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# Application of the Interpolation Method of Sequential Computation of the Fourier Spectrum to Sparse Images

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*Abstract*: **Interpolation Method of Sequential Computation of the Fourier spectrum (IMSCS) is used for the reconstruction of sparse digital images. Peculiarities of application of the method are investigated on different types of images with a large sparseness (from 90 to 99 percent of information is missing). To improve the work of IMSCS, when considering the large sparseness of the initial data, its totality includes a procedure for additional iterative refinement of each of the restored harmonics of the spatial spectrum. As an alternative approach, to determine the analysis by objective criteria, spline interpolation is chosen. The conducted study allows us to conclude that it is fundamentally possible to use IMSCS to restore rarefied images, both for the reconstruction of gaps, and in order to reduce the amount of data.**

*Keywords:* **remote sensing, sparse digital images, image processing, interpolation method of sequential computation of the Fourier spectrum**

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## 1. INTRODUCTION

The problem of restoring a signal distorted by various types of lacunae (unfilled data areas) is extremely relevant. It occurs in many practical problems of image and signal processing [1-9]. For example, during remote sensing of the Earth using aerospace vehicles, gaps often appear in the space-time data structure. These losses (areas that are not filled with data) may occur due to the technical features of the equipment and measurement techniques. This happens due to the specifics of the trajectories and because when we register images in the optical range, the image can be partially covered by clouds. In such cases, it is argued that the image is distorted by the loss of adjacent samples (continuous gaps).

In the work proposed here, sparse images (from unevenly spaced samples) are considered. That is, such images, throughout the field of which there are only a certain number of elements, while most of the elements are missing. Often, in remote sensing, a large amount of memory is required to record the measured data, which affects the processing speed and does not allow real-time observations. Therefore, at present, as a rule, they deliberately resort to reducing the number of measured signals by thinning it out.

An example here is the technology of ultrasonic testing using piezoelectric antenna arrays. It was shown in [10] that the recognition method with compression (Compressive Sensing (CS)) [11–13] makes it possible to increase the rate of recording echo signals by an average of five times by reducing the number measured echoes, and at the same time to obtain high resolution. Generally speaking, in almost all video surveillance systems, in order to save the amount of data, images are subjected to the procedure of "sparseness" (compression) according to various algorithms. The most common compression methods are Discrete Cosine Transformation (DCT), Discrete Wavelet Transformation (DWT), Gabor transformation, and others [14]. Each method that reduces the size of the amount of required memory leads to losses. They appear visually either in the manifestation of the block structure (for DCT) or in blurring of the image (for DWT). The challenge is to find a compromise between the degree of compression, which reduces file sizes, and the worsening of image quality.

The work presented here explores the application of the Interpolation Method of Sequential Computation of the Fourier spectrum (IMSCS) for the reconstruction of various types of images with a high degree of sparseness (90 to 99 percent of information is missing). In the article [15], when we first described the IMSCS, we already demonstrated the preliminary results of the operation of this method with sparse images. There, as the most "hard" case, an example was considered with the absence of about 70 percent of the original image data. And based on the processing examples, it has been argued that IMSCS retouching and restoration can give good results even with a significant proportion of the missing image. In [16], we showed that image inpainting methods, including those implemented using neural networks, do not allow restoring images in continuous gaps. The same applies to all types of image interpolation. At the same time, [16] demonstrates the ability of IMSCS to partially restore the contents of a continuous gap, while competing methods only retouch those places where image data is lost. Description of the algorithm of the Interpolation Method of Sequential Computation of the Fourier spectrum (IMSCS), according to [16], is given in the Appendix. In the work proposed here, in order to improve the operation of the IMSCS, with a strong sparseness of images, its algorithm includes an internal procedure for additional iterative refinement of each of the harmonics. A similar modification of the technique was used by us to reconstruct one-dimensional acoustic signals from incomplete data [17].

