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Numerical research of thermal changes influence to the ground in permafrost conditions

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Abstract: **The article is devoted to the numerical solution of the Stefan problem for studying the process of soil thawing in permafrost conditions. An enthalpy solution method was constructed, and the applicability of this method was considered. The numerical solution was found using the Pismen-Rekford scheme in 2D and 3D cases. The developed computational algorithms are parallelized for use on modern high performance computational systems. An approach has been implemented for modeling thermal processes in the thickness of an arbitrary array of substances, taking into account arbitrary initial conditions and environmental conditions. Mathematical modeling of the process of thawing of the upper layer of permafrost was carried out in two-dimensional and three-dimensional formulations, including the formulation with a gas reservoir located in the ground.**

Keywords: **mathematical modeling, permafrost, Stefan problem, enthalpy method, Pismen-Rekford scheme**

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1. INTRODUCTION

In recent years, the Arctic coast of our country has been actively studied and developed. According to experts, the northern regions store colossal reserves of hydrocarbons that can be extracted. However, both production and exploration of deposits in this region has its own characteristics and specifics. One of the first problems that engineers face when studying soil in the northern regions is the presence of a layer of permafrost. Permafrost is a layer of soil, the temperature of which has not risen above 0°C for a long time (from 2-3 years to millennia). Groundwater in such areas are ice sheets. The thickness of the permafrost layer can reach more than 1000 m. In Russia, 60-70% of the territory is located in the permafrost zone. Distribution zones - Eastern Siberia, Transbaikalia, northern regions near the coast of the Arctic Ocean.

A large number of natural and theoretical studies have been devoted to permafrost. The work [1] exhaustively describes permafrost, its properties and physical processes that occur in its thickness. In [2], the interaction between piles and permafrost is modeled. Also, one of the potential problems that threaten the safety of land exploitation in the permafrost zone are methane bombs.

This article proposes an approach to numerical modeling based on solving the problem of the evolution of a system with different phase states of matter and changing the location of the boundary between these phases – the Stefan problem [3-9]. Works [10-13] are devoted to the numerical solution of problems with phase transitions. In [10, 11], the method of lines was considered. The finite element method and finite difference methods are also often used in practice [12,13].

In this paper, we use the enthalpy approach [14], which has already been applied by the authors to solve the problem of melting an artificial ice island [15]. With its help, it was possible to obtain temperature distributions in the soil, as well as to simulate the process of its thawing.

2. MATERIALS AND METHODS

2.1. Mathematical model and numerical method

Thermal processes in the soil in the permafrost zone cannot be modeled solely on the basis of the heat conduction equation. It is necessary to take into account melting, a phase transition that requires thermal energy. This can be done if we go to the Stefan problem, that is, set the problem of heat conduction for each phase and the boundary condition, the Stefan condition. Using the law of conservation of energy, Fourier's law of heat conduction, numbering all the phases with separate indexes i and denoting the boundary between them $\Gamma(t)$ we arrive at the system:

$$\frac{\partial C_i^V T}{\partial t} = \vec{\nabla}\left(k_i \vec{\nabla} T\right), \qquad \vec{r} \notin \Gamma(t),$$

$$\lambda^V \frac{\partial \vec{r_\Gamma}}{\partial t} \cdot \overrightarrow{dS} = \left(\vec{q}_L - \vec{q}_S\right) \cdot \overrightarrow{dS}, \quad \vec{r_\Gamma} \in \Gamma(t),$$

$\vec{r}$ – radius vector, t – time, T – temperature field, $\vec{q}_i$ – heat flow. By C_i^V, k_i, λ^V marked coefficients of heat capacity per unit volume, thermal conductivity and heat of phase transition, $\overrightarrow{dS}$ – normal vector to the interface plane. At the boundary, the temperature is equal to the phase transition temperature T_p.

