DOI: 10.17725/rensit.2022.14.187

Computer simulation of unsteady elastic stress waves in a console and a ten-storey building under fundamental influence in the form of a Heaviside function

[1,2]Vyacheslav K. Musayev

[1]Moscow State University of Civil Engineering, https://mgsu.ru/
Moscow 129337, Russian Federation
[2]Russian University of Transport, https://www.miit.ru/
Moscow 127994, Russian Federation
E-mail: musayev-vk@yandex.ru

Received February 14, 2022, peer-reviewed February 21, 2022, accepted March 1, 2022

Abstract: **The aim of the work is to consider the problems of numerical simulation of seismic safety of a console and a ten-storey building with a base in the form of an elastic half-plane under non-stationary wave influences. Modeling of tasks of the transition period is an actual fundamental and applied scientific task. Methodology. To solve a two-dimensional plane dynamic problem of elasticity theory with initial and boundary conditions, the finite element method in displacements is used. Based on the finite element method in displacements, an algorithm and a set of programs have been developed for solving linear planar two-dimensional problems that allow solving problems with non-stationary wave effects on complex systems. The algorithmic language Fortran-90 was used in the development of the software package. Результаты. Results. The problem of the effect of a plane longitudinal wave in an elastic half-plane in the form of four trapezoids and in the form of two half-periods of a sinusoid is considered to assess the physical reliability and mathematical accuracy. A system of equations consisting of 8016008 unknowns is solved. The problem of the effect of a plane longitudinal elastic wave in the form of a Heaviside function on a console with a base (the ratio of width to height is one to ten) is considered. A system of equations consisting of 16016084 unknowns is solved. The problem of the effect of a plane longitudinal elastic wave in the form of a Heaviside function on a ten-storey building with a base in the form of a half-plane is considered. A system of equations consisting of 16202276 unknowns is solved. Contour stresses and components of the stress tensor are obtained in the characteristic areas of the problem under study. Based on the conducted research, the following conclusions can be drawn. Elastic contour stress on the sides of the console and a ten-story building are almost a mirror image of each other, that is, antisymmetric. The console and supporting structures of the building work like a beam during seismic action, that is, if there are tensile stresses on one side, then compressive stresses on the other. Bending waves mainly prevail on the contours of the console and supporting structures of the building under seismic influence.**

Keywords: **mathematical modeling, wave theory of seismic safety, seismic impact, fundamental impact, Heaviside function, verification of the numerical method, console, ten-storey building, elastic half-plane, contour stress, bending waves**

UDC 539.3

For citation: Vyacheslav K. Musayev. Computer simulation of unsteady elastic stress waves in a console and a ten-storey building under fundamental influence in the form of a Heaviside function. *RENSIT: Radioelectronics. Nanosystems. Information technologies*, 2022, 14(2):187-196e. DOI: 10.17725/rensit.2022.14.187.

Contents

1. INTRODUCTION

The aim of the work is to consider the problems of numerical modeling of the seismic safety of a cantilever and a ten-story building with a base in the form of an elastic half-plane under non-stationary wave actions. Modeling the problems of the transition period is an actual fundamental and applied scientific problem. In this work, to solve a two-dimensional plane dynamic problem of elasticity theory with initial and boundary conditions, the finite element method in displacements is used.

The formulation of some dynamic problems of the mechanics of a deformable solid body is given in [1-15].

Unique objects are subject to seismic impacts [6-7,12-15]. The safety of unique objects from seismic impacts is an urgent fundamental and applied scientific problem [6-7,12-15].

However, the reliability of stress waves in deformable bodies in published scientific papers is not presented enough for a correct assessment from the point of view of known knowledge about stress waves. On the other hand, due to scarce information about the seismic wave impact on the object under study, there may be different approaches and impact models [6-7,12-15].

In this paper, one of the approaches to solving the problem in the field of the wave theory of seismic safety is presented. Verification (physical reliability and mathematical accuracy) of the developed technique, algorithm and software package is considered in solving the problem of the propagation of plane longitudinal waves in the form of four trapezoids and two half-periods of a sinusoid in an elastic half-plane.

A numerical solution is given to the problem of modeling non-stationary seismic waves in a cantilever (the ratio of width to height is one to ten) and a ten-story building with a base in the form of a half-plane.

Transient processes are very important for assessing the safety of complex technical systems. The main stress state is formed during a transient process, that is, a non-stationary wave process. Therefore, the development of a methodology, algorithm and a set of programs for solving the problem posed is an urgent scientific task [6-11,14].

The application of the considered numerical method, algorithm and software package in solving non-stationary wave problems in deformable bodies is given in the following works [6-11,14]. Verification (assessment of accuracy and reliability) of the considered numerical method, algorithm and software package is given in the following works [6-11].

2. PROBLEM STATEMENT

To solve the problem of modeling elastic non-stationary stress waves in deformable regions of complex shape, we consider a certain body in a rectangular Cartesian coordinate system XOY, which is subjected to a mechanical non-stationary impulse influence at the initial time $t = 0$ [1-4,6-7,14].

