different, however, with multiple reflections between the same objects, the transition angles between them will be repeated. Thus, the index l in this case also determines the ordinal number of the angle on the jth propagation path.

Let us now consider a special case when waves with linear polarization $\dot{\bar{\boldsymbol{E}}}_{OX}, \dot{\bar{\boldsymbol{E}}}_{OY}$, are emitted, and also there is a single reflecting object in space, from which the wave is reflected once. Let us write the Jones vectors for direct and singly reflected waves:

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_LOS\_OX} = \boldsymbol{R}(\alpha)\times\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}C_\alpha\\-S_\alpha\end{bmatrix},$$

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_LOS\_OY} = \boldsymbol{R}(\alpha)\times\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}S_\alpha\\C_\alpha\end{bmatrix},$$

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_mp\_OX} = \boldsymbol{R}(\varphi)\times\begin{bmatrix}\dot{k}_1 & 0\\0 & \dot{k}_2\end{bmatrix}\times\boldsymbol{R}(\theta)\times\begin{bmatrix}1\\0\end{bmatrix} =$$

$$= \begin{bmatrix}C_\varphi\dot{k}_1C_\theta - S_\varphi\dot{k}_2S_\theta\\-S_\varphi\dot{k}_1C_\theta - C_\varphi\dot{k}_2S_\theta\end{bmatrix},$$

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_mp\_OY} = \boldsymbol{R}(\varphi)\times\begin{bmatrix}\dot{k}_1 & 0\\0 & \dot{k}_2\end{bmatrix}\times\boldsymbol{R}(\theta)\times\begin{bmatrix}0\\1\end{bmatrix} =$$

$$= \begin{bmatrix}C_\varphi\dot{k}_1S_\theta + S_\varphi\dot{k}_2C_\theta\\-S_\varphi\dot{k}_1S_\theta + C_\varphi\dot{k}_2C_\theta\end{bmatrix}.$$

Thus, the Jones vectors for the sum waves for both cases will look like this:

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_all\_OX} = \begin{bmatrix}C_\varphi\dot{k}_1C_\theta - S_\varphi\dot{k}_2S_\theta + C_\alpha\\-S_\varphi\dot{k}_1C_\theta - C_\varphi\dot{k}_2S_\theta - S_\alpha\end{bmatrix},$$

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_all\_OY} = \begin{bmatrix}C_\varphi\dot{k}_1S_\theta + S_\varphi\dot{k}_2C_\theta + S_\alpha\\-S_\varphi\dot{k}_1S_\theta + C_\varphi\dot{k}_2C_\theta + C_\alpha\end{bmatrix}.$$

It can be seen that when the bases of all elements coincide, i.e., when $\alpha = \varphi = \theta = 0$, in the basis of the receiving antenna, we can get the sum of the direct and reflected waves with the preservation of linear polarization:

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_all\_OX} = \begin{bmatrix}\dot{k}_1 + 1\\0\end{bmatrix},$$

$$\dot{\bar{\boldsymbol{E}}}^{IN}_{rx\_all\_OY} = \begin{bmatrix}0\\\dot{k}_2 + 1\end{bmatrix}.$$

2.2 Using the coherence matrix to estimate the parameters of an electromagnetic wave

There are three main types of wave polarization [2]:

- completely polarized waves;
- absolutely unpolarized waves;
- partially polarized waves.

Fully polarized and absolutely unpolarized waves represent the limiting states of a partially polarized wave. A monochromatic wave is considered to be fully polarized if for a long time (compared to the oscillation period) its components have a constant phase difference and a constant ratio of amplitudes. If the completely polarized states of the monochromatic components of the spectrum can differ in an unpredictable way, then the resulting wave will be partially polarized. With absolute independence (in the statistical sense) of the sequence of completely polarized states, the wave becomes absolutely unpolarized. Matrix $\dot{\boldsymbol{K}}$ is called the coherence matrix of a plane quasi-monochromatic wave [5]:

$$\dot{\boldsymbol{K}} = \left\langle \dot{\bar{\boldsymbol{E}}}(t) \oplus \dot{\bar{\boldsymbol{E}}}^{(+)}(t) \right\rangle =$$

$$= \begin{bmatrix}\left\langle \dot{E}_x(t)\dot{E}_x^\bullet(t)\right\rangle & \left\langle \dot{E}_x(t)\dot{E}_y^\bullet(t)\right\rangle\\\left\langle \dot{E}_y(t)\dot{E}_x^\bullet(t)\right\rangle & \left\langle \dot{E}_y(t)\dot{E}_y^\bullet(t)\right\rangle\end{bmatrix} = \begin{bmatrix}\dot{K}_{xx} & \dot{K}_{xy}\\\dot{K}_{yx} & \dot{K}_{yy}\end{bmatrix}, \quad (9)$$

where

$$\dot{\bar{\boldsymbol{E}}}(t) = \begin{bmatrix}\dot{E}_x(t)\\\dot{E}_y(t)\end{bmatrix} = \begin{bmatrix}\dot{E}_x(t)\exp(j\Phi_x(t))\\\dot{E}_y(t)\exp(j\Phi_y(t))\end{bmatrix}. \quad (10)$$

