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Mathematical modeling of temperature changes impact on artificial ice islands

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Abstract: **The article is devoted to the numerical solution of the Stefan problem for thermal effects on an artificial ice island. For modern tasks of the development of the Arctic, associated with the exploration and production of minerals, it is important to create artificial ice islands in the Arctic shelf, due to the speed of their construction, economic feasibility and other factors. The most important task for the exploitation of such islands is their stability, including against melting. This paper discusses the issue of the stability of ice islands to melting. For this, the Stefan problem on the change in the phase state of matter is formulated. An enthalpy solution method is constructed, and the applicability of this method is considered. For the numerical solution, the Peasman-Reckford scheme is used, which is unconditionally spectrally stable in the two-dimensional case, which allows to freely choose the time step. In addition, the developed approach takes into account the flow of water and the flow of melted water, which is important in the task at hand. The developed computational algorithms are parallelized for use on modern multiprocessor computing systems An approach is implemented for modeling thermal processes in the thickness of an arbitrary mass of substances, taking into account arbitrary initial conditions, environmental conditions, tidal currents of water, and solar radiation. This approach was used to calculate the temperature distribution in the thickness of the ice island, as well as to study the impact of seasonal temperature changes on the stability of the island.**

Keywords: **mathematical modeling, ice island, Stefan problem, enthalpy method, Peasman-Rechford scheme**

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Contents

1. INTRODUCTION

Modern tasks of the development of the Arctic set new requirements for the offshore infrastructure facilities. Artificial ice islands provide a low-cost and environmentally friendly alternative to conventional drilling platforms for oil and gas production in the Arctic. The approach has already been successfully implemented in Canada [1]. Ice islands and their advantages in the development of the Arctic are considered in [2]. In [3], a numerical solution of the problems of stability of ice islands to elastic impact was carried out. But during operation, in addition to mechanical stability, it is necessary to take into account resistance to thermal influences (melting of an ice island). To correctly solve the problem of stability of an ice island to static and dynamic elastic effects, it is necessary to know the temperature distribution inside the ice island. It is also important to study the island's resistance to seasonal temperature fluctuations. Approaches for taking into account climatic impacts on the ice island were considered in [4-6].

This article proposes an approach to numerical modeling of the melting of ice islands, based on solving the problem of the evolution of a system with different phase states of matter and changing the location of the boundary between these phases - Stefan's problem [7-12]. The works [13-16] are devoted to the numerical solution of problems with phase transitions. In [13,14], the method of lines was considered. The finite element method and finite difference methods are also often used in practice [15,16].

In this work, the enthalpy approach is used [17]. For this, the task is reduced to the function of heat content. The developed approach also takes into account water flow, melted water runoff, and other important phenomena. With its help, it was possible to obtain the temperature distributions in the ice island, as well as to conduct a study on resistance to seasonal temperature fluctuations.

2. MATERIALS AND METHODS

2.1. Mathematical model

Consider a three-dimensional computational domain of arbitrary volume V. Let us write down the first law of thermodynamics and pass from internal energy to enthalpy:

$$\delta Q = dH - VdP, \tag{1}$$

where δQ – heat flow, dH – enthalpy change, V – volume of matter, dP – pressure change. For sufficiently slow (practically equilibrium) processes with constant pressure, one can (1) takes the form:

$$\delta Q = dH. \tag{2}$$

Enthalpy is a function of state. Let us select in the computational domain a small element of volume V. For it, the first law of thermodynamics can be rewritten as:

$$-\int_{S=\partial V} \vec{q} \cdot d\vec{S} = \int_V \frac{\partial H}{\partial t} dV, \tag{3}$$

where $\vec{q}$ – heat flow per unit area. Using Gauss's theorem, we turn to the continuity equation for heat:

$$div(\vec{q}) + \frac{\partial H}{\partial t} = 0. \tag{4}$$

Applying the Fourier thermal conductivity formula, we obtain from (4):

$$\vec{\nabla}(-k(H,x,y,z)\vec{\nabla}T) + \frac{\partial H}{\partial t} = 0, \tag{5}$$

where $k(H,x,y,z)$ – coefficient of thermal conductivity, which depends on the substance and its phase state.