Let us assume that some body is made of a homogeneous isotropic material obeying Hooke's elastic law at small elastic deformations [1-4,6-7,14]. The exact equations of the two-dimensional (plane stress state) dynamic theory of elasticity have the following form [1-4,6-7,14]:

$$\begin{gathered}
\frac{\partial \sigma_x}{\partial X} + \frac{\partial \tau_{xy}}{\partial Y} = \rho \frac{\partial^2 u}{\partial t^2}, \\
\frac{\partial \tau_{yx}}{\partial X} + \frac{\partial \sigma_y}{\partial Y} = \rho \frac{\partial^2 v}{\partial t^2}, \ (x, y) \in \Gamma, \\
\sigma_x = \rho C_p^2 \varepsilon_x + \rho (C_p^2 - 2C_s^2) \varepsilon_y, \\
\sigma_y = \rho C_p^2 \varepsilon_y + \rho (C_p^2 - 2C_s^2) \varepsilon_x, \\
\tau_{xy} = \rho C_s^2 \gamma_{xy}, \ \varepsilon_x = \frac{\partial u}{\partial X}, \ \varepsilon_y = \frac{\partial v}{\partial Y}, \\
\gamma_{xy} = \frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X}, \ (x, y) \in (\Gamma \bigcup S),
\end{gathered} \tag{1}$$

where: σ_x, σ_y and τ_{xy} are the components of the elastic stress tensor; εx, ε_y and γ_{xy} are the components of the elastic strain tensor; u and v are the components of the elastic displacement vector along the OX and OY axes, respectively; ϱ is the density of the material; $C_p = \sqrt{\dfrac{E}{\rho(1-\nu^2)}}$ is the velocity of the longitudinal elastic wave; $C_s = \sqrt{\dfrac{E}{2\rho(1+\nu)}}$ is the velocity of the transverse elastic wave; ν is Poisson's ratio; E is the modulus of elasticity; $S\ (S_1 \cup S_2)$ – the boundary contour of the body G.

System (1) in the area occupied by the body Γ should be integrated under initial and boundary conditions [1-4,6-7,14].

3. DEVELOPMENT OF THE METHOD AND ALGORITHM

To solve a two-dimensional plane dynamic problem of elasticity theory with initial and boundary conditions (1), we use the finite element method in displacements [6-7,14].

Taking into account the definition of the stiffness matrix, the vector of inertia and the vector of external forces for the body Γ, we write down the approximate value of the equation of motion in the theory of elasticity [6-7,14]:

$$\boldsymbol{H}\ddot{\boldsymbol{\Phi}} + \boldsymbol{K}\boldsymbol{\Phi} = \boldsymbol{R},\ \boldsymbol{\Phi}\big|_{t=0} = \boldsymbol{\Phi}_0,\ \dot{\boldsymbol{\Phi}}\big|_{t=0} = \dot{\boldsymbol{\Phi}}_0, \tag{2}$$

where: H is the diagonal matrix of inertia; K is the stiffness matrix; $\boldsymbol{\Phi}$ is the vector of nodal elastic displacements; $\dot{\boldsymbol{\Phi}}_0$ is the vector of nodal elastic displacement velocities; $\ddot{\boldsymbol{\Phi}}$ is the nodal elastic accelerations vector; R is the vector of external nodal elastic forces.

To integrate equation (2) with a finite element version of the Galerkin method, we bring it to the following form [6-7,14]:

$$\boldsymbol{H}\frac{d}{dt}\dot{\boldsymbol{\Phi}} + \boldsymbol{K}\boldsymbol{\Phi} = \boldsymbol{R},\ \frac{d}{dt}\boldsymbol{\Phi} = \dot{\boldsymbol{\Phi}}. \tag{3}$$

Integrating relation (3) over the time coordinate using the finite element version of the Galerkin method, we obtain an explicit two-layer scheme for internal and boundary nodal points [6-7,14]:

$$\begin{aligned} \dot{\boldsymbol{\Phi}}_{i+1} &= \dot{\boldsymbol{\Phi}}_i + \Delta t \boldsymbol{H}^{-1}(-\boldsymbol{K}\boldsymbol{\Phi}_i + \boldsymbol{R}_i), \\ \boldsymbol{\Phi}_{i+1} &= \boldsymbol{\Phi}_i + \Delta t \dot{\boldsymbol{\Phi}}_{i+1}. \end{aligned} \tag{4}$$

The main relations of the finite element method in displacements are obtained using the principle of possible displacements and the finite element version of the Galerkin method [6-7,14]. The general theory of numerical equations of mathematical physics requires for this the imposition of certain conditions on the ratio of steps along the time coordinate Δt and along the spatial coordinates, namely [6-7,14]:

$$\Delta t = k\frac{\min \Delta l_i}{C_p},\ (i = 1, 2, 3, ...), \tag{5}$$

where Δl is the length of the side of the finite element.

The results of the numerical experiment showed that, at k = 0.5, the stability of the explicit two-layer scheme is ensured for internal and boundary nodal points on quasi-regular grids [6–7, 14].

For the area under study, which consists of materials with different physical properties, the minimum step along the time coordinate (5) is selected.

Based on the finite element method in displacements, a technique has been developed, an algorithm has been developed, and a set of programs has been compiled for solving two-dimensional wave problems of the dynamic theory of elasticity [6-7,14].

