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Intensity of Electromagnetic Wave into Layers with Fluctuations of Dielectric Permittivity

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Abstract: The study is made of the intensity of a plane electromagnetic wave propagating into the layer with random discrete irregularities of the dielectric permittivity. The mean intensity of scattered field as a function of the parameters of random irregularities of rectangular and triangular forms is analyzed. It is shown that the deviation of the average intensity from the unperturbed value increases both the average amplitude and its standard of fluctuations. It is found that the amplitude of the intensity oscillations for a layer with irregularities of the rectangular shape is significantly greater than for fluctuations with the triangular profile.

Keywords: electromagnetic wave, scattering, random medium, fluctuations of dielectric permeability, average field intensity

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1. INTRODUCTION

A considerable amount of work has been devoted the scattering of electromagnetic waves in randomly inhomogeneous media with large-scale or delta-correlated disturbances of the medium [1–5]. The average intensity of the field of a wave incident along the normal to a layer with narrow (compared to the wavelength in vacuum) random discrete one-dimensional inhomogeneities under the condition of uniform distribution of the phase of the reflection coefficient of the wave from each inhomogeneity was analyzed in [6]. In [7], using the method of geometric optics, the mean electromagnetic field in a randomly inhomogeneous atmosphere was calculated. Taking into account the fractal properties of randomly inhomogeneous media, multiple scattering of waves was studied in [8]. The scattering of acoustic waves by a set of spherical inhomogeneities randomly located on a horizontal seabed was analyzed in [9]. Despite of the large amount of research presented in these works, the properties of the mean intensity of the wave field in a medium with random rarefied fluctuations of the dielectric constant of an arbitrary shape have not been studied yet. It is possible to use these layers to model the propagation and reflection of electromagnetic waves

in the problems of sounding the Earth in shallow geophysics [10], as well as during the passage of radio emission through the troposphere (for example, [11, 12]).

In [13], the average intensity of the field of a plane wave incident on a one-dimensional layer with random rarefied rectangular dielectric permittivity inhomogeneities with zero mean amplitude is analyzed. In this paper, we generalize the results obtained in [13] to the case of a medium with rectangular permittivity perturbations with a nonzero mean value, and also consider wave propagation in a medium with triangular permittivity fluctuations. The dependence of the average intensity on the parameters of fluctuations (average width, average amplitude and standard of fluctuations in amplitude) is studied.

2. PROBLEM STATEMENT AND CALCULATION METHOD

The paper investigates the average intensity of a harmonic plane electromagnetic wave with an electric field strength $\vec{E}_0(x) = \vec{n}E_0 \exp(i\omega t - ikx)$ which is falled falling on a layer ($0 \leq x \leq L$) with random one-dimensional inhomogeneous fluctuations of the dielectric constant of various shapes (**Fig. 1**). Here \vec{n} is the unit normal vector perpendicular to the

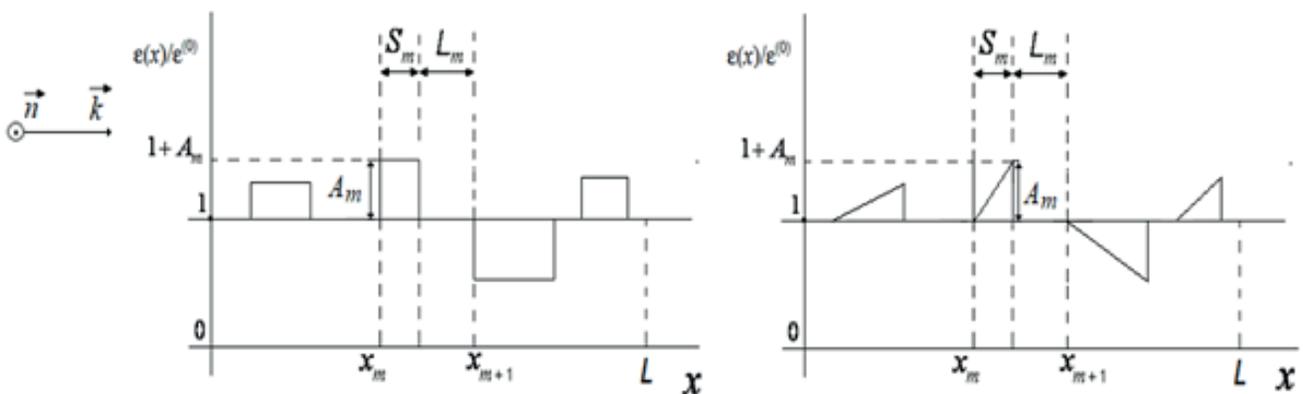


Fig. 1. The dielectric constant of the layer $\epsilon(x)/\epsilon^{(0)}$.

