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Mathematical model for spacecrafts identification

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Abstract: **Proposed below is a mathematical model for reception and processing of spurious ("uncontrolled" radiation) from constantly operating units of the special complex installed onboard a spacecraft. This model makes it possible to implement a new technique for identification of such radiation thus improving the capabilities of the space monitoring. At the same time creation of stand-alone radio systems solely for identification of the spacecrafts based on the proposed mathematic model would require significant expenses therefore it is worthwhile to add new identification hardware to the existing ground-based radio systems used for space flight control. Identifying features in the proposed mathematical model are the parameters of signals of the uncontrolled radiation from constantly operating units installed onboard spacecrafts that "leak" through antenna systems (master generators, heterodyne oscillators in the spacecraft radio receivers). Identification of spacecrafts by uncontrolled radiation from the receiving master generators and heterodyne oscillators involves keeping track of the behavior of oscillation parameters as well as identifying signs that distinguish the oscillations of one generator from another. Since the uncontrolled radiation from heterodyne oscillators is a harmonic oscillation, it features such parameters as amplitude, frequency and initial phase. It is impossible to use the amplitude and initial phase of the signal for identification purposes because the propagation medium strongly affects these parameters. The most informative for identification purposes is the frequency of oscillations, or rather, the behavior of the frequency changes over time. These changes are due to the frequency instability of the onboard master generators. The behavior of the frequency change depends on the characteristics of each onboard generator, which serves as a basis for identification. It should be noted that the identification process can be conditionally divided into two stages: the first stage includes validation of models and processing (evaluation) algorithms while the second stage involves classification of the results of processing (evaluation).**

Keywords: **identification, spacecraft, radio engineering system, space control, uncontrolled radiation, effective antenna area**

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Contents

1. INTRODUCTION

For many space systems it is typical to have onboard transmitting devices of the spacecraft disconnected when outside the visibility range of its radio systems. This fact significantly complicates identification of maneuvering spacecrafts.

Based on the above fact, we found it worthwhile to develop such an identification technique that would eventually improve operational quality of a space monitoring system.

In doing so, for the identification parameters we decided to use signals associated with spurious "uncontrolled" radiation (UCR) coming from constantly operating units of the onboard equipment (i.e. heterodyne oscillators, master generators).

The proposed identification technique is based on the algorithm for evaluating parameters of the correlated component of the instability process of master generators installed onboard the spacecraft, with the process itself regarded as an identifying factor [1].

2. MAIN PART

To solve the problem in question we evaluated the feasibility of reception and processing of spurious signals radiated from certain radio systems (RS) of a ground-based complex. Results of such evaluation showed that antenna arrangements of these systems could be used for SC identification purposes as well [4].

The only signal received from an "enclosed" SC could be an UCR from onboard reciever's heterodyne oscillators and master generators. Sources of frequency instability in the onboard RS can be conditionally classified by the following criteria [1,9]:

Systematic changes of frequency caused by drifts. These changes are due to aging of the resonator material and are extremely slow. They are also called "long-term" instability and they are measured in frequency chage per hour, day, month or year depending on the device type and application specifics.

Deterministic periodic frequency deviations caused by incidental frequency modulation from surrounding processes, such as instability of power supplies, crosstalks, temperature variations, vibrations, pressure etc.

Frequency deviations triggered by random fluctuations due to the use of electronic components in the devices. Such frequency fluctuations are called "short-term" instability.

Instability will include three main components. The first one is a slowly varying component $g(t)$ (dashed line in **Fig. 1**) that describes non-stationarity of the process and can be considered as a deterministic component (at least throughout a certain sampling period).

This component is described by a polynomial

$g(t) = C_0 + c_1 t + c_2 t^2 + \ldots$.

The second component $m(k)$ accounts for slowly changing fluctuations $\varphi(k)$ relative to component $g(k)$. This component can be considered random local stationary process with long correlation time. In this case

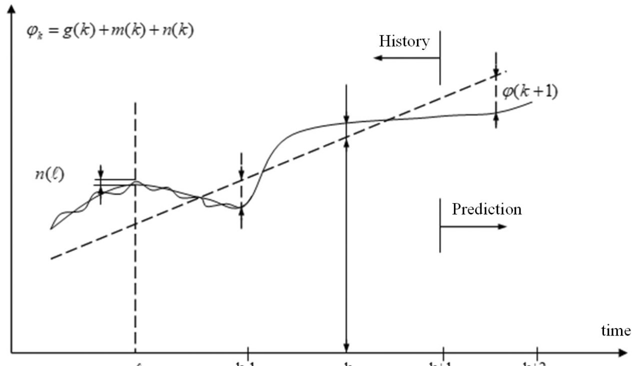

Fig. 1. *Illustration of frequency instability process components.*

when $g(k) = g_0 = \text{const}$, it is also subject to prediction.