Expressions for trace $Sp\dot{\boldsymbol{K}}$ and determinant $\det\dot{\boldsymbol{K}}$ coherence matrix are presented below [5]:

$$Sp\dot{\boldsymbol{K}} = \dot{K}_{xx} + \dot{K}_{yy}$$

$$\det\dot{\boldsymbol{K}} = \dot{K}_{xx}\dot{K}_{yy} - \dot{K}_{xy}\dot{K}_{yx}.$$

Using these parameters, it is possible to obtain a quantitative estimate m of the degree of coherent relationship between the components of the vector (10), called the degree of polarization [5]:

$$m = \left[1 - \frac{4\det\dot{\boldsymbol{K}}}{Sp^2\dot{\boldsymbol{K}}}\right]^{0.5}. \quad (11)$$

The degree of polarization of a fully polarized wave is equal to one, and that of an absolutely unpolarized wave is zero.

There is also another parametric way of representing polarization - the Stokes parameters. Below are expressions for obtaining these parameters [5]:

$$S_0 = P_0 = \left|\dot{E}_x\right|^2 + \left|\dot{E}_y\right|^2, \tag{12}$$

$$S_1 = Q = \left|\dot{E}_x\right|^2 - \left|\dot{E}_y\right|^2, \tag{13}$$

$$S_2 = U = \dot{E}_x \dot{E}_y^* + \dot{E}_x^* \dot{E}_y, \tag{14}$$

$$S_3 = V = i\left(\dot{E}_x \dot{E}_y^* - \dot{E}_x^* \dot{E}_y\right). \tag{15}$$

In this case, these parameters are related to the degree of wave polarization as follows [5]:

$$Q^2 + U^2 + V^2 = m^2 P_0^2 \le P_0^2. \tag{16}$$

In addition, using these parameters, it is possible to determine the phasor parameters γ and δ [5]:

$$Q = mP_0 \cos 2\gamma, \tag{17}$$

$$U = mP_0 \sin 2\gamma \cos 2\delta, \tag{18}$$

$$V = mP_0 \sin 2\gamma \sin 2\delta. \tag{19}$$

Where can you get the following expressions:

$$\cos 2\gamma = \frac{Q}{\sqrt{Q^2 + U^2 + V^2}}, \tag{20}$$

$$tg 2\delta = \frac{V}{U}. \tag{21}$$

Also, using the Stokes parameters, one can determine the parameters of the correlation coefficient of the orthogonal wave components [5]:

$$|\mu| = \sqrt{\frac{U^2 + V^2}{P_0^2 - Q^2}}, M = \pi - \arctan\left(\frac{V}{U}\right). \tag{22}$$

From the analysis of statements indirectly related to this, it can be assumed that the sum of two completely polarized waves forms a partially polarized wave if:

- angle between components Ex1 and Ex2 of these waves is not equal to 0° or 90° (otherwise it will turn out to be completely linear or completely elliptical, respectively),
- waves correspond to the same signal, but at the receiving point they have a different delay and phase

If this assumption is correct, then as a result of re-reflections, the total wave in the basis of the receiving antenna is composed of a set of fully or partially polarized waves and, thus, is also partially polarized. This means that in this situation it is possible to obtain an estimate of the degree of polarization from its coherence matrix estimate $\hat{m}$. Then this estimate $\hat{m}$ will tend to 1 when the reflected signals are small or absent, i.e. when at the input of the receiving antenna there is only a forward-propagating beam, which is always fully polarized. In the case when the reflected signals are large, the estimate will tend to zero. Based on this statement, two more interesting thoughts can be formulated:

- With multiple reflection, as mentioned above, the wave is depolarized. In the limiting case, such a re-reflected wave will become absolutely unpolarized [2], i.e. as a result, noise will be added to the direct propagation beam, the distribution law of which tends to normal. At the same time, it will turn out to be uncorrelated on different orts of the receiving antenna.
- When forming a set of reflected signals, it may turn out that they will have similar distribution laws. As is known from the central limit theorem [7], the distribution law of the total signal from such a set will tend to normal as the number of reflected signals increases.

These ideas make it possible to synthesize optimal algorithms for estimating the phase of the carrier frequency for some simplified conditions.

3. SYNTHESIS OF OPTIMAL ALGORITHMS FOR ESTIMATING THE PHASE OF THE CARRIER FREQUENCY UNDER CONDITIONS OF STRONG MULTIPATH AND MULTIPLE SIGNAL RE-REFLECTIONS

3.1. Optimal estimation of the phase of the carrier frequency of the signal in the presence of N receiving channels

From [6,10], with a known signal structure and carrier frequency, it is possible to obtain an optimal algorithm for estimating the signal parameter under the influence of additive interference. Assume that the receiver contains a single complex signal at the input:

$$y(t) = s(t,\lambda) + n(t).$$

Here $s(t, \lambda)$ - useful signal depending on time t and estimated parameter λ, $n(t)$ – noise. Imagine that this analog signal is uniformly sampled with a step ΔT:

$$y_i = s_i(\lambda) + n_i, \tag{23}$$

where $s_i(\lambda) = s(t_i,\lambda)$ – useful signal at time t_i, dependent from estimated parameter λ, $n_i = n(t_i)$ – noise value at time t_i. Values $i = 1, ..., M$ constitute an ensemble of discrete values.