Let's introduce the function of heat content:

$$Q = \begin{cases} \rho_S C_S T, & T < T_0, \\ \rho_L C_L (T - T_0) + \rho_S C_S T_p + \rho_S \lambda, & T > T_0, \end{cases} \tag{6}$$

where ρ_S and C_S – density and specific heat of solid, and ρ_L and C_L – ones of liquid state, T_0 – the phase transition temperature, λ – specific heat of fusion in a given volume.

The inverse transition to temperature is possible by the formula:

$$T = \begin{cases} Q \cdot \rho_S^{-1} C_S^{-1}, & Q < \rho_S C_S T_S = Q_-, \\ T_0, & Q_- < Q < Q_+, \\ \dfrac{Q + (\rho_L C_L - \rho_S C_S) T_0 - \rho_S \lambda}{\rho_L C_L}, & Q > \rho_S C_S T_S + \rho_S \lambda = Q_+, \end{cases} \tag{7}$$

where Q_- and Q_+ – limits of the value of heat content at the phase transition temperature.

We believe that the thermophysical constants are determined by the substance, which is constant at a given point. Note that mapping (6) is not continuous with respect to temperature. This correlates with the fact that at the melting point, a substance can be in different phase states.

Enthalpy and heat content coincide up to a constant. Substitute expressions (6) and (7) into (5) and obtain a hyperbolic equation for the evolution of the system through the heat content:

$$\frac{\partial}{\partial x}\left(k(Q)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(Q)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k(Q)\frac{\partial T}{\partial z}\right)=\frac{\partial H}{\partial t}. \quad (8)$$

Note that the thermal conductivity coefficients also depend on the coordinate of the point, but in our model, at each point, the substance remains constant and only its phase state changes, so this dependence is not reflected in this equation.

The value of thermal conductivity $k(Q)$ for each phase of the substance is known. For intermediate values, during phase transitions, for thermal conductivity, we can write:

$$k(Q)=\begin{cases} k_S, & Q<Q_-, \\ k_S+(\mathrm{k}_L-\mathrm{k}_S)\cdot\dfrac{Q-Q_-}{Q_+-Q_-}, & Q_-<Q<Q_+, \\ k_L, & Q>Q_+, \end{cases}$$

where k_S – thermal conductivity of a solid state of matter, k_L – thermal conductivity of a liquid state.

2.2. Numerical method

For the numerical solution of equation (8), the Peasman-Reckford scheme (in the two-dimensional case, referred to as the longitudinal-transverse scheme) [18,19] was chosen since it showed the best results in terms of operating speed. In the two-dimensional case, it is unconditionally spectrally stable, which makes it possible to freely choose the time step, but at the same time it can be solved in a time linear in the number of nodes. The computation is carried out in two steps: during the first, the scheme is explicit in one direction, and implicit in the second; during the second step, the directions change:

$$\frac{2\Delta V}{\tau}\left(Q^{j+1}-Q^{j+\frac{1}{2}}\right)=\Lambda_x Q^{j+\frac{1}{2}}+\Lambda_y Q^{j+1},$$

$$\Lambda_x Q_{i_x,i_y}=k_{i_x+\frac{1}{2}}\frac{T(Q_{i_x+1,i_y})-T(Q_{i_x,i_y})}{h_x}+k_{i_x-\frac{1}{2}}\frac{T(Q_{i_x-1,i_y})-T(Q_{i_x,i_y})}{h_x}, \quad (9)$$

$$\Lambda_y Q_{i_x,i_y}=k_{i_y+\frac{1}{2}}\frac{T(Q_{i_x,i_y+1})-T(Q_{i_x,i_y})}{h_y}+k_{i_y-\frac{1}{2}}\frac{T(Q_{i_x,i_y-1})-T(Q_{i_x,i_y})}{h_y}.$$

The schematic template is shown in **Fig. 1**.

When modeling thermal processes in an ice island, it is necessary to take into account that ice melting in the upper part of the island can flow downward, thus changing the substance at the mesh point. For this, at each time step, all nodes of the computational mesh are checked for the fulfillment of the melting condition. If it is satisfied, the substance is replaced in the node. In addition, the model takes into account the flow in the liquid, as a result of which mixing occurs. The realization of all these effects implies the modification of the values of heat content and matter at the grid nodes after solving equations (8) when certain conditions are reached.