The third component $n(k)$ represents fast frequency fluctuations that are stationary throughout the sampling period but have low correlation time. Therefore, the instability process due to phase drifts will be as follows

$$\varphi(t) = g(t) + m(t) + n(t). \quad (1)$$

However, some peculiarities of the process components (1) require development of special algorithms for their analysis.

Component $g(t)$ of the process (1) features relatively slow changes within the observation time interval. Therefore, it is worthwhile to regard it as constant within this interval and exclude from the analysis of the instability models.

Component $m(t)$ of the process (1) is a locally stationary process with a relatively long correlation time. To determine its parameters, we find it appropriate to use the least squares method [4,5] or dynamic filtration method [6,7]. In order to lower computation time and increase speed we deem necessary to use their recurrent modifications with relatively small portion of the gathered information being involved in the processing.

Component $n(t)$ of the process (1) describes rapid phase fluctuations over time which are the most informative in terms of detecting peculiarities of master generators. $n(t)$ is more informative than $m(t)$ because component $m(t)$ has constituents of unknown Doppler frequency shift. Presence of the Doppler shift often make it impossible to ensure identification by component $m(t)$.

We will analyze component $m(t)$ of the phase shift process $\varphi(t)$ (1) by using a model as defined by state and watch equations

$$\begin{cases} \dot{x}(t) = Fx(t) + Gg(t), \\ m(t) = Hx(t), \end{cases} \quad (2)$$

$$\begin{cases} z(t) = H_1 m(t) + n(t), \\ z(t) = \varphi(t) - g(t), \end{cases} \quad (3)$$

while taking into account that process n(t) is regarded as the white noise because its correlation time is considerably lower than correlation time of component m(t). This model is used for making a modification to the dynamic filtration algorithm.

Additionally, evaluation of the model parameters of component m(t) for the least squares method was considered

$$Z(t) = AM(t) + n(t), \quad (4)$$

where A is a constraint matrix; $M(t)$ is a vector of the linear model parameters that are being evaluated using the least squares method (LSM); $n(t)$ is a component in (1), $z(t) = \varphi(t) - q(t) = m(t) + n(t)$.

The experience in research of stationary quickly fluctuating processes similar to $n(t)$ shows that they can be highly effectively described by autoregressive models [2,3,6]. Therefore, further description and analysis of component $n(t)$ was done in terms of autoregressive models:

$$\begin{cases} y(t) = \sum_{i=1}^{n} a_i y(t-i) + e_1(t), \\ n(t) = e_1(t) + e_2(t), \end{cases} \quad (5)$$

where a_i are coefficients of an autoregressive model; $e_1(t)$ is a generating noise, $e_1(t) = \sum_{i=1}^{n} a_i n(t-i) + e_1(t-i)$, $e_2(t)$ is a white noise of measurements, $e_2(t) = e_1(t) - \sum_{i=1}^{n} a_i e_2(t-i)$, $y(t)$ is a process of oscillation phase change caused by poorly correlated component of the instability; $n(t)$ is a component of process $\varphi(t)$ (2).

We will develop equations that describe algorithms for evaluation of components (1) $m(t)$ and $n(t)$ for models (2)-(5).

Taking into account the peculiarities of the instability model (1), we will analyze the feasibility of developing a recurrent algorithm of LSM using a "sliding" window. To do so we will first investigate a linear model of component $m(t)$ of the phase change process $\varphi(t)$ (1), presented as follows

$Z(t) = AH + n(t)$

with a measurement vector, where $z_1,\ldots, z_N$ are measurements of the signal phase at moments $t_1,\ldots, t_N$, matrix $A=\left[A_1^T,\ldots,A_n^T\right]$, where $A_i^T=[a_i,\ldots,a_{ir}]$ is i-th row of matrix A, vector of parameters in question $M=[M_1,\ldots,M_r]^T$, and vector of random measurement errors $[n_1,\ldots,n_n]^T$, where $n_1, \ldots, n_n$ is a value of component $n(t)$ at time moment $t_1, \ldots, t_n$. Evaluation using the least squares method $\widehat{M}(n,n+1)$ can be made based on measurements $z_{n+1}, \ldots, z_{n+\ell}$ while using matrix rows from $(n + 1)$ to $(n + \ell)$.