In general, thermophysical coefficients may depend on water saturation and soil type and may vary in space and time. For our tasks, we believe that the entire geological section can be divided into areas with constant values, where the type of soil and its water saturation does not change.

To go to specific characteristics (C_i, λ) we use formulas:

$\rho_i C_i = C_i^V$, $\rho_i \lambda = \lambda^V$.

The coefficient k is effective and takes into account the heat exchange in the ground due to the mechanisms of heat conduction between solid particles, steam convection and other processes.

After adding boundary and initial conditions, the system becomes correct. But it is difficult to solve it in such a formulation. It is possible to formulate an equation for the entire region if we pass from temperature to heat content. We denote the heat content or enthalpy as H. The phase transition occurs in the range

$C_s^V T_p = H_- < H < H_+ = C_s^V T_p + \lambda^V$.

Then return to the temperature is possible with:

$$T = \begin{cases} \dfrac{H}{C_s^V}, & H < H_-, \\ T_p, & H_- < H < H_+, \\ \dfrac{H + (C_L^V - C_s^V)\mathrm{T}_p - \lambda^V}{C_L^V}, & H > H_+. \end{cases}$$

It remains to determine the coefficient of thermal conductivity in the intermediate region:

$$k(H)=\begin{cases} k_s, & H<H_-, \\ k_s+(k_L-k_s)\cdot\dfrac{H-H_-}{H_+-H_-}, & H_-<H<H_+, \\ k_L, & H>H_+. \end{cases}$$

Then for all interior points we arrive at the equation:

$$\frac{\partial C_i^V T}{\partial t}=\frac{\partial H}{\partial t}=\vec{\nabla}(k_i\vec{\nabla}T).$$

For the boundary, the right equality holds automatically (can be obtained from the continuity equation for heat). In total, taking into account the boundary conditions:

$$\frac{\partial H}{\partial t}=\vec{\nabla}_r(k(H)\vec{\nabla}_r T(H)),$$

$$\left(\alpha T(H)+\beta\frac{\partial T(H)}{\partial n}\right)\Bigg|_{\partial\Omega}=\gamma,$$

$$H(\vec{r},t)\big|_{t=0}=H(T_0(\vec{r})).$$

The system can be solved in one-dimensional, two-dimensional and three-dimensional form. To solve it numerically, it is necessary to pass to the grid function of heat content and rewrite the differential operators in the form of difference operators. The main difficulty here is the development of an efficient, that is, unconditionally stable method, with the algorithmic complexity of the transition to the next step in time linear in the total number of nodes; as well as the quasi-linear character of the equation.

It is proposed to switch to multilayer schemes, which are implicit in each of the fractional steps in only one direction. For each time step, carry out several iterations using the thermal conductivity coefficients from previous iterations. Then it is possible to solve linear equations by sweep at each iteration.

In the two-dimensional case, the Pismen-Rekford scheme is used (longitudinal-transverse scheme):

$$\frac{u_{ml}^{n+1/2}-u_{ml}^{n}}{\tau/2}=\Lambda_{xx}u_{ml}^{n+1/2}+\Lambda_{yy}u_{ml}^{n},$$

$$\frac{u_{ml}^{n+1}-u_{ml}^{n+1/2}}{\tau/2}=\Lambda_{xx}u_{ml}^{n+1/2}+\Lambda_{yy}u_{ml}^{n+1}.$$

Differential operators (the value of k at fractional steps is chosen as the arithmetic mean):

$$\Lambda_{xx}u=k_{m+\frac{1}{2}\,l}\frac{u_{m+1\,l\,p}-u_{m\,l}}{h_x^2}+k_{m-\frac{1}{2}\,l}\frac{u_{m-1\,l}-u_{m\,l}}{h_x^2},$$

$$\Lambda_{yy}u=k_{m\,l+\frac{1}{2}}\frac{u_{m\,l+1\,p}-u_{m\,l\,p}}{h_x^2}+k_{m\,l-\frac{1}{2}}\frac{u_{m\,l-1\,p}-u_{m\,l\,p}}{h_x^2}.$$