## 2. APPLICATION OF IMSCS TO SPARSE IMAGES

As a first example, we use the original digital image from the database: https://www.goodfon.ru/download/lods-franck-portret-lea/1280x1024/. Let's give this image the name "Portrait" and for the convenience of processing we will make its size 512 by 512 pixels, in addition, we will convert its color in greyscale from 1 - conditionally black, to 255 - white. The result of the transformations is shown in **Fig. 1***a*. Based on this, we will model a rarefied image. That is, according to a random uniform law, we will remove ninety percent of the information from Fig. 1*a*, thus obtaining Fig. 1*b*. Black field (gradation of brightness = 0) in Fig. 1*b* corresponds to the missing data, and the remaining ten percent of the informative elements of the image have their original values as in Fig. 1*a* (from 1 to 255).

In the absence of 90 percent of the data for image reconstruction using the Interpolation Method of Sequential Computation of the Fourier spectrum, it was not necessary to apply the procedure for additional iterative refinement of each of the harmonics. Result of sparse image restoration Fig. 1*b* is shown in Fig. 1*c*. "Portrait" is quite recognizable. When comparing Fig. 1*a* and Fig. 1*c*, it can be established that the reconstructed image contains a certain number

*a* *b* *c*

**Fig. 1.** *Original digital image "Portrait" sized 512 by 512 pixels - (a); Sparse image (10 percent of the total data volume is known Fig. 1a) - (b); IMSCS recovery Fig. 1b (256 harmonics 1 iteration) - (c).*

of interfering artifacts. In addition, a visually noticeable decrease in the overall sharpness. This circumstance can be fixed according to objective criteria – "average contrast" and "sharpness rating" [18]. **Table 1** shows these quantitative characteristics. In the left column of Table 1 are

**Table 1**

Image quality scores for Fig. 1.

| Original digital image 512 by 512 pixels "Portrait" | Original digital image (100 percent of the total data is known) | IMSCS reconstruction (10 percent of the total data is known) |
|---|---|---|
| Sharpness rating | 1.665 | 1.007 |
| Average contrast | 0.063 | 0.054 |

shown the results of evaluations for the original image, in the right column – for the reconstruction using IMSCS. The sharpness estimate and average contrast of the reconstructed image are significantly lower than those of the original image. Nevertheless, the information content and quality of Fig. 1*c* can be considered satisfactory.

For additional control over the preservation of the information content of the reconstructed image, we use Content-based image retrieval (CBIR) – a section of computer vision that solves the problem of finding images that have the required content in a large set of digital images. The Yandex intelligent recognition system will help us with this, which does not experience problems with searching for images on the Internet by the content of the uploaded image Fig. 1*c*.

Let's consider an even more complicated case. A sparse image with a known one percent of the total data. Let's remove ninety-nine percent of the information from Fig. 1*a* thus obtaining **Fig. 2***a*. As before, the black field (brightness gradation = 0) in Fig. 2*a* corresponds to the missing data, and the remaining one percent of the informative elements of the image have their original values as in Fig. 1*a* (from 1 to 255). Fig. 2*b* demonstrates the result

*a* *b* *c* *d*

**Fig. 2.** *Sparse image (1 percent of the total data volume is known Fig. 1a) - (a); Restoration of IMSCS (256 harmonics, 1 iteration) - (b); IMSCS recovery (256 harmonics, 20 iterations) - (c); Restoration by spline interpolation - (d).*

of applying IMSCS to Fig. 2*a* without additional iterative refinement of the reconstructed harmonics of the spatial spectrum. With such a reconstruction, the unsatisfactory quality of restoration is obvious. This manifests itself in the form of a "spotted" structure (Fig. 2*b*). Those, some small recovery area is formed around each informative pixel, and the rest of the image field is filled with average brightness values (grey background). To overcome this limitation, we propose a modification of the IMSCS, which consists in the procedure of additional iterative refinement of each of the harmonics. The number of iterations is chosen empirically and is limited when an acceptable result is achieved. We believe that the sufficient number of iterations is such that the "grey" background of the reconstructed image is completely filled with significant interpolation data. A further increase in the number of iterations not only does not improve the quality of the reconstruction, but also increases the number of interfering artifacts. In the considered example Fig. 2*c* the number of iterations is 20.