4. RESULTS

4.1. Modeling of impulse propagation in the form of four trapezoids in an elastic half-plane

The problem of the impact of a plane longitudinal wave in an elastic half-plane (**Fig. 1**) in the form of four trapezoids (**Fig. 2**) is considered to assess the physical reliability and mathematical accuracy [6-7,14]. The problem under study was first solved by

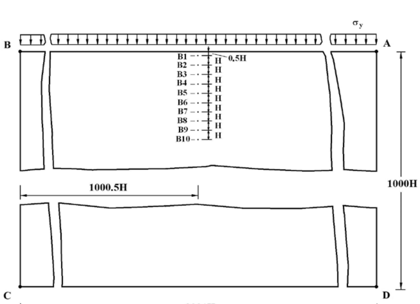

Fig. 1. *Statement of the problem of the propagation of plane longitudinal waves in the form of four trapezoids in an elastic half-plane.*

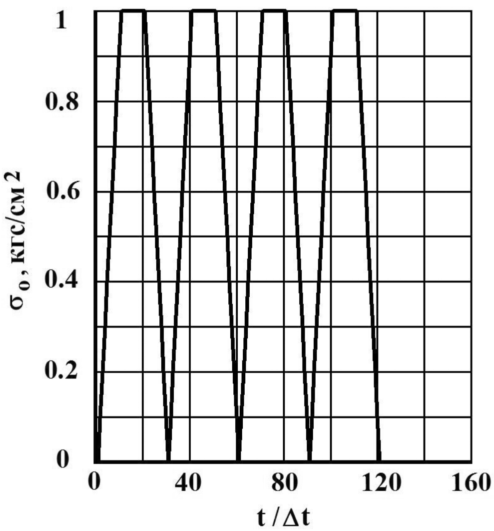

Fig. 2. *Impulse action in the form of four trapezoids.*

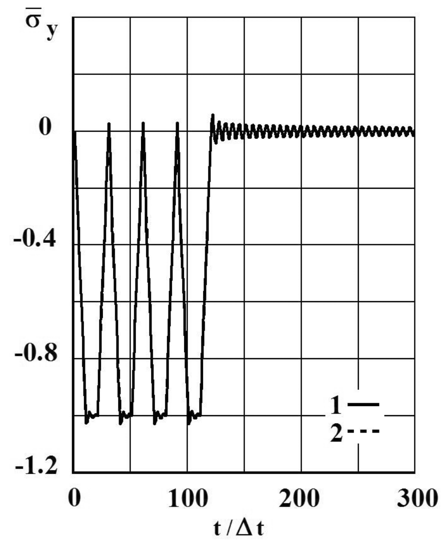

Fig. 3. *Variation of the elastic normal voltages* $\bar{\sigma}_y$ *(the problem of the propagation of plane longitudinal waves in the form of four trapezoids in an elastic half-plane) in time* $t/\Delta t$ *at point B1: 1 – numerical solution; 2 - analytical solution.*

Musayev V.K. using the developed methodology, algorithm and software package [6-7,14].

Calculations were carried out with the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). For conversion to other units of measurement, the following assumptions were made: 1 kgf/cm^2 ≈ 0.1 MPa; 1 kgf·s^2/cm^4 ≈ 10^9 kg/m^3.

At the boundary of the half-plane *AB* (Fig. 1), a normal stress σ_y is applied, which varies from $0 \le n \le 121$ ($n = t/\Delta t$) and the maximum value is P ($P = \sigma_0$, $\sigma_0 = -1$ MPa (–1 kgf/cm^2)). Boundary conditions for the BCDA contour at $t > 0$ $u = v = \dot{u} = \dot{v} = 0$. Reflected waves from the BCDA contour do not reach the studying points at $0 \le n \le 300$.

The calculations were carried out with the following initial data: $H = \Delta x = \Delta y$; $\Delta t = 1.862\cdot 10^{-6}$ s; $E = 2.1\cdot 10^5$ MPa ($2.1\cdot 10^6$ kgf/cm^2); $\nu = 0.3$; $\rho = 0.8\cdot 10^4$ kg/m^3 ($0.8\cdot 10^{-5}$ kgf·s^2/cm^4); $C_p = 5371$ m/s; $C_s = 3177$ m/s.

The computational domain under study has 2004002 nodal points. The system of equations is solved from 8016008 unknowns.

The calculation results were obtained at characteristic points *B*1 – *B*10 (Fig. 1).

As an example in **Fig. 3** shows the change in the normal stress $\bar{\sigma}_y$ ($\bar{\sigma}_y = \sigma_y / |\sigma_0|$) (Fig. 2) in time *n* at point B1 (1 – numerical solution; 2 – analytical solution).

In this case, we can use the conditions at the front of a plane wave, which are described in [2]. For a plane stress state at the front of a plane longitudinal wave, there are the following analytical dependences $\sigma_y = -|\sigma_0|$.

Hence we see that the exact solution of the problem corresponds to the influence σ_0 (Fig. 2).

4.2. Simulation of pulse propagation in the form of two half-cycles of a sinusoid in an elastic half-plane

The problem of modeling plane waves in an elastic half-plane (Fig. 1) in the form of two half-periods of a sinusoid (**Fig. 4**) is considered.