Based on [10], provided that the parameter постоянен, one can write the likelihood function:

$$L(\lambda) = p(y_1,..., y_M \mid \lambda). \tag{24}$$

Now imagine that the receiver contains N inputs, each of which has independent noise. Таким образом, Thus, the overall ensemble contains $(M \cdot N)$ independent noise samples. Then, in accordance with the property of the multidimensional probability density [7], we can write expression (24) as follows:

$$\begin{aligned} L(\lambda) &= p(y_1,..., y_M \mid \lambda) = \\ &= p_1(y_1^{(1)},..., y_M^{(1)} \mid \lambda) \cdot ... \cdot p_1(y_1^{(N)},..., y_M^{(N)} \mid \lambda), \end{aligned} \tag{25}$$

where $y_1, ..., y_{M \cdot N}$ – general ensemble obtained with N receiver inputs, $y_1^{(j)},..., y_M^{(j)}$ – ensemble received from the j-th input of the receiver.

Since the noise is additive [10], and also in the case when it has a normal distribution law at each input j, (25) can be written in the following form:

$$L(\lambda) = \eta \prod_{j=1}^{N} \exp\left\{ -\left[\sum_{i=1}^{M} \left(y_i^{(j)} - s_i(\lambda) \right)^2 / 2\sigma_j^2 \right] \right\}, \tag{26}$$

where σ_j^2 – additive noise variance in the jth channel, η – normalized constant.

With a uniform spectral density band ΔF of the process $n(t)$ uncorrelated samples are obtained, as is known, at $\Delta T = (1/2)\Delta F$. For a different form of the spectrum, the samples turn out to be practically uncorrelated if we take $\Delta t > \tau_c$, where τ_c, – process $n(t)$ correlation period [6].

For a function with a limited band, its values at times separated by an interval $\Delta T = (1/2)\Delta F$, represent the coefficients of the expansion in terms of the Kotelnikov functions, and the more accurate the expansion, the greater the product of the observation interval T to the width of the band $m = T\Delta F$. Pushing the band of noise to infinity, $\Delta F \to \infty$, $m \to \infty$, using Parseval's equality and taking into account that $\sigma_j^2 = G_o^{(j)} \Delta F = G_o^{(j)} / 2\Delta T$, we obtain the likelihood function of the useful signal when observed against the background of white noise with spectral density $G_o^{(j)}$ in every channel:

$$L(\lambda) = \eta \prod_{j=1}^{N} \exp\left\{ -\frac{1}{G_o^{(j)}} \int_0^T \left(y(t)^{(j)} - s(t,\lambda) \right)^2 dt \right\}, \tag{27}$$

Let us expand further the integrand:

$$\begin{aligned} L(\lambda) &= \\ &= \eta \prod_{j=1}^{N} \exp\left\{ \frac{1}{2\sigma_j^2} \left[\sum_{i=1}^{M} \left(-\left[y_i^{(j)} \right]^2 + 2 y_i^{(j)} s_i(\lambda) - s_i^2(\lambda) \right) \right] \right\}, \end{aligned} \tag{28}$$

$$\begin{aligned} L(\lambda) = \eta \prod_{j=1}^{N} \exp\{ \frac{1}{G_0^{(j)}} \int_0^T (-[y(t)^{(j)}]^2 + \\ +2 y^{(j)} s(t,\lambda) - s^2(t,\lambda)) dt \}, \end{aligned} \tag{29}$$

Since, in this implementation, under the mixtures of the components $\left[y_i^{(j)} \right]^2$ in (28) and $\left[y(t)^{(j)} \right]^2$ in (29) do not depend on λ, they can be introduced into a constant normalizing factor, after which (28) and (29) can be written as follows:

$$L(\lambda) = \eta \prod_{j=1}^{N} \exp\left\{ \frac{1}{2\sigma_j^2} \left[\sum_{i=1}^{M} \left(2 y_i^{(j)} s_i(\lambda) - s_i^2(\lambda) \right) \right] \right\}, \tag{30}$$

$$L(\lambda) = \eta \prod_{j=1}^{N} \exp\left\{ \frac{1}{G_o^{(j)}} \int_0^T \left(2 y(t)^{(j)} s(t,\lambda) - s^2(t,\lambda) \right) dt \right\}. \tag{31}$$

Let us rewrite expression (31) as follows:

$$L(\lambda) = \eta \exp\left(\sum_{j=1}^{N} \left[\frac{1}{G_o^{(j)}} \int_0^T \left(2 y(t)^{(j)} s(t,\lambda) - s^2(t,\lambda) \right) dt \right] \right). \tag{32}$$