In this work, for a comparative analysis with the proposed Interpolation Method of Sequential Computation of the Fourier spectrum, the missing data are also filled in using the spline interpolation (spline) described in [19,20]. The physical meaning of this algorithm is that for an arbitrary set of reference points (nodes) a system of linear equations is solved that simulates the behavior of a curved elastic plate. The result is a relation that describes a two-dimensional spline surface. Fig. 2 shows the result of applying the spline to the image Fig. 2*a*. Compared to IMSCS, with iterations (Fig. 2*c*), spline interpolation reveals a smoother recovery. However, IMSCS, with iterations is better at restoring image details. This becomes apparent if one carefully examines the eyes of the "portrait" in Fig. 2*b* and Fig. 2*c*. Of course, the quality of reconstruction in Fig. 2 compared to Fig. 1 is not good enough from an expert point of view. This is natural since the last case is the limit (missing 99 percent of relevant information). Nevertheless, the Yandex intelligent system (search on the Internet by image) unmistakably finds the original. And not only does it find the original of this photo, this system recognizes the face of a particular person, since it gives out the same girl among the search options, but from a different angle.

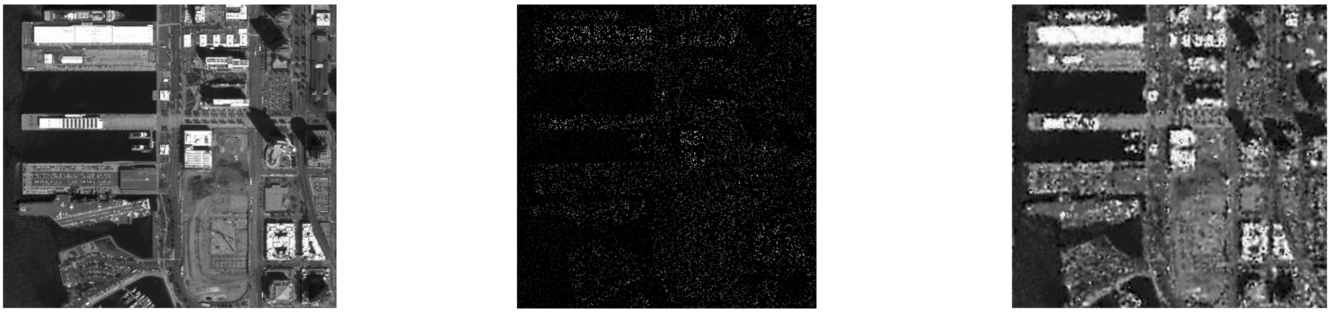
In this example (one percent of the data), the spline procedure is comparable in processing time to IMSCS, with iterations. However, to implement calculations of spline interpolation of an image of 512 by 512 pixels, significant computing power is required, therefore, in the seemingly simpler case – the first case (with ten percent of the information), where using IMSCS,with only one iteration, the restoration of a sparse image is much faster and it is better. **Table 2** shows the results of objective methods for evaluating reconstructed images – sharpness evaluation and average contrast. Specified in Table 2 IMSCS, means IMSCS, with 20 iterations. Compared with the data in Table 1 in Table 2 shows a more significant drop in the estimated parameters for IMSCS, compared to the original. In addition, in Table 2 shows the results of estimates for spline interpolation. The estimates of sharpness and average contrast in the reconstruction of the image by spline are even further from the original digital image than the estimates for IMSCS.