In the three-dimensional case, the Pismen-Rekford scheme is rewritten as:

$$\frac{u_{mlp}^{n+1/3}-u_{mlp}^{n}}{\tau/3}=a^2\Lambda_{xx}u_{mlp}^{n+1/3}+a^2\Lambda_{yy}u_{mlp}^{n}+a^2\Lambda_{zz}u_{mlp}^{n},$$

$$\frac{u_{mlp}^{n+2/3}-u_{mlp}^{n+1/3}}{\tau/3}=a^2\Lambda_{xx}u_{mlp}^{n+1/3}+a^2\Lambda_{yy}u_{mlp}^{n+2/3}+a^2\Lambda_{zz}u_{mlp}^{n+1/3},$$

$$\frac{u_{mlp}^{n+1}-u_{mlp}^{n+2/3}}{\tau/3}=a^2\Lambda_{xx}u_{mlp}^{n+2/3}+a^2\Lambda_{yy}u_{mlp}^{n+2/3}+a^2\Lambda_{zz}u_{mlp}^{n+1}.$$

Sustainability requires:

$$\frac{4\tau a^2}{h^2}\le 1,$$

$$h=min\left(h_x,h_y,h_z\right),$$

$$a=\sqrt{\max\left(\left(\rho_i C_i\right)_q^{-1}\right)\cdot\max\left(k_j\right)_r}\ .$$

where i, j are indices of phase states, q, r are indices of substances.

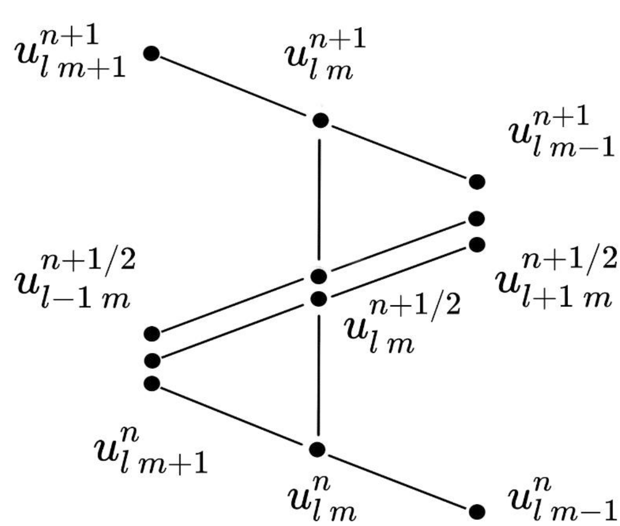


Fig. 1. *Template of the Pismen-Rekford scheme.*

2.2. Software implementation

The software developed on the basis of the mathematical model for solving the multidimensional Stefan problem in the enthalpy formulation can be divided into two groups. On the one hand, the main cycle with the transition between time layers is implemented in C++ using the OpenMP library to maximize performance. The initial and boundary conditions, the distribution of substances in the computational domain, and other parameters are transferred to the program from other files, which are generated separately using scripts in the slower, but more convenient for practical use, Python language.

The package of auxiliary programs includes scripts for generating a grid from geometric shapes, initial temperature distributions of their objects of complex shape, such as gradients, an XML file for the task configuration, and scripts for processing computation results.

The main program includes a module for parsing this data and a module for computation itself. It is possible to record intermediate results for further processing in VTK format files.

3. THE RESULTS

3.1. Formulation of the problem

Consider the structure of the soil, including peaty loam, loam and sand. The thermophysical characteristics of the layers [1] are given in **Table 1.**

The density of peaty loam is taken equal to ρ = 1500 кг/м^3, loam ρ = 1600 кг/м^3, sand ρ = 1300 кг/м^3.

Schemes of the computational domain for studying the process of thawing the soil are shown in **Fig. 2.** Computations were carried out on two-dimensional (Fig. 2*a*) and three-dimensional (Fig. 2*b*) models.