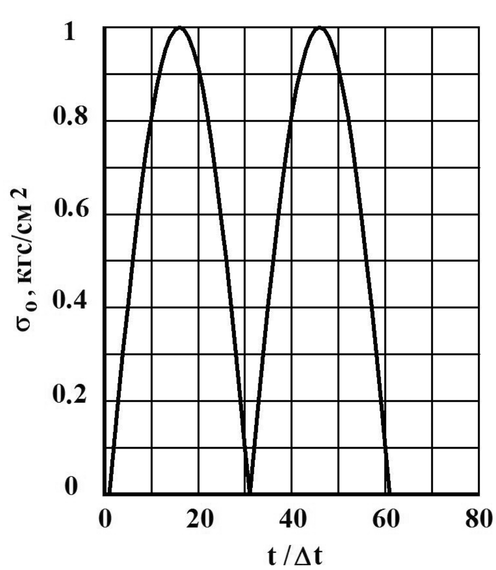

Fig. 4. *Impact in the form of two half-cycles of a sinusoid.*

Calculations were carried out with the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). For conversion to other units of measurement, the following assumptions were made: 1 kgf/cm^2 ≈ 0.1 MPa; 1 kgf·s^2/cm^4 ≈ 10^9 kg/m^3.

The problem is solved to assess the physical reliability and mathematical accuracy of the considered numerical method [6-7,14]. The problem under study was first solved by Musaev V.K. using the developed methodology, algorithm and software package [6-7,14].

At the boundary of the half-plane AB (Fig. 1), a normal stress σy is applied, which varies from $0 \le n \le 61$ ($n = t/\Delta t$) and the maximum value is P ($P = \sigma_0$, $\sigma_0 = -0.1$ MPa (–1 kgf/cm^2)). Boundary conditions for the *BCDA* contour at $t > 0$. Reflected waves from the *BCDA* contour do not reach the points under study at $0 \le n \le 200$.

The calculations were carried out with the following initial data: $H = \Delta x = \Delta y$; $\Delta t = 1.862 \cdot 10^{-6}$ *s*; $E = 2.1 \cdot 10^5$ MPa ($2.1 \cdot 10^6$ kgf/cm^2); $\nu = 0.3$; $\rho = 0.8\ 10^4$ kg/m^3 ($0.8 \cdot 10^{-5}$ kgf·s^2/cm4); $C_p = 5371$ *m/s*; $C_s = 3177$ *m/s*.

The system of equations is solved from 8016008 unknowns.

The calculation results were obtained at characteristic points *B*1 – *B*10 (Fig. 1).

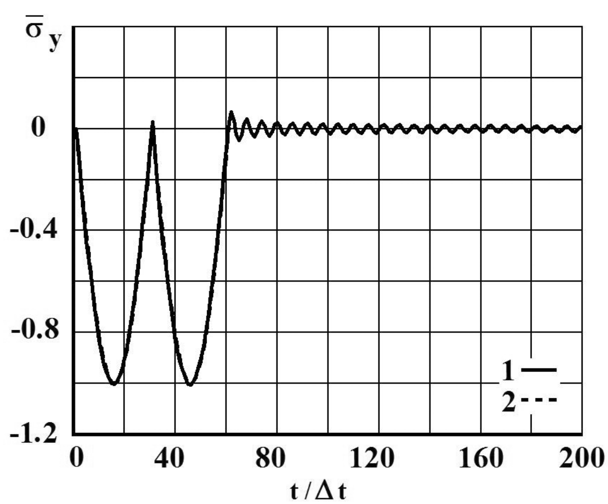

Fig. 5. *Change of elastic normal voltages* $\bar{\sigma}_y$ *(problem of propagation of plane longitudinal waves in the form of two half-cycles of a sinusoid in an elastic half-plane) in time t/Δt at point B1: 1 – numerical solution; 2 - analytical solution.*

As an example in **Fig. 5** shows the change in the normal stress $\bar{\sigma}_y$ ($\bar{\sigma}_y = \sigma_y / |\sigma_0|$) (Fig. 4) in time *n* at point *B*1 (1 – numerical solution; 2 – analytical solution).

In this case, one can use the conditions at the front of a plane wave, which are described in [2]. At the front of a plane longitudinal wave, there are the following analytical dependences for a plane stress state $\sigma_y = -|\sigma_0|$. Hence, we see that the exact solution of the problem corresponds to the action σ_0 (Fig. 4).

4.3. Seismic stress waves in a console with a base

The problem of the impact of a plane longitudinal non-stationary seismic wave on the cantilever (the ratio of width to height is one to ten) in an elastic half-plane (**Fig. 6**) in the form of a Heaviside function (**Fig. 7**) is considered.

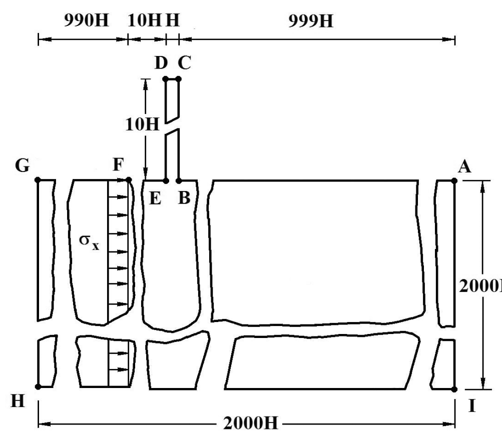

Fig. 6. *Statement of the problem for the console (the ratio of width to height is one to ten) with an elastic base (half-plane).*