As a second example, let's take an optical image received from a satellite – a fragment of the city of San Diego in the USA. For this, a public Yandex map is used. We translate the cut fragment 512 by 512 pixels into a black and white image, where the brightness is distributed

**Table 2**

Image quality scores for Fig. 2.

| Original digital image 512 by 512 pixels "Portrait" | Original digital image (100 percent of the total data is known) | IMSCS reconstruction (1 percent of the total data is known) | Spline reconstruction (1 percent of the total data is known) |
|---|---|---|---|
| Sharpness score | 1.665 | 0.339 | 0.241 |
| Average contrast | 0.063 | 0.032 | 0.027 |



*а* *б* *в*

**Fig. 3.** *The original digital image of "San Diego", 512 by 512 pixels in size - (a); sparse image (10 percent of the total data volume is known Fig. 3a) - (b); IMSCS recovery Fig. 1b (256 harmonics 1 iteration) - (c).*

from 1 - conditionally black, to 255 - white. The initial sample thus formed is shown in **Fig. 3***a*, let's call it "San Diego". This aerospace image clearly shows the USS "Midway" aircraft carrier moored in the port, which has been turned into a museum ship since 1998 (located in the middle of the left side of the picture). Based on Fig. 3*a* we simulate a sparse image. That is, as in the first example "Portrait", according to a random uniform law, we will remove ninety percent of the information from Fig. 3*a*, thus we get Fig. 3*b*. Black field (gradation of brightness = 0) in Fig. 3*b* corresponds to the missing data, and the remaining ten percent of the informative elements of the image have their original values as in Fig. 3*a* (from 1 to 255).

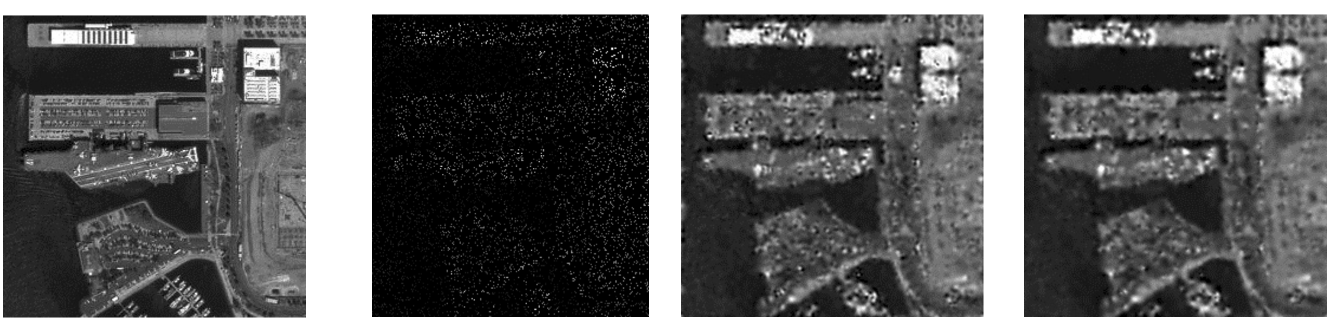
Restoration of Fig. 3*b* using IMSCS, is shown in Fig. 3*c*. As in the case with Fig. 1*c*, to reconstruct a sparse image by interpolating a sequentially calculated Fourier spectrum, it is enough to calculate 256 harmonics of the spectrum and additional iterations of refinement of each harmonic are not required. Despite small artifacts, all significant objects (city buildings, the outline of the port, an aircraft carrier) have a completely recognizable appearance. However, small image elements (cars, aircraft on board an aircraft carrier, etc.) are not restored. To implement spline interpolation (with ten percent of meaningful information present) requires extremely large computing power. Therefore, the calculations of statistical characteristics were carried out with an image of a smaller size (1/4 part of Fig. 3 is cut out) 256 by 256 pixels **Fig. 4.**

Visually, that is, by expert evaluation, it is difficult to determine which of the Fig. 4*c* or 4*d* is better. **Table 3** shows the data of objective assessments of image quality. It can be seen that the sharpness and average contrast of the