Table 1

Physical characteristics of the soil

Substance	$C_s^V, \frac{kJ}{m^3 \cdot °C}$	$C_L^V, \frac{kJ}{m^3 \cdot °C}$	$\lambda, \frac{kJ}{kg}$	$k_s, \frac{W}{m \cdot °C}$	$k_L, \frac{W}{m \cdot °C}$
Peaty loam	2350	3150	71957	2.73	2.56
Loam	2350	3150	71957	1.7	1.51
Sand	1670	2010	60437	1.86	1.51

The index S denotes the frozen state, and L denotes the thawed one.

3.2. The results of modeling the thawing of frozen soil under temperature changes in the two-dimensional case

The nature of permafrost thawing for a small area (for example, under a structure or a pipeline with a constant positive temperature) was studied for the model presented in Fig. 2*a*. For all computations, the temperature of the upper boundary is assumed to be +20°C. The boundary condition of zero heat flux is set on the side and lower boundaries. The results of computations are shown in **Fig. 3.**

Next, a soil with a gas cavity located at a depth of 3 m was considered (Fig. 2*a*). The results of the calculation for the thawing of this geological section are shown in **Fig. 4**.

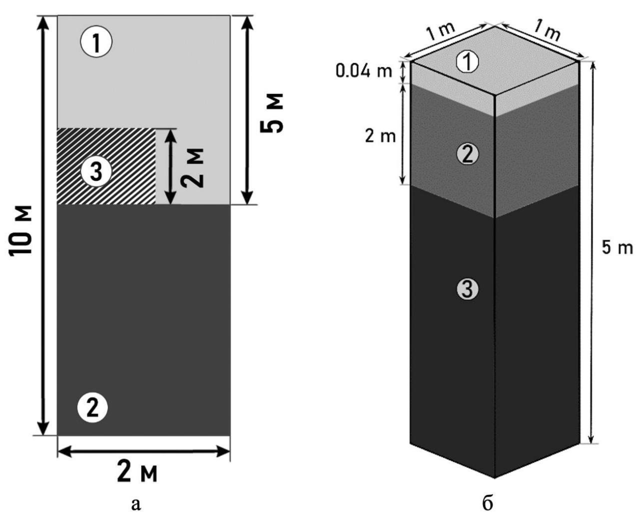
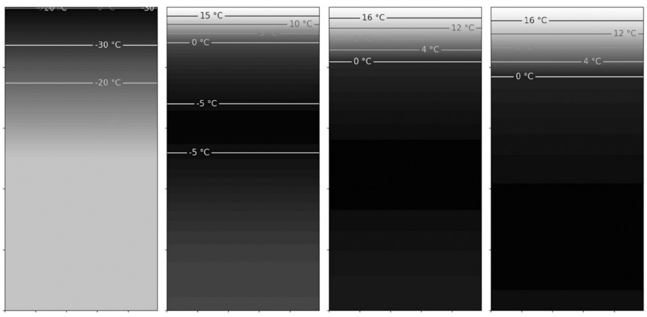
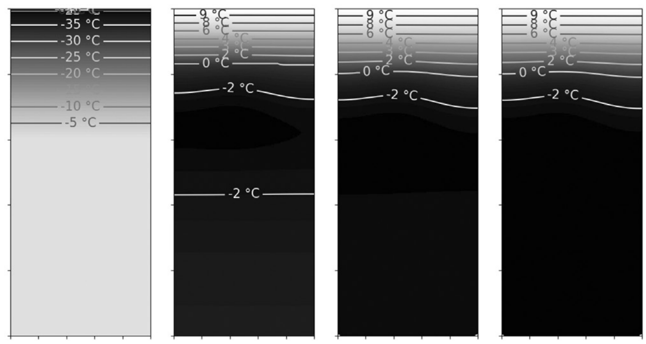