The integral of the difference can be represented as the difference of the integrals, and the summation operation can also be performed separately for the integrands. So we get

$$\begin{aligned} L(\lambda) = \eta \exp\Bigg(2 \sum_{j=1}^{N} \left[\frac{1}{G_o^{(j)}} \int_0^T \left(y(t)^{(j)} s(t,\lambda) \right) dt \right] - \\ - \sum_{j=1}^{N} \left[\frac{1}{G_o^{(j)}} \int_0^T \left(s^2(t,\lambda) \right) dt \right] \Bigg) \end{aligned} \tag{33}$$

or

$$L(\lambda) = \eta \exp\left(2q(\lambda) - E(\lambda)\right), \quad (34)$$

where

$$q(\lambda) = \sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(y(t)^{(j)} s(t,\lambda)\right)dt\right],$$

$$E(\lambda) = \sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(s^2(t,\lambda)\right)dt\right].$$

In the general case, the desired phase estimation algorithm could be found by the maximum likelihood method by solving the following equation:

$$\frac{\partial L(\lambda)}{\partial \lambda} = 0.$$

However, in the case of using the PNS as a useful signal $PN(t,F,\tau)$, where F – code frequency, τ – code delay, Ω – carrier frequency, the expression describing such a signal is as follows:

$$s(t) = aPN(t,F,\tau)\cos(\Omega t - \varphi).$$

Then, according to [6,9], the phase φ is a non-energy parameter, and in order to obtain an algorithm for its estimation, it is only necessary to find the maximum of the correlation term $q(\lambda)$, since with the chosen type of signal modulation

$$E(\varphi) = \sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(aPN(t,F,\tau)\cos(\Omega t - \varphi)\right)^2 dt\right] \to$$

$$\to \sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \frac{\left(aPN(t,F,\tau)\right)^2}{2} dt\right],$$

that is, does not depend on φ.

Since in this case we are interested in the optimal algorithm for estimating the phase of the carrier frequency, we assume that other parameters are known, namely PN code delay τ и frequency , and carrier frequency . Then the correlation term can be represented as follows:

$$q(\varphi) = a\sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(y(t)^{(j)} PN(t,F,\tau)\cos(\Omega t - \varphi)\right)dt\right]. \quad (35)$$

Before directly searching for the maximum, we expand the cosine of the difference according to the well-known trigonometric formula:

$$q(\varphi) = a\sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(\cos(\varphi) y(t)^{(j)} PN(t,F,\tau)\cos(\Omega t) + \right.\right.$$
$$\left.\left. + \sin(\varphi) y(t)^{(j)} PN(t,F,\tau)\sin(\Omega t)\right)dt\right].$$

We take out the constant parameters from under the sign of the integral and the sum, and also expand the sum integral into the sum of integrals:

$$q(\varphi) =$$
$$= a\left\{\begin{array}{l}\cos(\varphi)\sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(y(t)^{(j)} PN(t,F,\tau)\cos(\Omega t)\right)dt\right] + \\ + \sin(\varphi)\sum_{j=1}^{N}\left[\frac{1}{G_o^{(j)}}\int_0^T \left(y(t)^{(j)} PN(t,F,\tau)\sin(\Omega t)\right)dt\right]\end{array}\right\}. \quad (36)$$

Let's make a change of variables:

$$X_j = \int_0^T \left[y(t)^{(j)} PN(t,F,\tau)\cos(\Omega t)\right]dt,$$

$$Y_j = \int_0^T \left[y(t)^{(j)} PN(t,F,\tau)\sin(\Omega t)\right]dt.$$

Then expression (36) can be represented in a more visual form:

$$q(\varphi) = a\left\{\cos(\varphi)\sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}} + \sin(\varphi)\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}}\right\}. \quad (37)$$

Taking the partial derivative of φ is no longer a problem:

$$\frac{\partial q}{\partial \varphi} = a\left\{-\sin(\varphi)\sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}} + \cos(\varphi)\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}}\right\} = 0.$$

Thus:

$$\cos(\varphi)\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}} = \sin(\varphi)\sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}},$$

$$\frac{\sin(\varphi)}{\cos(\varphi)} = \sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}} / \sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}} = tg(\varphi). \quad (38)$$

Then from (38) one can obtain an algorithm for the optimal phase estimation for the case under consideration:

$$\varphi^* = arctg\left(\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}} / \sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}}\right) + p\pi,$$
$$p = -\frac{1}{2}\left[sign\left(\sum_{j=1}^{N}\frac{X_j}{G_0^{(j)}}\right)sign\left(\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}}\right) - sign\left(\sum_{j=1}^{N}\frac{Y_j}{G_0^{(j)}}\right)\right], \quad (39)$$

where the parameter p is needed to estimate the phase, which takes values from π to 2π.