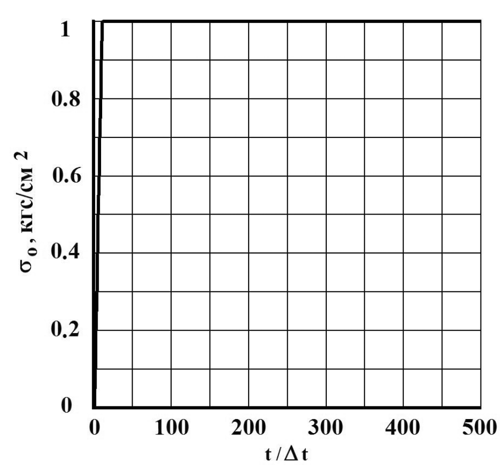

Fig. 7. *Impulse influence in the form of a Heaviside function.*

The problem under study was first solved by Musayev V.K. using the developed methodology, algorithm and software package [6-7,14].

Calculations were carried out with the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). For conversion to other units of measurement, the following assumptions were made: 1 kgf/cm^2 ≈ 0.1 MPa; 1 kgf·s^2/cm^4 ≈ 10^9 kg/m^3.

The initial conditions are assumed to be zero. From point *F* parallel to the free surface *ABEFG*, a normal stress σ_x is applied, which at $0 \leq n \leq 11$ ($n = t/\Delta t$) changes linearly from 0 to *P*, and at $n \geq 11$ it is equal to *P* ($P = \sigma_0$, σ_0 = 0.1 MPa (1 kgf/cm^2)). The calculations were carried out with the following initial data: $H = \Delta x = \Delta y$; $\Delta t = 1.393 \cdot 10^{-6}$ s; E = 3.15 10^4 MPa ($3.15 \cdot 10^5$ kgf/cm^2); $\nu = 0.2$; $\rho = 0.255 \cdot 10^4$ kg/m^3 ($0.255 \cdot 10^{-5}$ kgf·s^2/cm^4); C_p = 3587 *m/s;* C_s = 2269 *m/s.*

Boundary conditions for the *GHIA* contour at $t > 0$, $u = v = \dot{u} = \dot{v} = 0$. Reflected waves from the *GHIA* contour do not reach the points under study at $0 \leq n \leq 500$. The *ABCDEFG* contour is free from loads, except for point *F*.

The system of equations is solved from 16016084 unknowns.

On **Fig. 9-13** shows the change in the loop stresses in the console (**Fig. 8)** in time t/Δt.

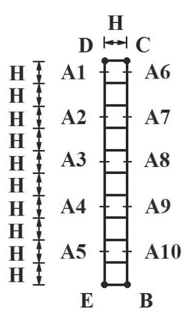

Fig. 8. *Points at which contour stresses were obtained in the console.*

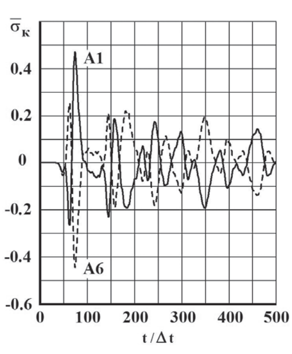

Fig. 9. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A1 and A6 on the console contour in time t/*Δ*t.*

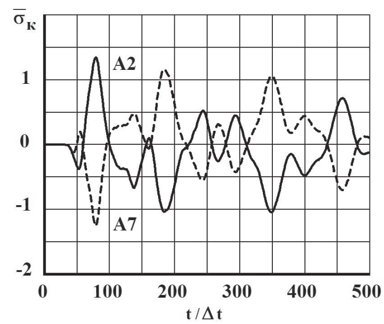

Fig. 10. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A2 and A7 on the console contour in time t/*Δ*t.*

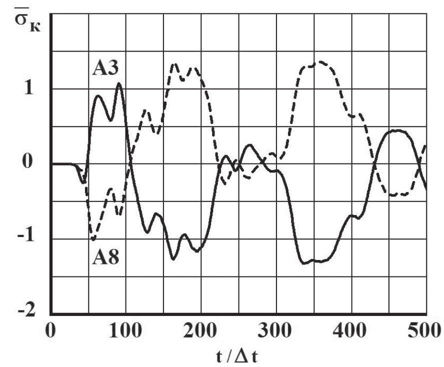

Fig. 11. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A3 and A8 on the console contour in time t/*Δ*t.*

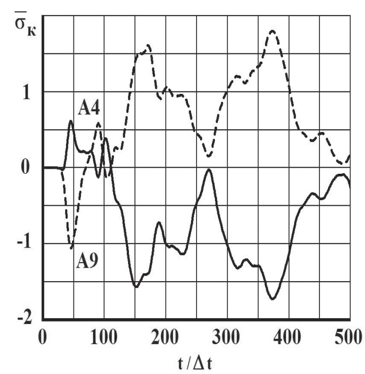

Fig. 12. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A4 and A9 on the console contour in time t/*Δ*t.*

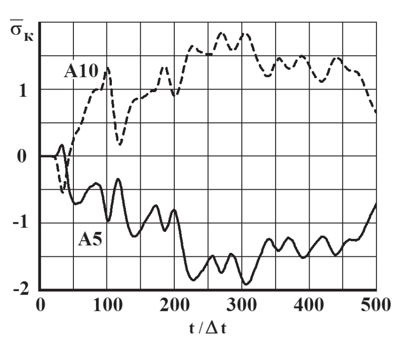

Fig. 13. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A5 and A10 on the console contour in time t/*Δ*t.*

4.4. Seismic stress waves in a ten-story building with a foundation

The problem of the influence of a plane longitudinal non-stationary seismic wave on a ten-story building in an elastic half-plane (**Fig. 14**) in the form of a Heaviside function (**Fig. 15**) is considered.