As a result, expressions were developed to determine transfer coefficients when a new measurement is selected

$$K^{(1)}(n,n+\ell)=\sum\nolimits_{n=1}^{T}(n,n+\ell)A_{n+\ell+1}\times \\ \times\left[J+A_{n+\ell+1}^T\sum\nolimits_{n=1}^{T}(n,n+\ell)A_{n+\ell+1}\right]^{-1}, \quad (6)$$

$$\sum\nolimits_{n=1}^{T}(n,n+\ell+1)= \\ =\left(J-K^{(1)}(n,n+\ell)A_{n+\ell+1}^T\right)\sum\nolimits_{n=1}^{T}(n,n+\ell), \quad (7)$$

where ℓ is a unity matrix, or in detail as follows

$$\sum\nolimits_{n=1}^{T}(n,n+\ell+1)=\sum\nolimits_{n=1}^{T}(n,n+\ell)- \\ -\sum\nolimits_{n=1}^{T}(n,n+\ell)A_{n+\ell+1}\times \\ \times\left(A_{n+\ell+1}^T\sum\nolimits_{n=1}^{T}(n,n+\ell)A_{n+\ell+1}+1\right)^{-1}\times \\ \times A_{n+\ell+1}^T\sum\nolimits_{n=1}^{T}(n,n+\ell), \quad (8)$$

and when forgetting the outdated information

$$K^{(2)}(n,n+\ell+1)=\sum\nolimits_{n=1}^{T}(n,n+\ell+1)\times \\ \times A_{n+1}\left[J-A_{n+1}^T\sum\nolimits_{n=1}^{T}(n,n+\ell+1)A_{n+1}^T\right], \quad (9)$$

$$\sum\nolimits_{n=1}^{T}(n+1,n+\ell+1)= \\ =\left(J+K^{(2)}(n,n+\ell+1)A_{n+\ell+1}^T\right)\times \\ \times\sum\nolimits_{n=1}^{T}(n,n+\ell+1). \quad (10)$$

or in detail as follows

$$\sum\nolimits_{n=1}^{T}(n+1,n+\ell+1)=\sum\nolimits_{n=1}^{T}(n,n+\ell+1)+ \\ +\sum\nolimits_{n=1}^{T}(n,n+\ell+1)\times \\ \times A_{n+1}\left[J-A_{n+1}^T(n,n+\ell+1)A_{n+1}\right]^{-1}\times \\ \times A_{n+1}^T\sum\nolimits_{n=1}^{T}(n,n+\ell+1). \quad (11)$$

Vectors K(1) and K(2) having length of $r + 1$ are called transfer coefficients when introducing a new measurement and when forgetting, respectively. Another more PC-friendly presentation of the transfer coefficient K(1) [7] can be used

$$K^{(1)}(n,n+\ell)=\sum\nolimits_{n=1}^{T}(n,n+\ell+1)A_{n+\ell+1} \quad (12)$$

and K(2) coefficient

$$K^{(2)}(n,n+\ell+1)=\sum\nolimits_{n=1}^{T}(n+1,n+\ell+1)A_{n+1}. \quad (13)$$

Furthermore, by doing a block-by-block multiplication of matrix $A^T(n,\ n+l+1)$ and vector $z(n,\ n+l+1)$, we find that

$$A^T(n+1,n+\ell+1)z(n+1,n+\ell+1)= \\ =A(n,n+\ell+1)z(n,n+\ell+1)-A_{n-1}z_{n+1}. \quad (14)$$

So, using recurrent procedure of the LSM over the sliding window makes it possible to establish a relatively simple procedure for evaluation of $m(t)$ component parameters of process $\varphi(t)$ (1). This procedure will be used for processing the measurement results of UCR signal phase.

For complete research and synthesis of an optimum algorithm for identification of SC UCR, we performed an analysis of the evaluation procedure by using a linear Kalman filter with a finite memory of component $m(t)$ of process $\varphi(t)$ (1) as an alternative to modification of LSM (7)-(14).