2.3. Software implementation

The developed program accepts as input a rectangular two-dimensional uniform mesh, as well as configuration files and initial temperature distribution. In the configuration file, it is possible to specify all the characteristics of substances, the parameters of the task, such as the time step and the total number of steps, as well as the frequency of recording the fields of temperatures, heat content and aggregate state into the VTK file for subsequent visualization.

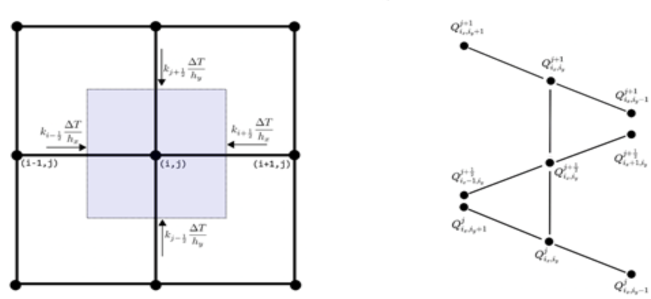

Fig. 1. *Template (right) and illustration of the method working on a two-dimensional mesh (left).*

The program solves the problem in several stages. At the beginning, the correctness of the input data is checked and a uniform mesh is built according to the transmitted data. After this, the initial field of heat content is computed using equation (6). Then equations (9) are solved by the sweep method. Moreover, method (11) allows for parallel execution. To do this, each step is divided into $i_x = 1,2, ..., N_x$ for the first half step and $i_y = 1,2, ..., N_y$ for the second separate iterations, which can be solved in parallel. OpenMP is used for parallelization.

The solution of the systems is found using the sweep method (Thomas's algorithm). For systems of this kind, a solution can be obtained in $O(n)$ operations. This algorithm will parallelize quite well using block decomposition:

- The original matrix is represented as a product of a block matrix and some modified matrix. In this case, the block matrix can be scattered between processes in blocks.
- Computation of blocks independently by different processes.
- Return to the original matrix.

After each time step, the grid nodes are checked for the fulfillment of the phase transition conditions. With a user-defined frequency, the values of the grid function of heat content are converted into temperature (7) and written to the VTK file.

Subsequent rendering of VTK files is possible using the open source ParaView software, as well as custom developed scripts.

To set the initial conditions, scripts have been implemented for constructing meshes from arbitrary substances of various shapes (circle, rectangle, etc.), as well as initial temperatures from VTK files of previous computations and arbitrary figures with constant temperatures.

3. RESULTS

3.1. Problem formulation

The integration area is an ice island with a height of 10 *m* and a horizontal length of 300 *m* [3]. The island is submerged in water at a depth of 8 m and lies on the bottom. The geometrical dimensions of the area and the layout are shown in **Fig. 2**. The characteristics of the substances are presented in **Table 1**. We also assume that the melting point of ice is $T_0 = 0°C$, and its specific heat of fusion λ = 334 kJ/kg.

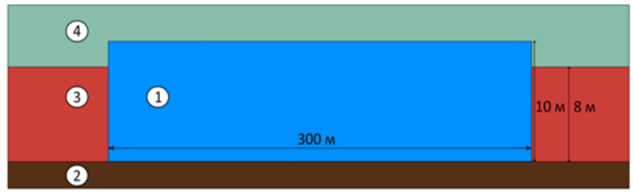

Fig. 2. *Model of ice island.*

In Fig. 2, the designations of substances are introduced: 1 – ice (artificial ice island), 2 – bottom soil, 3 – water, into which the ice island is immersed, 4 – air.

The distribution of temperatures in the ice island was found at average temperatures in winter. Computations were also carried out to study the stability of the ice island to melting during seasonal temperature changes.

The computations were carried out using three main models:

- setting without taking into account the flow of water and freezing of the soil;
- setting taking into account water flow, but without soil freezing;
- setting taking into account water flow and soil freezing.

3.2. The results of modeling the problem in the formulation without taking into account water flow and soil freezing

Modeling were carried out to determine the equilibrium temperature distribution in the ice island at an air temperature of $-40°C$,

Table 1.
Thermophysical properties of substances.