**Table 3**

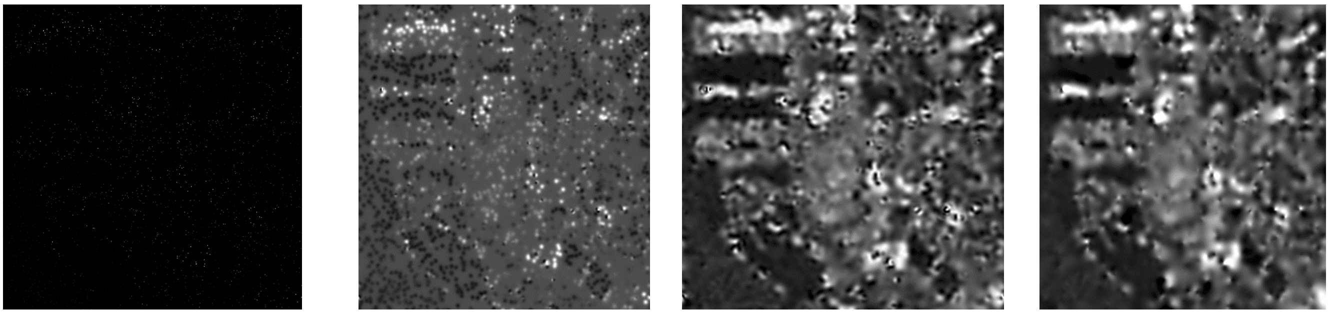
Image quality estimations for Fig. 4

| Original digital image 256 by 256 pixels "San Diego" | Original digital image (100 percent of the total data is known) | IMSCS reconstruction (10 percent of the total data is known) | Spline reconstruction (10 percent of the total data is known) |
|---|---|---|---|
| Sharpness rating | 14.071 | 4.908 | 3.374 |
| Average contrast | 0.16 | 0.117 | 0.093 |



*а* *б* *в* *г*

**Fig. 4.** *Original digital image 256 by 256 pixels - (a); Sparse image (10 percent of the total data volume is known) - (b); Reconstruction of IMSCS (128 harmonics, 1 iteration) - (c); Spline reconstruction - (d).*



*а* *б* *в* *г*

**Fig. 5.** *Sparse image (1 percent of the total amount of data in Fig. 3a is known) - (a); Restoration of IMSCS (256 harmonics, 1 iteration) - (b); Reconstruction of IMSCS (256 harmonics, 20 iterations) - (c); Reconstruction by spline interpolation - (d).*

reconstructed images, relative to the original sample, for the "San Diego" example suffered a much more significant decrease than for the "Portrait" example (compare with Table 1). At the same time, the overall sharpness of the original images 1.665 for "Portrait" and 14.071 for "San Diego" differ by almost an order of magnitude. A significant difference in values is also the average contrast of 0.063 for "Portrait" and 0.16 for "San Diego".

**Fig. 5** shows the results of the reconstruction of a sparse image when one percent of the total data volume is known of Fig. 3*a*. It should be noted that with only one percent of the initial data known, the procedure of iterative additional refinement of each of the restored harmonics is necessary. In our work, the number of iterations is 20.

Statistical quality estimates for reconstructions of the 99 percent sparse San Diego image are shown in **Table. 4.**

If the restoration of the sparse image "San Diego" when it's known 10 percent of the total amount of data still passes more or less acceptable, then if 99 percent of the original information is lost, it is no longer necessary to talk about any reconstruction, see Fig. 5. With a certain amount of imagination, you can see the rough outline of the coastline. The Yandex intelligent system for searching images on the Internet based on Fig. 5*c* or Fig. 5*e* can't even classify what it is. While for 90 percent sparsity recovery (Fig. 3*c*), Yandex classifies this as an aerospace image and offers "similar" ones. We remind you that with one percent of the original data for the "Portrait" example, Yandex search unmistakably recognized the image. The point is that the two images taken here as initial ones have significantly different autocorrelation functions (ACF). **Fig. 6** shows the normalized ACF for the original image "Portrait". **Fig. 7** - normalized ACF for "San Diego".

**Table 5** shows the number of pixels at which the ACF decreases to a certain level (0.8; 0.7; 0.5) relative to the maximum of the normalized ACF.