The problem under study was first solved by Musayev V.K. using the developed methodology, algorithm and software package [6-7,14].

Calculations were carried out with the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). For conversion to other units of measurement, the following assumptions were made: 1 kgf/cm^2 ≈ 0.1 MPa; 1 kgf·s^2/cm^4 ≈ 10^9 kg/m^3.

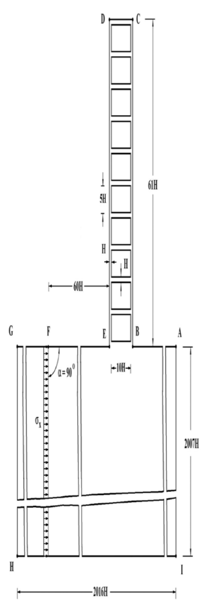

Fig. 14. *Statement of the problem for a ten-story building with an elastic foundation (half-plane).*

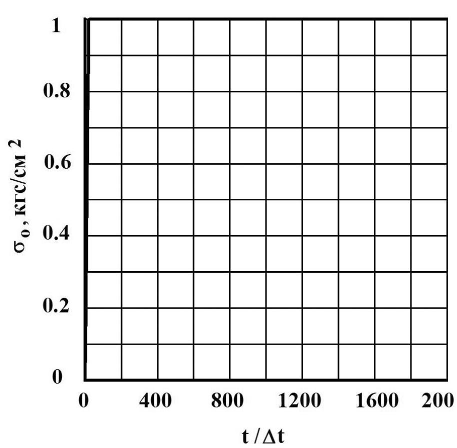

Fig. 15. *Impulse influence in the form of a Heaviside function.*

The initial conditions are assumed to be zero. From point F parallel to the free surface $ABEFG$, a normal stress σx is applied, which at $0 \le n \le 11$ ($n = t/\Delta t$) changes linearly from 0 to P, and at $n \ge 11$ it is equal to P ($P = \sigma_0$, $\sigma_0 = 0.1$ MPa (1 kgf/cm^2)). The calculations were carried out with the following initial data: $H = \Delta x = \Delta y; \Delta t = 1.393 \cdot 10^{-6}$ *s*; $E = 3.15$ 10^4 MPa ($3.15 \cdot 10^5$ kgf/cm^2); $\nu = 0.2$; $\rho = 0.255 \cdot 10^4$ kg/m^3 ($0.255 \cdot 10^{-5}$ kgf·s^2/cm^4); $C_p = 3587$ *m/s*; $C_s = 2269$ *m/s*.

Boundary conditions for the *GHIA* contour at $t > 0$, $u = v = \dot{u} = \dot{v} = 0$. Reflected waves from the GHIA contour do not reach the points under study at $0 \le n \le 2000$.

The contour *ABCDEFG* is free from loads, except for point *F*. The system of equations is solved from 16202276 unknowns. On **Fig. 17-21** shows the change in the loop stresses $\bar{\sigma}_k$ in the console (**Fig. 16**) in time $t/\Delta t$.

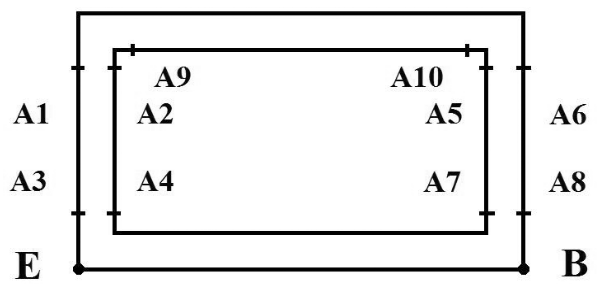

Fig. 16. *Points at which contour stresses were obtained in the console.*

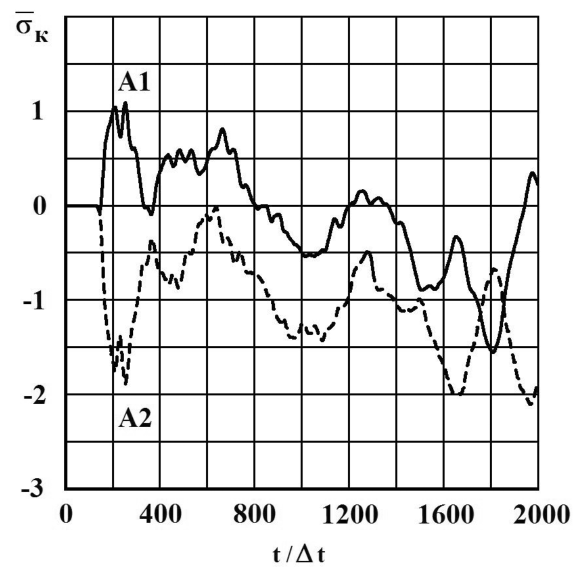

Fig. 17. *Change in the elastic contour stresses $\bar{\sigma}_k$ at points A1 and A2 on the console contour in time $t/\Delta t$.*