To do so, we analyzed a dynamic system with discrete time being analogous to (2), (3) and described by equations of state

$$x(k+1) = \Phi(k+1, k)x(k) + G(k+1)\xi(k+1),$$
$$m(k+1) = Hx(k+1) \qquad (15)$$

and equations of observation

$$z(k) = H_1 m(k) + n(k),$$
$$z(k) = \varphi(k) - q(k) = m(k) + n(k), \qquad (16)$$

where $\Phi(k + 1, k)$ is a transfer matrix; k is a variable denoting filtration iteration index.

Practical implementation of this identification method was carried out in the following order. Preliminary frequency search is done using a Fourier-transform processor within 50 kHz band based on a sequential scanning method for a given frequency band. After detection of a signal and estimating frequency with an accuracy of 25 kHz, the signal is then directed to the digital spectrograph that uses the same Fourier-transform processor. Data from the analyzer output are then transmitted to a minicomputer for further processing.

Sequence of subsequent operations performed on the received signal is shown as a block diagram in **Fig. 2**.

Detected signal with phase change pattern denoted as $\varphi(t)$ is then directed to a device that subtracts long-term instability component $g(t)$ from $\varphi(t)$. It should be noted that this procedure is performed by using a method that involves finding mean value over the observation interval with subsequent calculation of the result from each measurement.

In accordance with model (1), calculation of $g(t)$ from $\varphi(t)$ results in the fact that now we need to process a sum of two components of process $\varphi(t)$: $m(t)$ and $n(t)$ (see Fig. 2). Component $m(t)$ is a slowly varying component due to both instability of a generator and unknown Doppler frequency component. Since the Doppler component is unknown, component $m(t)$ of process $\varphi(t)$ cannot be used for SC identification. Therefore, we deem necessary to evaluate parameters of model $m(t)$, and estimate its state vector and then exclude $m(t)$ from further analysis.

To accomplish this purpose we use a procedure for linear optimal filtration and recurrent LSM with a sliding window.

Selection of a finite memory procedure is impeded by not having accurate information about the order of model $m(t)$. It was believed that presence of sliding windows would slightly alleviate this problem.

Evaluation results for component $m(t)$ are subtracted from sum $m(t) + n(t)$ (see Fig. 2) and further processing is performed on the fast-changing component $n(t)$. This component is governed by the peculiarities of a master generator. Its values are then fed into a device that implements time-series analysis procedure. The device outputs

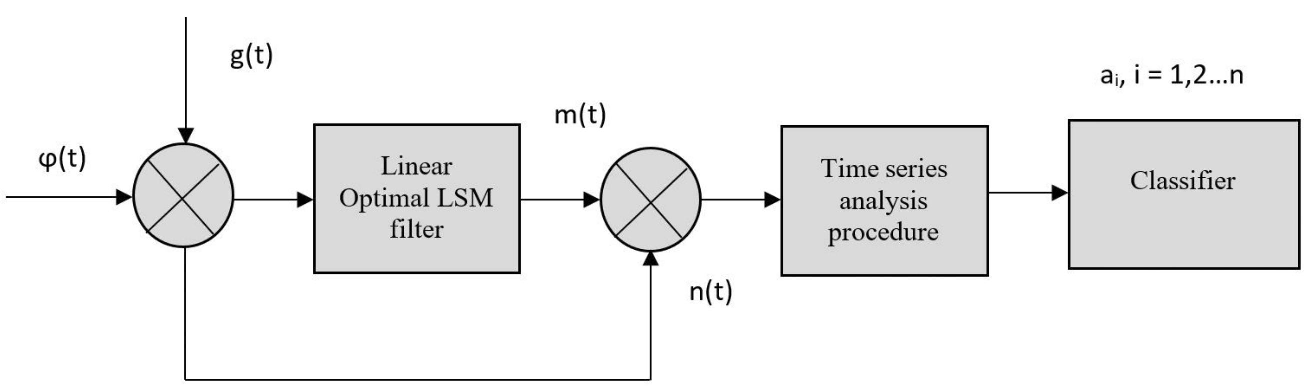

Fig. 2. *Sequence of operations performed on the received signal during SC identification.*