Let's call the data Table. 5 conditional correlation radii of the original tested images for different ACF levels. By definition,

**Table 4**

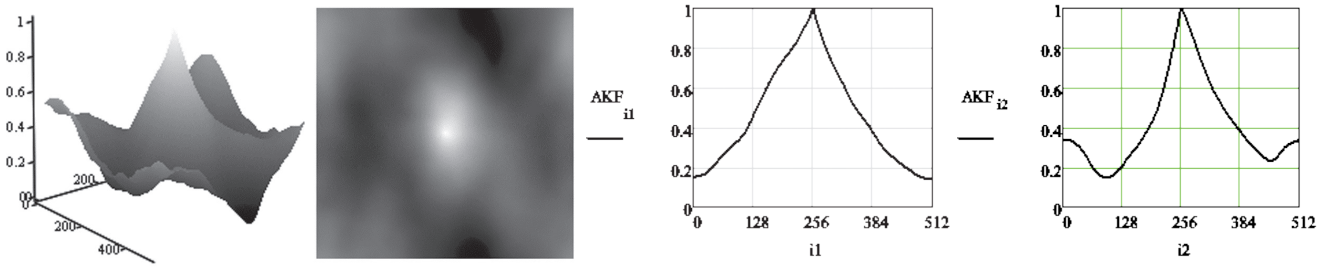
Image quality estimations for Fig. 5.

| Original digital image 512 by 512 pixels "San Diego" | Original digital image (100 percent of the total data is known) | IMSCS reconstruction (1 percent of the total data is known) | Spline reconstruction (1 percent of the total data is known) |
|---|---|---|---|
| Sharpness score | 16.441 | 1.364 | 0.798 |
| Average contrast | 0.176 | 0.072 | 0.053 |

**Table 5**

Conditional image correlation radius.

| Decrease from the maximum of the normalized ACF | 0.8 | 0.7) | 0.5 |
|---|---|---|---|
| "Portrait" 512*512 Distance in pixels from maximum ACF | 28 | 50 | 91 |
| "San Diego" 512*512 Distance in pixels from maximum ACF | 3 | 6 | 25 |



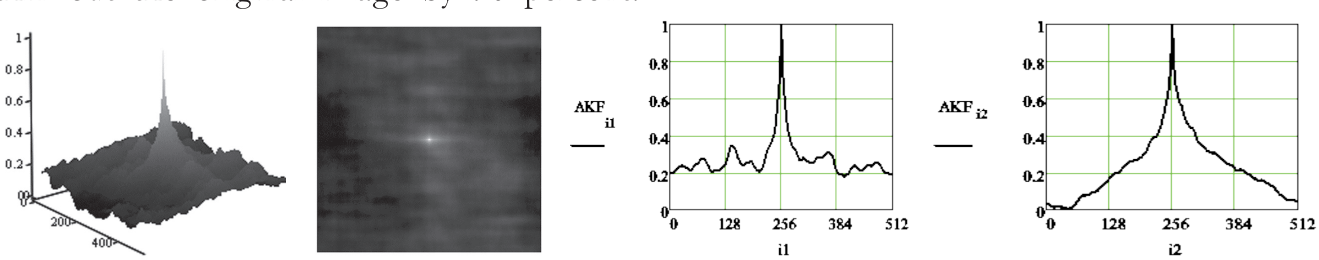

*a* *b* *c* *d*

**Fig. 6.** *Autocorrelation function for the original image "Portrait" (Fig. 1a). Volumetric image of the ACF - (a); Volumetric image of the ACF top view - (b); Cross section of the ACF - (c); Cross section of the ACF orthogonal to the section Fig. 6c - (d).*

the signal correlation radius indicates at what distance the signal samples can be considered statistically independent. The more homogeneous is the area image, the larger the correlation radius for it. Areas with a small variation in brightness in the image field, from the point of view of correlation analysis, are of little information. For "Portrait" it is a slowly changing background, cheeks, forehead, shoulder, etc., for "San Diego" it is the sea. And vice versa, the more frequent brightness changes in the image, the narrower the autocorrelation function becomes, indicating to the researcher that sparsity should be used with great care to save memory space. Otherwise, if the number of pixels of a sparse image sufficient for the reconstruction of small image details does not fit into the radius of the ACF, then restoration will not occur. If, in the San Diego aerospace example, the most informative areas are the city, roads, port outlines with moored large ships, then it is possible to thin out the original image by 90 percent. However, if the target is individual cars and small boats, then meaningful information needs to be added to the sparse image. Thus, the allowable sparseness must be chosen for each specific technical task, and for a certain type of image.