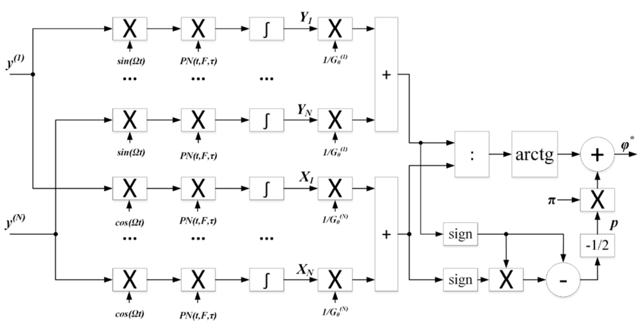

Fig. 2. *Functional diagram of the optimal phase estimation system.*

Also in **Fig. 2**, we can depict the functional diagram of the optimal phase estimation system (39) as follows.

It should be emphasized that the synthesized phase estimation algorithm is optimal only in the case when the reflected wave, or the sum of the reflected waves, has completely lost its polarization properties. Let us further consider the situation when the wave remains partially polarized.

3.2 Optimal carrier phase estimation in the presence of two correlated noise components

Since the case when the reflected waves completely lose their polarization properties is the limiting case of strong multipath, it is logical to consider the situation when the reflected wave remains partially polarized, i.e. interference in different receiving channels will be correlated. However, in order not to immediately overcomplicate the problem, we assume that the signal belonging to the direct propagation beam is, for some reason, not correlated with the reflected.

Let us assume that the receiver may contain two inputs, each of which contains the same useful signal, but the effect of interference is correlated. In this case, the noise y will be described by the two-dimensional probability density $p_2(\lambda_1,\lambda_2)$, then you can get the following likelihood function for a mixture of z signal and noise in the n-th channel:

$$L = L(\lambda) = p(\boldsymbol{y}^{(1)}, \boldsymbol{y}^{(2)} \mid \lambda), \tag{40}$$

implying that $\boldsymbol{y}^{(j)} = \left[\boldsymbol{y}_1^{(j)}, ..., \boldsymbol{y}_M^{(j)}\right]$ – is the noise ensemble of in the j-th channel.

In this case, relying on [6,7,8], with additive correlated noise, the likelihood function can be written in the following form:

$$L(\lambda) = \frac{1}{|K|\sqrt{4\pi^2}} \times$$
$$\times \exp\left(\frac{1}{2|K|} \begin{vmatrix} k_{11} & k_{12} & \sum\limits_{i=1}^{M}\left(y_i^{(1)} - s_i(\lambda)\right) \\ k_{21} & k_{22} & \sum\limits_{i=1}^{M}\left(y_i^{(2)} - s_i(\lambda)\right) \\ \sum\limits_{i=1}^{M}\left(y_i^{(1)} - s_i(\lambda)\right) & \sum\limits_{i=1}^{M}\left(y_i^{(2)} - s_i(\lambda)\right) & 0 \end{vmatrix} \right), \tag{41}$$

where

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

- noise correlation matrix, and $k_{12} = k_{21} = k$.

Next, we can get the following expression for the exponent:

$$\frac{1}{2|K|}\left[\begin{array}{l} 2k_{22}\sum\limits_{i=1}^{M}\left(y_i^{(1)}s_i(\lambda)\right) + 2k_{11}\sum\limits_{i=1}^{M}\left(y_i^{(2)}s_i(\lambda)\right) \\ -2k\sum\limits_{i=1}^{M}\left(y_i^{(1)}s_i(\lambda) + y_i^{(2)}s_i(\lambda)\right) + 2k\sum\limits_{i=1}^{M}\left(y_i^{(1)}y_i^{(2)}\right) - \\ -k_{22}\sum\limits_{i=1}^{M}\left(y_i^{(1)}\right)^2 - k_{11}\sum\limits_{i=1}^{M}\left(y_i^{(2)}\right)^2 + 2k\sum\limits_{i=1}^{M}\left(s_i(\lambda)\right)^2 \\ -k_{22}\sum\limits_{i=1}^{M}\left(s_i(\lambda)\right)^2 - k_{11}\sum\limits_{i=1}^{M}\left(s_i(\lambda)\right)^2 \end{array}\right]. \tag{42}$$

Further, by analogy with (31), we can take out into the normalizing factor the components that do not depend on λ, and get an expression like:

$$L(\lambda) = \eta \exp\left(\frac{1}{2|K|}\left(q(\lambda) - E(\lambda)\right) \right), \tag{43}$$

where

$$q(\lambda) = 2k_{22}\sum_{i=1}^{M}\left(y_i^{(1)}s_i(\lambda)\right) + 2k_{11}\sum_{i=1}^{M}\left(y_i^{(2)}s_i(\lambda)\right) - $$
$$-2k\sum_{i=1}^{M}\left(y_i^{(1)}s_i(\lambda) + y_i^{(2)}s_i(\lambda)\right), \tag{44}$$

$$E(\lambda) = \left[k_{22} + k_{11} - 2k\right]\sum_{i=1}^{M}\left(s_i(\lambda)\right)^2, \tag{45}$$

or in case when $\Delta F \to \infty$, by analogy with (30):

$$q(\lambda) = 2k_{22}\int_0^T\left(y(t)^{(1)}s(t,\lambda)\right)dt +$$
$$+2k_{11}\int_0^T\left(y(t)^{(2)}s(t,\lambda)\right)dt - \tag{46}$$
$$-2k\int_0^T\left(y(t)^{(1)}s(t,\lambda) + y(t)^{(2)}s(t,\lambda)\right)dt,$$

$$E(\lambda)=\left[k_{22}+k_{11}-2k\right]\int_0^T\left(s(t,\lambda)\right)^2 dt. \qquad (47)$$

According to [6], the phase is a non-energy parameter, and in order to obtain an algorithm for estimating it, it is necessary to find the maximum of the correlation term $q(\lambda)$.