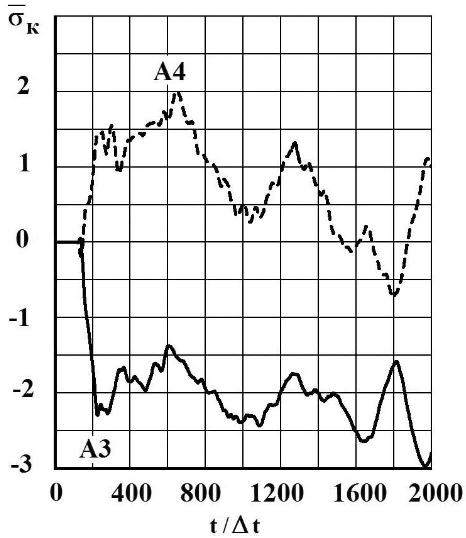

Fig. 18. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A3 and A4 on the console contour in time* $t/\Delta t$.

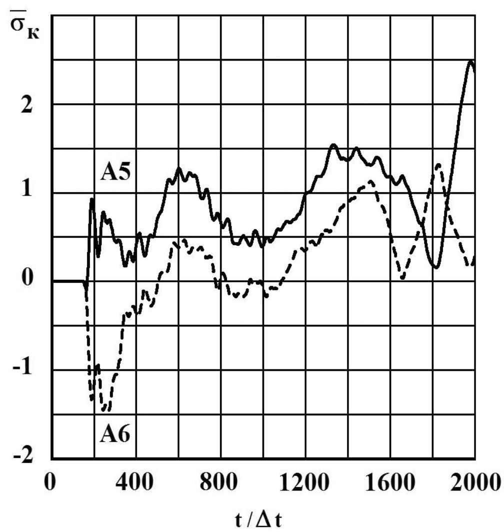

Fig. 19. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A5 and A6 on the console contour in time* $t/\Delta t$.

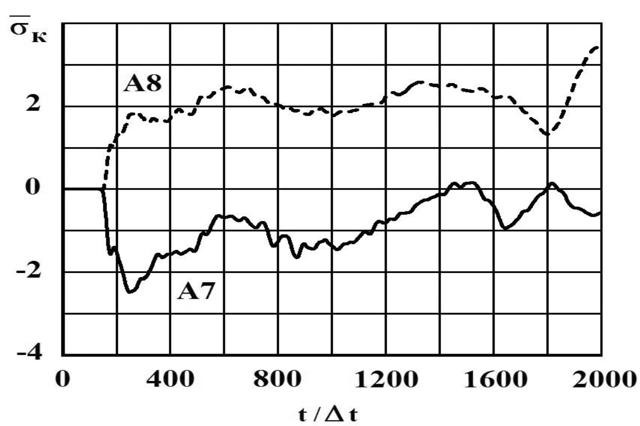

Рис. 20. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A7 and A8 on the console contour in time* $t/\Delta t$.

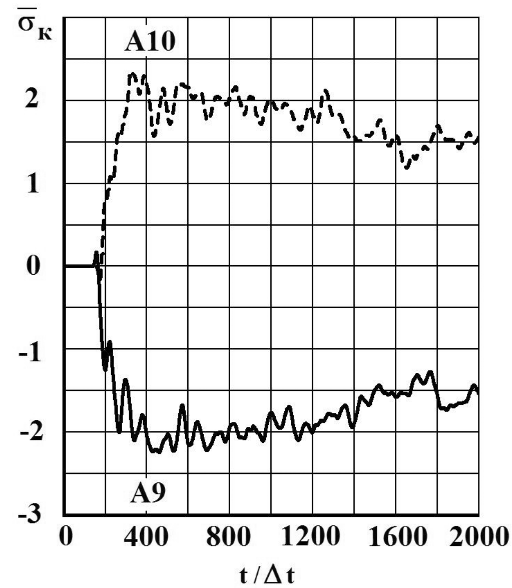

Fig. 21. *Change in the elastic contour stresses* $\overline{\sigma}_k$ *at points A9 and A10 on the console contour in time* $t/\Delta t$.

4. CONCLUSION

On the basis of mathematical modeling (finite element method), a technique, an algorithm and a set of programs for solving linear two-dimensional plane problems have been developed, which allow solving complex problems with non-stationary wave effects.

The main relations of the finite element method are obtained using the principle of possible displacements. The elasticity matrix is expressed in terms of P-wave velocity, S-wave velocity, and density. A linear dynamic problem with initial and boundary conditions in the form of partial differential equations, for solving problems under wave influences, using the finite element method, is reduced to a system of linear ordinary differential equations with initial conditions, which is solved by an explicit two-layer scheme.

To predict the seismic safety of an object, with non-stationary wave effects, numerical modeling of the equations of the wave theory of elasticity is used.

The problem of the impact of a plane longitudinal wave in the form of four trapezoids and two half-periods of a sinusoid on an elastic half-plane is solved. The system of equations from 8016008 unknowns is solved. A comparison was made with the results of the analytical solution, which showed a quantitative match.