No	Substance	Density, kg\/m³	Thermal conductivity, W/(m·K)	Specific heat, J/(kg·K)
1	Ice	917	0.591	2100
2	Bottom soil	2500	0.8	750
3	Watter	1000	2.22	4180
4	Air	1.60	0.022	-

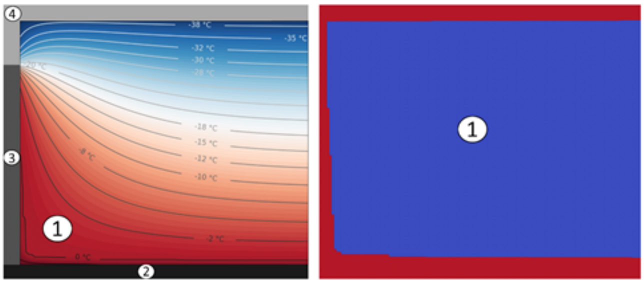

Fig. 3. *Temperature distribution in an ice island (left) and a map of phase states (right) in the problem of establishing the temperature in an ice island in the setting without taking into account soil freezing.*

a bottom soil temperature of 5°C and a water temperature of 3°C. The problem of establishing was solved, the initial temperature of the ice island was 10°C, the contacting substances (water, air, soil) were set in the form of boundary conditions with given temperatures, heat capacities, and thermal conductivity coefficients.

Fig. 3 shows the results of computing the temperature settling. A period of 150 days was simulated. On the left is a picture of the distribution of temperatures in the ice island, on the right is a map of phase states (blue – ice, red – water and air).

Further, the study of the stability of the ice island was carried out with seasonal temperature changes. The air temperature has been changed to 3°C. The results after 150 days are shown in **Fig. 4**. It can be seen that from the edges the ice island practically did not melt – since we do not take into account the flow of water, a temperature gradient simply established there. From above, the island melted significantly, since after melting the water immediately flowed down and the boundary condition with air was set on the remaining ice mass. Even a relatively low coefficient of thermal conductivity of the air provided a serious heat flow, which melted the island.

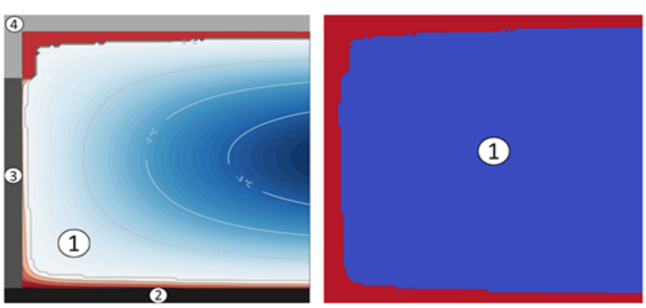

Fig. 4. *Temperature distribution in an ice island (left) and a map of phase states (right) in the problem of melting an ice island under the influence of a change in air temperature in the formulation without taking into account the water flow and soil freezing.*

3.3. The results of modeling the problem in the formulation taking into account the water flow and without soil freezing

Obviously, without taking into account the flow of water, the ice island will practically not melt. Let's take into account the water flow as follows. For any melted water with a temperature strictly greater than zero, at the end of the time step, we explicitly set the temperature equal to three degrees Celsius. As a result, it will be possible to take into account the flow of water. We use the distribution of substances, the temperature distribution after freezing, and the thermophysical constants obtained in the problem of establishing the temperature distribution in Section 3.2. Similar to the previous calculation, the results after 150 days of melting of the ice island are shown in **Fig. 5**. It is clearly seen that the island has practically melted. This was due to the fact that a thin layer of water appeared in its bottom part during the establishment. Further, the constant water temperature created a large temperature difference with the ice and, as a consequence, an abnormally large heat flux through the lower edge.