## 3. CONCLUSION

The work proposed here investigates the application of the interpolation method of a sequentially calculated Fourier spectrum for the reconstruction of different types of images with a high degree of sparseness (in the absence of 90 to 99 percent of significant elements). To improve the operation of the IMSCS, with a strong sparseness of the initial data, its algorithm includes an internal procedure for additional iterative refinement of each of the reconstructed harmonics of the spatial spectrum. The possibility of restoring an image distorted by sparseness, with the degree of information content necessary for a specific technical task, must be evaluated by the radius




*a* *b* *c* *d*

**Fig. 7.** *Autocorrelation function for the original image "San Diego" (Fig. 3a). Volumetric image of the ACF - (a); Volumetric image of the ACF top view - (b); Cross section of the ACF - (c); Cross section of the ACF orthogonal to the section Fig. 7c - (d).*

ACF correlation characteristic for the types of images used. According to the objective criteria – "average contrast" and "sharpness estimation" [18], it was found that the spline interpolation used for comparative analysis [19, 20] is significantly inferior to IMSCS. The conducted study allows us to conclude that it is fundamentally possible to use IMSCS to restore sparse images both for the reconstruction of lacunae and for the reduction of the amount of data.

## 4. APPLICATION

### Algorithm for the method of interpolation of a sequentially calculated Fourier spectrum (IMSCS)

The complete image $Y_p$ can be written as the sum of the images outside the gap $Y_{-L}$ and inside it $Y_L$:

$$Y_p = Y_{-L} + Y_L. \tag{A1}$$

Let the mask that can be used to obtain an image with a gap from the full image is denoted as $L$, and this mask is equal to 1 inside the gap and zero outside the gap.

Then expression (A1) can be written as

$$Y_p = Y_{-L} + LY_p. \tag{A2}$$

Let us find the spectrum (A2) and obtain

$$(1 - fL)*fY_p = fL_{-L}, \tag{A3}$$

where $fL$ – is the spectrum of the mask $L$ to obtain a gap; $fY_p$ – full image spectrum; $fL_{-L}$ is the spectrum of the image with a gap, and the sign (*) denotes the convolution operation. Solution (A3) can be produced iteratively.

Thus, the work of the method of interpolation of the sequentially calculated Fourier spectrum (IMSCS) is as follows:

1. Localize the gap, i.e. we determine the coordinates of all the pixels of the image that need to be filled using IMSCS.
2. Calculate the average brightness of the image using only reliably known pixels (without gap pixels). Thus, the zero harmonic of the spatial spectrum of the improved image is estimated.
3. Fill the gap with the brightness values calculated in the previous paragraph, that is, we get the first approximation of filling the gap. In this case, the undistorted part of the image (originally reliably known) does not undergo any changes.
4. Calculate the full spatial spectrum of the image obtained in the previous paragraph (with the gap filled).
5. Having limited the spectrum from the previous paragraph to zero and first harmonics, we calculate the second approximation of the brightness values to fill the gap.
6. Fill the gap with the brightness values calculated in the previous paragraph, and we get the second approximation of filling the gap. In this case, the undistorted part of the image (originally reliably known) does not undergo any changes.
7. Next, cycle through steps 4, 5, 6, each time sequentially, in step 5, increasing the number of harmonics in the spectrum to calculate the gap filling brightnesses by one, up to the highest possible. Thus, the gap is consistently filled, and the undistorted part of the image (originally known for certain) does not undergo any changes.

As a result of the proposed algorithm, the retouched gap is gradually filled with an image that is more and more consistent with the spectrum of the surrounding image (initially known for certain).

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