In the case of use as a useful signal PNS $PN(t,F,\tau)$, where F – code frequency, τ – code delay, Ω – carrier frequency, The expression describing such a signal looks like this:

$$s(t)=aPN(t,F,\tau)\cos(\Omega t-\varphi).$$

Since in this case we are interested in the optimal algorithm for estimating the phase, we assume that other parameters are known, namely PN code delay τ and frequency F, and carrier frequency Ω. Then the correlation term can be represented as follows:

$$q(\varphi)=a\begin{bmatrix} 2k_{22}\int_0^T\left(y(t)^{(1)}PN(t,F,\tau)\cos(\Omega t-\varphi)\right)dt+ \\ +2k_{11}\int_0^T\left(y(t)^{(2)}PN(t,F,\tau)\cos(\Omega t-\varphi)\right)dt- \\ -2k\int_0^T\begin{pmatrix} y(t)^{(1)}PN(t,F,\tau)\cos(\Omega t-\varphi)+ \\ +y(t)^{(2)}PN(t,F,\tau)\cos(\Omega t-\varphi)\end{pmatrix}dt\end{bmatrix}. \qquad (48)$$

Before directly searching for the maximum, we expand the cosine of the sum according to the well-known trigonometric formula, expanding the sum integral into the sum of integrals:

$$q(\varphi)=$$
$$=a\begin{bmatrix} 2k_{22}\left(\cos\varphi X_1+\sin\varphi Y_1\right)+ \\ +2k_{11}\left(\cos\varphi X_2+\sin\varphi Y_2\right)- \\ -2k\left(\cos\varphi X_1+\sin\varphi Y_1+\cos\varphi X_2+\sin\varphi Y_2\right)\end{bmatrix},$$

where, for j = 1,2:

$$X_j=\int_0^T\left(y(t)^{(j)}PN(t,F,\tau)\cos(\Omega t)\right)dt,$$

$$Y_j=\int_0^T\left(y(t)^{(j)}PN(t,F,\tau)\sin(\Omega t)\right)dt.$$

Let us now take the partial derivative on φ:

$$\frac{\partial q}{\partial\varphi}=$$
$$=2a\begin{bmatrix} k_{22}(-\sin\varphi X_1+\cos\varphi Y_1)+ \\ +k_{11}(-\sin\varphi X_2+\cos\varphi Y_2)- \\ -k(-\sin\varphi X_1+\cos\varphi Y_1-\sin\varphi X_2+\cos\varphi Y_2)\end{bmatrix}=0$$

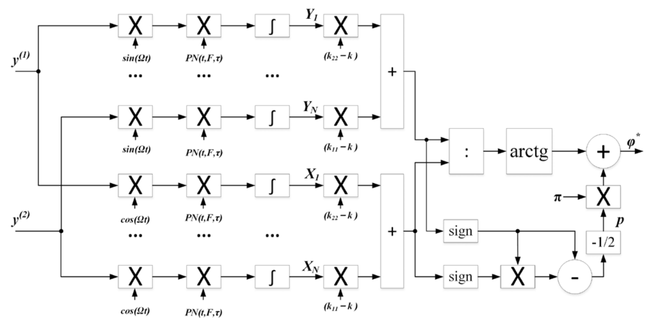

Fig. 3. *Functional diagram of the optimal phase estimation system.*

Select the elements depending on φ:

$$\sin\varphi\left(-k_{22}X_1-k_{11}X_2+k(X_1+X_2)\right)+$$
$$+\cos\varphi\left(k_{22}Y_1+k_{11}Y_2-k(Y_1+Y_2)\right)=0,$$

$$tg(\varphi)=\frac{\sin(\varphi)}{\cos(\varphi)}=\frac{k_{22}Y_1+k_{11}Y_2-k(Y_1+Y_2)}{k_{22}X_1+k_{11}X_2-k(X_1+X_2)}=$$
$$=\frac{Y_1\left(k_{22}-k\right)+Y_2\left(k_{11}-k\right)}{X_1\left(k_{22}-k\right)+X_2\left(k_{11}-k\right)}. \qquad (49)$$

Then from (49) one can obtain an algorithm for the optimal phase estimation for the case under consideration:

$$\varphi^*=arctg\left(\frac{Y_1\left(k_{22}-k\right)+Y_2\left(k_{11}-k\right)}{X_1\left(k_{22}-k\right)+X_2\left(k_{11}-k\right)}\right)+z\pi,$$
$$z=-\frac{1}{2}\begin{pmatrix} sign\left(X_1\left(k_{22}-k\right)+X_2\left(k_{11}-k\right)\right)\times \\ \times sign\left(Y_1\left(k_{22}-k\right)+Y_2\left(k_{11}-k\right)\right)- \\ -sign\left(Y_1\left(k_{22}-k\right)+Y_2\left(k_{11}-k\right)\right)\end{pmatrix}. \qquad (50)$$

In **Fig. 3**, we can depict the functional diagram of the optimal phase estimation system (50) as follows.