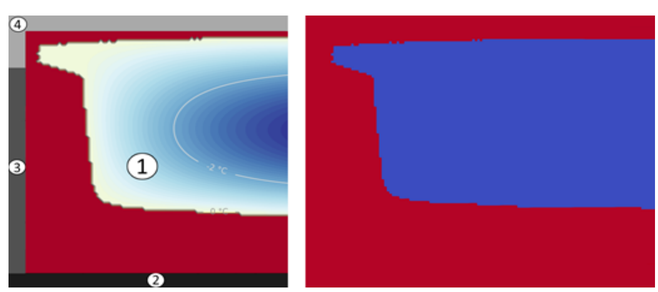

Fig. 5. *Temperature distribution in an ice island (left) and a map of phase states (right) in the problem of melting an ice island under the influence of air temperature changes in the formulation taking into account the water flow.*

3.4. The results of modeling the problem in the formulation taking into account the flow of water and soil freezing

In order to solve the problem that arose when taking into account the flow of water in paragraph 3.3, you can take into account the freezing of the soil. To do this, we use a computational area of 300 meters by 20 meters, where ice occupies the upper half, similar to the previous statements, and 10 meters below it is filled with soil (in previous calculations, the boundary of the island with the soil was set by the boundary condition). The initial temperature distribution is similar to paragraph 3.2, the soil has an initial temperature of $+5°C$. The boundary conditions for water and air are similar to the previous formulations with the addition of the condition of zero heat flux through the bottom and side faces of the soil massif.

The results of modeling the establishment of temperatures are shown in **Fig. 6.** It can be seen that the depth of soil freezing is about five meters. On the right in Fig. 6 there is a map of phase states in the computational domain. Dark blue denotes a solid medium (ice and soil), blue – liquid (water) and red – gaseous (air).

Further, for this temperature distribution, the problem of stability in extreme conditions with water flow was set. The air temperature was taken equal to $+10°C$. The simulation results are shown in **Fig. 7**. It can be seen that after 150 days the topside melted

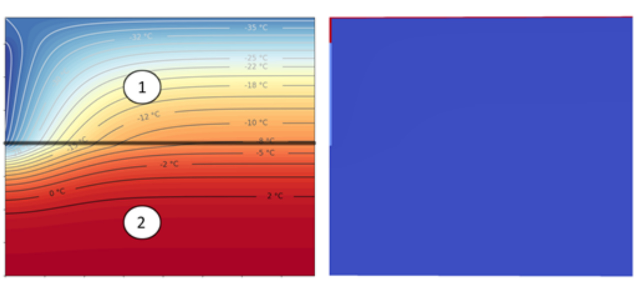

Fig. 6. *Temperature distribution in an ice island (left) and a map of phase states (right) in the problem of establishing the temperature in an ice island in the setting taking into account soil freezing.*

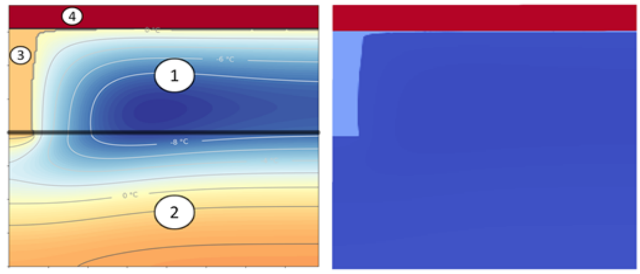

Fig. 7. *Temperature distribution in an ice island (left) and a map of phase states (right) in the problem of melting an ice island under the influence of a change in air temperature in the setting taking into account the water flow and freezing the soil.*

completely. The edge of the island has also seriously melted, the size of the thawed part is comparable to the size of the surface of the island. The temperature in the interior of the island and in the soil has practically not changed.

4. CONCLUSION

The enthalpy approach for solving the Stefan problem and the software for solving this problem in various settings were developed. On the basis of this program, various models of the ice island were tested and conclusions about its resistance to melting were drawn. The developed approach allows one to take into account various external influences, such as heat fluxes from man-made installations, water flow and seasonal temperature changes. Based on the data obtained, it is possible to more efficiently plan the development of the structure of future ice islands, and also an opportunity opens up for numerical modeling of existing ones in order to audit their integrity. Also, using this approach, it is possible to check the feasibility of using such materials as pykerite and other ice-composite materials, the introduction of additional heat-insulating layers, cooling the fins of the island and other ideas.

The developed methods can be applied in solving static and dynamic thermoelastic problems, which are a logical continuation of this study.

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