Fig. 2. *Schemes of the computational domain for studying the process of soil freezing. a – two-dimensional model: 1 – peaty loam, 2 – loam, 3 – gas cavity. b – three-dimensional model: 1 – surface layer with air temperature, 2 – peaty loam, 3 – loam.*



a b c d

Fig. 3. *Mpdeling results of the impact of seasonal air temperature changes on the soil on a two-dimensional model, thawing of the upper layer of permafrost, time points: a - initial moment, b - 45 days of positive temperature, c - 90 days of positive temperature: d - 135 days of positive temperature.*



a b c d

Fig. 5. *Modeling results of the impact of seasonal air temperature changes on the soil on a three-dimensional model, thawing of the upper layer of permafrost, time points: a - initial moment, b - 45 days of positive temperature, c - 90 days of positive temperature: d - 135 days of positive temperature.*

3.3. The results of modeling the thawing of frozen soil under temperature changes in the three-dimensional case

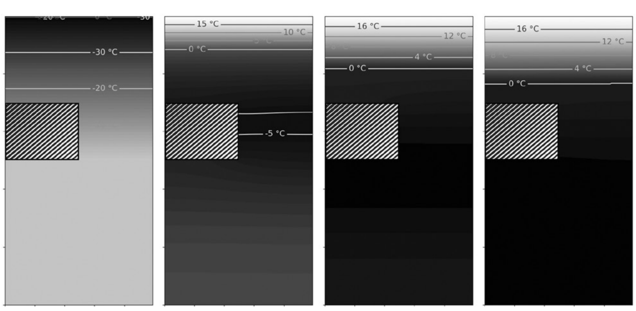
For the three-dimensional case, problems were solved in formulations similar to the two-dimensional case. The model of the geological section is shown in Fig. 2*b*. The case without a gas cavity was considered, the modeling results for which are shown in **Fig. 5**.

Fig. 6 shows the simulation results in the form of two-dimensional pictures on the plane of the vertical section of the temperature distribution fields for the case of a geological section with a parallelepiped gas reservoir.

4. CONCLUSION

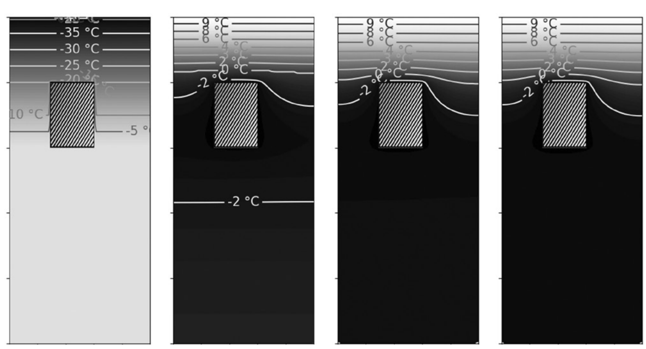
In this work, an enthalpy approach for solving the Stefan problem is developed, and software is developed for solving this problem in various formulations. On the basis of this program, numerical modeling of the thawing of the upper layer of permafrost was carried out and the temperature distribution in the frozen ground was found.

The logical continuation of this study is the solution of a thermoelastic problem for soil with



a b c d

Fig. 4. *Modeling results of the impact of seasonal air temperature changes on the soil with a gas reservoir on a two-dimensional model, thawing of the upper layer of permafrost, time points: a - initial moment, b - 45 days of positive temperature, c - 90 days of positive temperature: d - 135 days of positive temperature.*



a b c d

Fig. 6. *Modeling results of the impact of seasonal air temperature changes on the soil with a gas reservoir on a three-dimensional model, thawing of the upper layer of permafrost, time points: a - initial moment, b - 45 days of positive temperature, c - 90 days of positive temperature: d - 135 days of positive temperature.*

permafrost in order to study the resistance of upper layers to gas pressure in gaseous regions during seasonal thawing, as well as longer processes of temperature change, such as global warming.

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