3.3. Investigation of characteristics of synthesized phase estimation algorithms

To study the characteristics of the algorithm (39), a simulation model was created in the MATLAB environment using the Quadriga multipath simulator. The standardized variants 3GPP-38.901-Indoor-LOS and WINNER-Indoor-A1-LOS were chosen as multipath scenarios. The receiver moved linearly in the direction away from the transmitter. On the transmitting and receiving sides, antennas with orthogonal linear polarization are implemented, which make it possible to

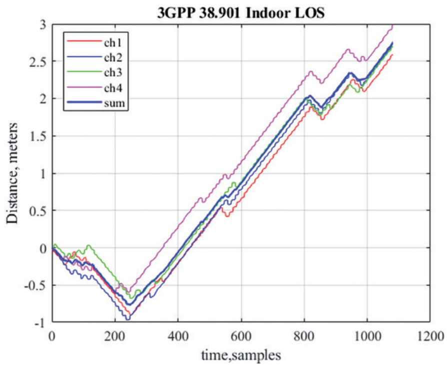

Fig. 4. *The result of modeling the algorithm (39) under the scenario 3GPP 38.901 Indoor LOS.*

obtain 4 tracking channels due to their various combinations:

- tx-1 → rx-1 – channel 1,
- tx-1 → rx-2 – channel 2,
- tx-2 → rx-1 – channel 3,
- tx-2 → rx-2 – channel 4.

The simulation results are shown in **Fig. 4, 5**.

Based on the simulation results, it can be seen that algorithm (39) does not receive more than the total effect, and the number of cycle slips in the phase estimates turns out to be some average. From this we can conclude that the formulation of the synthesis problem was oversimplified.

4. EMPIRICAL MULTIPATH MITIGATION ALGORITHM

Considering the large number of cycle slips in the estimation of the phase of a single signal, an attempt was made to create redundancy in the emitted signal itself. Since in this frequency range within the band up to 50 MHz the multipath is

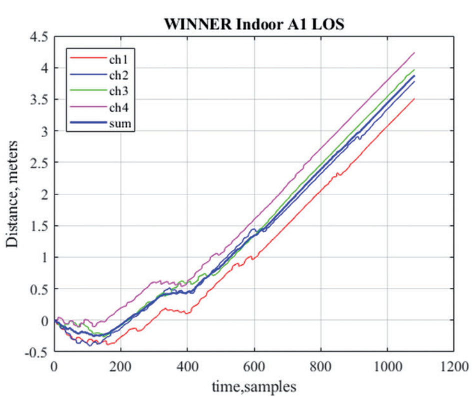

Fig. 5. *The result of modeling the algorithm (39) under the scenario WINNER Indoor A1 LOS.*

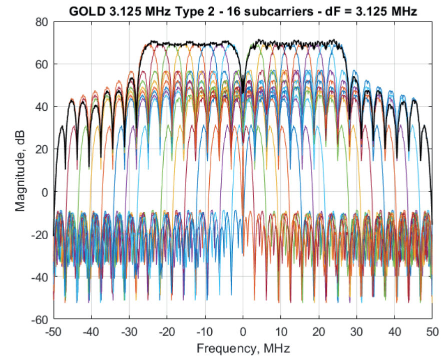

Fig. 6. *Spectral representation of the signal. Each subcarrier is modulated with the same Gold code (indicated in different colors), the result corresponds to the sum of the subcarriers (indicated in black).*

frequency-selective, an OFDM signal with Gold's PSS was created. The spectral representation of this signal is shown in **Fig. 6**. In addition, to further increase the redundancy, all combinations available from orthogonal polarization at transmitters and receivers were also used.

Phase estimation when using this signal was made independently for each subcarrier, using the same algorithms as described in [1]. However, it was noticed that when converting the phase estimates to meters, it is possible to implement an algorithm for detecting jumps in the estimates. To do this, it is necessary to calculate the phase difference between all possible pairs of subcarriers, and then, for each time point, select an estimate of the phase of the subcarrier that is not currently multipathed. These operations must be performed for each transmitter independently. In this way, estimates of the phase increments can be obtained, from which most of the jumps will be excluded. On **Fig. 7** shows the difference in phase estimates between subcarriers. It can be seen that this parameter contains only noise and jumps, since the influence