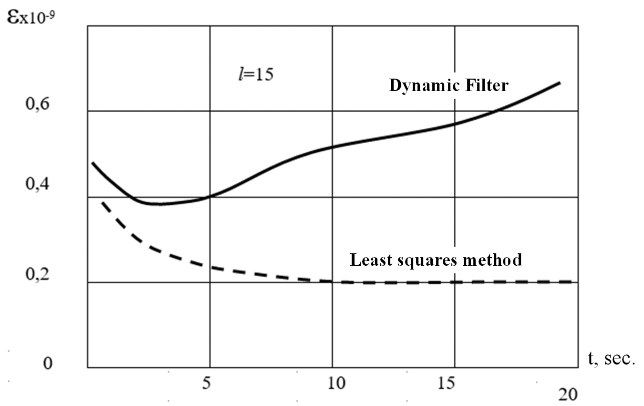

Fig. 3. *Algorithm comparison results for "sliding window" with l = 15.*

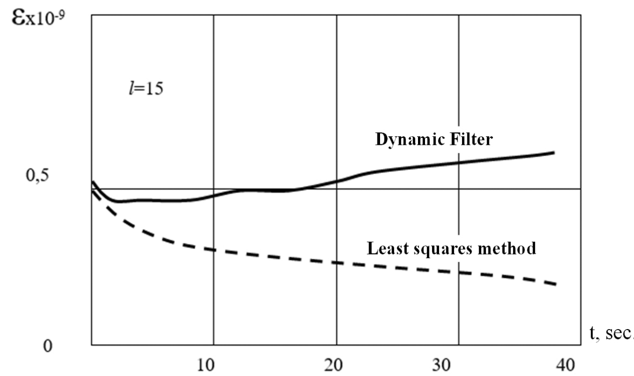

Fig. 5. *Algorithm comparison results for "sliding window" with l = 15.*

estimations of a_1 parameters of process $n(t)$. Values of these parameters are then directed to a classifier where a decision is made about a type and identity of the detected SC.

Fig. 3 and **4** for an observation time of 20 sec. show results of comparison between evaluation algorithms based on the least squares method and dynamic filtration method. Y-axes show mean values $\varepsilon(t)$ that are determined as sum of squared differences between measurement results $f(t_i)$ at moments t_i and evaluations of process $\hat{m}(t_i)$ as divided by number i.

$$\varepsilon(t) = \frac{1}{i}\sum_{k=1}^{i}[f(t_i) - \hat{m}(t_i)]^2,\ i \sim 1,\ldots,N,$$

where N is the number of measurements. Parameter i determines the size of the "sliding" window.

Analysis of curves (see Fig. 3 and 4) shows that dynamic filtration algorithm turns out to be unstable under conditions of high prior uncertainty about parameters of the process being filtered. Increasing window size to 15 makes it possible to slightly improve evaluation results obtained using the dynamic filtration algorithm as shown in **Fig. 5** for an observation interval of 40 sec.

However, further increasing window size l again results in increased ε, which is most likely due to some inconformity of the model with the process (**Fig. 6**)

Unlike the dynamic filter, recurrent LSM procedure over the sliding window turns out to be more stable and yields appropriate

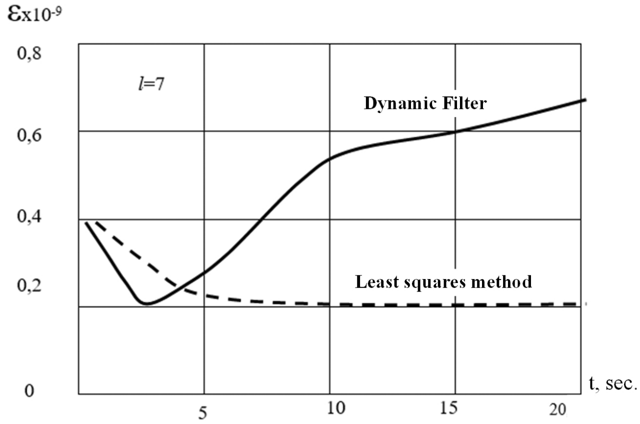

Fig. 4. *Algorithm comparison results for "sliding window" with l = 7.*

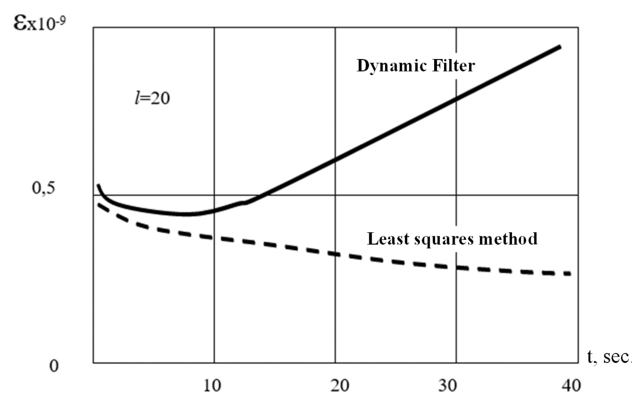

Fig. 6. *Algorithm comparison results for "sliding window" with l = 20.*

results. Analysis of graphs shows that LSM algorithm is sensitive to the window size l.