The problem of mathematical modeling of non-stationary elastic stresses waves in the console (the ratio of width to height is one to ten) with an elastic half-plane under seismic influence is solved. The system of equations from 16016084 unknowns is solved.

The problem of mathematical modeling of non-stationary elastic stresses waves in a ten-story building with an elastic half-plane under seismic influence is solved. The system of equations is solved from 16202276 unknowns.

The elastic contour tensions on the edges of the console and the ten-story building is almost a mirror image of one another, that is, antisymmetric. A console and a ten-story building under seismic influence work like a beam, that is, if there are tensile influences on one side, then compressive influences on the other. On the contours of the console and the ten-story building under seismic influence, bending waves mainly prevail.

REFERENCES

1. Davis R. *Tension Waves in Solids*. Moscow, Inostrannaya literatura Publ., 1961, 104 p.
2. Timoshenko SP, Goodier D. *Theory of elasticity*. Moscow, Nauka Publ., 1975, 576 p.
3. Ionov VI, Ogibalov PM. *Stresses in bodies under impulsive loading*. Moscow, Vysshaya shkola Publ., 1975, 464 p.
4. Bate K, Wilson E. *Numerical methods of analysis and finite element method*. Moscow, Stroyizdat Publ., 1982, 448 p.
5. Nemchinov VV. Numerical methods for solving flat dynamic elasticity problems. *International Journal for Computational Civil and Structural Engineering*, 2013, 9(1):90-97.
6. Musayev VK. Estimation of accuracy of the results of numerical simulation of unsteady wave of the stress in deformable objects of complex shape. International Journal for Computational Civil and Structural Engineering, 2015, 11(1):135-146.
7. Musayev VK. On the mathematical modeling of nonstationary elastic waves stresses in corroborated by the round hole. *International Journal for Computational Civil and Structural Engineering*, 2015, 11(1):147-156.
8. Dikova EV. Reliability of the numerical method, algorithm and software package Musayeva V.K. when solving the problem of the propagation of plane longitudinal elastic waves (the ascending part is linear, the descending part is a quarter of a circle) in a half-plane. *International Journal of Experiential Education*, 2016, 12-3:354-357.
9. Starodubtsev VV, Akatiev SV, Musayev AV, Shiyanov SM, Kurantsov OV. Modelirovanie uprugikh voln v vide impul'snogo vozdeystviya (voskhodyashchaya chast' - chetvert' kruga, niskhodyashchaya chast' - chetvert' kruga) v poluploskosti s pomoshch'yu chislennogo metoda Musayeva V.K. [Modeling of elastic waves in the form of impulse action (ascending part - a quarter of a circle, descending part - a quarter of a circle) in a half-plane using the numerical method Musayeva V.K.]. *Problemy bezopasnosti rossiyskogo obshchestva,* 2017, 1:36-40 (in Russ.).
10. Starodubtsev VV, Akatiev SV, Musayev AV, Shiyanov SM, Kurantsov OV. Modelirovanie c pomoshch'yu chislennogo metoda Musayeva V.K. nestatsionarnykh uprugikh voln v vide impul'snogo vozdeystviya (voskhodyashchaya chast' - chetvert' kruga, srednyaya - gorizontal'naya, niskhodyashchaya chast' - lineynaya) v sploshnoy deformiruemoy srede [Modeling using the numerical method Musayeva V.K. non-stationary elastic waves in the form of impulsive action (the ascending part is a quarter of a circle, the middle part is horizontal, the descending part is linear) in a continuous deformable medium]. *Problemy bezopasnosti rossiyskogo obshchestva,,* 2017, 1:63-68 (in Russ.).
11. Kurantsov VA, Starodubtsev VV, Musayev AV, Samoilov SN, Kuznetsov ME. Modelirovanie impul'sa (pervaya vetv': voskhodyashchaya chast' - chetvert' kruga, niskhodyashchaya chast' - lineynaya; vtoraya vetv': treugol'nik) v uprugoy poluploskosti s pomoshch'yu chislennogo metoda Musayeva V.K. [Simulation of an impulse (first branch:

ascending part - a quarter of a circle, descending part - linear; second branch: triangle) in an elastic half-plane using the numerical method Musayev V.K.]. *Problemy bezopasnosti rossiyskogo obshchestva,* 2017, 2:51-55 (in Russ.).

12. Favorskaya AV, Petrov IB. Calculation of seismic resistance of various structures by the grid-characteristic method. *RENSIT: Radioelectronics. Nanosystems. Information Technologies*, 2019, 11(3):345-350. DOI: 10.17725/rensit.2019.11.345.
13. Avershyeva AV, Kuznetsov SV. Numerical simulation of Lamb wate propagation isotropic layer. *International Journal for Computational Civil and Structural Engineering*, 2019, 15(2):14-23.
14. Musayev VK. Mathematical modeling of non-stationary elastic waves stresses under a concentrated vertical exposure in the form of delta functions on the surface of the half-plane (Lamb problem). *International Journal for Computational Civil and Structural Engineering,* 2019, 15(2):111-124.
15. Bratov VA, Ilyashenko AV, Kuznetsov SV, Lin NK, Morozov NF. Homogeneous horizontal and vertical seismic barriers: mathematical foundations and dimensional analysis. *Materials Physics and Mechanics,* 2020, 44(1):61-65.