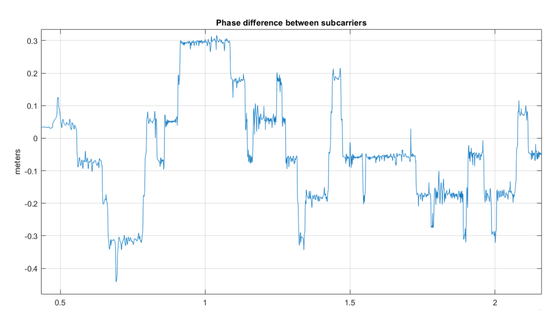

Fig. 7. *Difference between subcarrier phase estimates.*

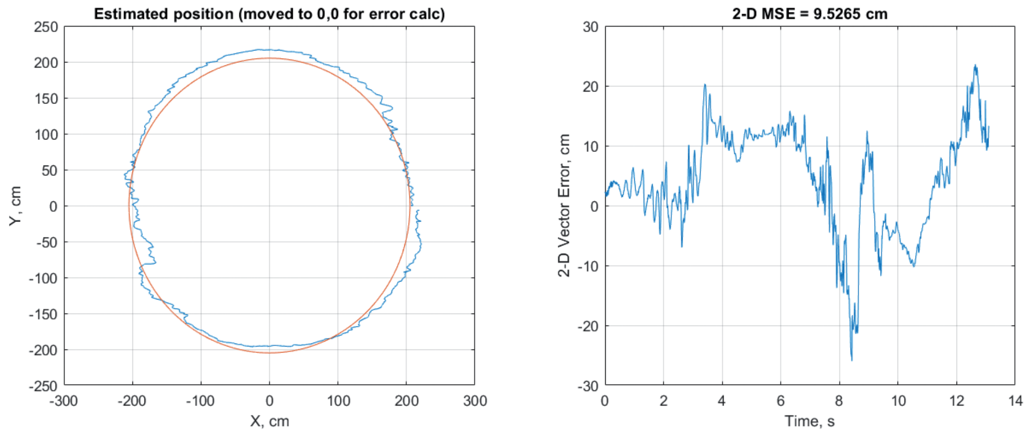

Fig. 8. *The result of the evaluation of the position and the standard deviation of the positioning error on the first lap.*

of the RF part, movement, and other hardware effects are the same on both subcarriers and are mutually exclusive. A detailed description of this algorithm is presented in [11,12,13].

As a result of applying this algorithm to the experimental data, it was possible to achieve an RMS estimate of the 2-D position of the order of 10 cm (see **Fig. 8-9**).

5. CONCLUSION

The paper presents a technique for describing reflected signals using the Jones vector, as well as the synthesis of optimal algorithms for estimating signal parameters under complex conditions for simplified scenarios. With the help of modeling, it is shown that the synthesized optimal algorithms are poorly applicable to real situations, however, they allow a better understanding of the physics of processes. Based on this understanding, a signal shaping technique is proposed to create a large redundancy of measurements in the receiver, as well as an empirical method for rejecting false measurements. With the help of this method, it has been experimentally shown that indoors in difficult conditions and in the presence of multipath, it is possible to achieve the RMS positioning error of less than 10 centimeters.

In addition, the conducted research allowed to achieve several additional results:

1. A system was created that allows one to form complex signal structures and evaluate the quality of the phase estimate.
2. An understanding has been reached of exactly what effects multipath has on signals of different frequencies and polarizations.
3. It was possible to experimentally show that, using the navigation signal with OFDM, it is possible to achieve good results in eliminating jumps from phase estimates. Thus, it is probably possible to formulate the problem of synthesizing a hop-free phase estimation algorithm using a similar navigation signal with frequency and polarization multiplexing,

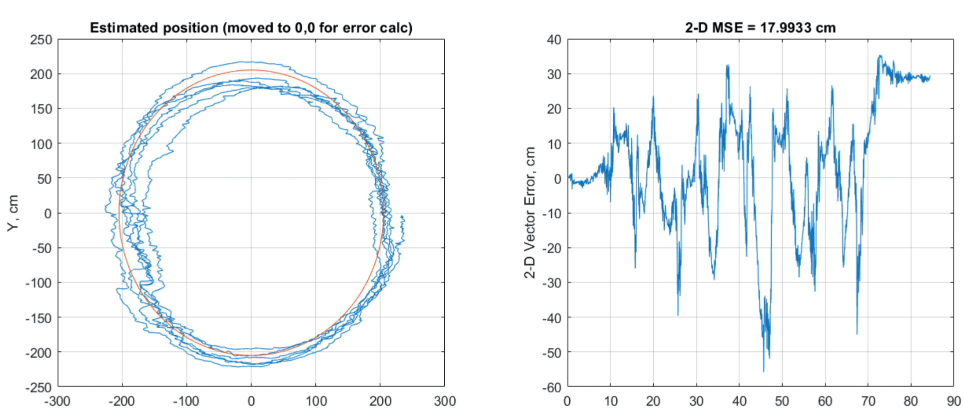

Fig. 9. *The result of the assessment of the position and RMS of the location error for the entire time of movement.*

which can potentially solve the problem of relative positioning with centimeter accuracy in difficult conditions.

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