3. CONCLUSION

Developed data processing algorithm for instability processes of master generators of onboard radio systems makes it possible to significantly simplify evaluation of the model parameters in question. Additionally, use of a "sliding window" when evaluating the parameters being processed improves quality and speed of SC identification.

REFERENCES

1. Kozelkov SV. Primenenie planetnogo radiolokatora dlya zadach navigatsii kosmicheskih apparatov. *Sistemy obrabotki informacii,* Khar'kov, NANU, PANM, HVU, 1999, 1(5);139-142. [Kozelkov SV. The use of planetary radar for spacecraft navigation tasks (in Russ.). Sistemy obrabotki informacii, Kharkiv, NANU, PANM, HVU, 1999, 1(5);139-142].
2. Issledovanie i razrabotka printsipov postroeniya, putej apparaturnoj realizatsii i povysheniya effektivnosti funktsionirovaniya spetsial'nyh sistem navigatsii i peredachi informatsii radiosignalami millimetrovogo diapazona voln. *Otchet o NIR* (zaklyuchitel'nyj) p/ya M-5068. "Raduga", №398; inv. №2586, 1985, 74 s. [Research and development of the principles of construction, ways of hardware implementation and improvement of the efficiency of special navigation systems and information transmission by millimeter wave radio signals. Research report (final), the post M-5068, "Raduga", n. 398; inv. n. 2586, 1985, 74 p.].
3. Kantor LYa. *Spravochnik po sputnikovoj svyazi i veshchaniyu*. Moscow, Radio i svyaz' Publ., 1986, 528 s. [Kantor LYa. Handbook of Satellite Communications and Broadcasting. Moscow, Radio i svyaz'Publ., 1986, 528 p.].
4. Chaki F. *Sovremennaya teoriya upravleniya: Nelinejnye, optimal'nye i adaptivnye sistemy*. Moscow, Mir Publ., 1975, 251 s. [Chaki F. Modern control theory: Nonlinear, optimal and adaptive systems. Moscow, Mir Publ., 1975, 251 p.].
5. Puhov GE. Preobrazovaniya Tejlora i ikh primenenie v elektrotekhnike i elektronike. Kiev, Naukova dumka Publ., 1978, 180 s. [Puhov GE. Taylor transformations and their application in electrical engineering and electronics. Kiev, Naukova dumka Publ., 1978, 180 p.].
6. Pupkov KA, Kapalin VI, Yushchenko AS. *Funkcional'nye ryady v teorii nelinejnyh sistem*. Moscow, Nauka Publ., 1978. 448 s. [Pupkov KA, Kapalin VI, Yushchenko AS. Functional series in the theory of nonlinear systems. Moscow, Nauka Publ., 1978, 448 p.]
7. Kozelkov SV. Progress kosmicheskoj tekhniki – ot "Shattla" do kosmicheskoj stancii. *Trudy IIER,* 1987, №3, 192 s. [Kozelkov SV. The progress of space technology - from the "Shuttle" to the space station (in Russ.). *Trudy IIER*, 1987, no. 3, 192 p.]
8. Pogorelov AI, Kupchenko LF. *Obshchie voprosy postroeniya signalov izmeritel'nyh radiosistem. Prostranstvenno-vremennaya obrabotka signalov.* Khar'kov, HAI Publ., 1986, s. 3-11. [Pogorelov AI, Kupchenko LF. General issues of building signals of measuring radio systems (in Russ.). Prostranstvenno-vremennaya obrabotka signalov. Khar'kov, HAI Publ., 1986, pp. 3-11.]
9. Kozelkov SV, Bogdanovskij AN, Polyakov AL, Rachinskij AP. Matematicheskaya model' identifikatsii kosmicheskikh

ob'ektov po nekontroliruemym izlucheniyam bortovoj apparatury kosmicheskikh apparatov. *Sistemy obrabotki informacii*, 2007, Vyp. 4, s. 51-55. [Kozelkov SV, Bogdanovskij AN, Polyakov AL, Rachinskij AP. Mathematical model of identification of space objects by uncontrolled radiation of onboard spacecraft equipment (in Russ.). *Sistemy obrabotki informacii,* 2007, vol. 4, pp. 51-55.].