X axis, E_0 is the amplitude of the incident wave, $k = k_0 \sqrt{\varepsilon^{(0)}}$, where $k_0 = 2\pi/\lambda_0$ is the wavenumber in vacuum, $\varepsilon^{(0)}$ is the relative permittivity of the medium in the absence of disturbances.

The relative dielectric constant of the layer in an individual realization is given in the form

$$\varepsilon(x) = \varepsilon^{(0)} \left(1 + \sum_{m=1}^N f_m(x) [H(x-x_m) - H(x-x_m-S_m)] \right). \quad (1)$$

Here $H(z)$ is the Heaviside unit function, the function $f_m(x)$ determines the disturbance profile, N is the number of irregularities in the realization, x_m is the coordinate of the beginning of the m th irregularity, S_m and $|A_m| = \max |f_m(x)|$ – its width and amplitude, respectively; the distance between the inhomogeneities is $L_m = x_{(m+1)} - x_m - S_m$. We assume that x_m , S_m , and A_m are independent random variables (RVs). Parameters S_m and x_m have a truncated Gaussian distribution with average values $S = \langle S_m \rangle$ and $\langle x_m \rangle = \langle x_i \rangle + (m-1)L$, (where $L = S + \langle L_m \rangle$), as well as fluctuation standards σ_S and σ_x . We take the probability density for RV A_m in (1) in the form of a Gaussian distribution with an average value $\langle A_m \rangle = A$ and a fluctuation standard σ_A . The following restrictions are assumed to be met

$$\{L, S\} \ll L, \sigma_x \ll L - S, |A| + 3\sigma_A \ll 1, (k\sigma_x)^2 \gtrsim 1. \quad (2)$$

As in [13], we will assume that $\text{Re } \varepsilon(x) > 0$ and $\text{Im } \varepsilon(x) = 0$. Note that the reflection of electromagnetic waves from a randomly inhomogeneous medium with fluctuations of the complex permittivity was considered in [14] under the condition that fluctuations are delta-correlated and mutually independent.

The intensity of the field of a plane wave propagating in a layer with one-dimensional inhomogeneities, under conditions (2), is

determined by the following expression (see [13]):

$$I(x) = \pi I_0 \exp[\kappa(0.75L - x)] \times \int_0^{+\infty} \frac{sh(\pi t)}{ch^2(\pi t)} \{2t \cos[2\kappa(L-x)t] + \sin[2\kappa(L-x)t]\} \exp(-\kappa L t^2) dt. \quad (3)$$

Here $I_0 = \frac{1}{2} E_0^2 \sqrt{\varepsilon_0 \varepsilon^{(0)} \mu_0^{-1}}$ is the intensity of the plane wave incident on the layer (where ε_0 and μ_0 are electric and magnetic constants); the parameter κ depends on the reflection coefficient R_m as follows: $\kappa = \mathcal{L}^{-1} \langle |R_m|^2 / (1 - |R_m|^2) \rangle$, the symbol $\langle \dots \rangle$ means averaging over the ensemble of realizations. Below, we proceed to calculating the coefficient κ in formula (3). To determine the value of κ , we first calculate the reflection coefficient R_m of a plane wave from a separate inhomogeneity with number m . The electric field strength E inside the m -th inhomogeneity with permittivity $\tilde{\varepsilon}(\zeta) = \varepsilon_0 [1 + f_m(\zeta)]$, set on the interval $0 \leq \zeta \leq S_m$ (where $\zeta = x - x_m$) is determined by the wave equation

$$E''(\zeta) + k^2 \tilde{\varepsilon}(\zeta) E(\zeta) = 0, \quad (4)$$

where the prime means differentiation with respect to the argument; for $\zeta < 0$ and $\zeta > S_m$, one should set $\tilde{\varepsilon}(\zeta) = \varepsilon^{(0)}$. Let us denote two independent solutions of Eq. (4) as functions $\Phi_{1,2}(\zeta)$. Taking into account the conditions for the tangential components of the electromagnetic field at the boundaries of the inhomogeneity ($\zeta = 0$ and $\zeta = S_m$), we obtain the expression for the reflection coefficient R_m

$$R_m = \frac{k^2 C(S_m) - b(S_m) + ik[\alpha_1(S_m) + \alpha_2(S_m)]}{k^2 C(S_m) + b(S_m) + ik[\alpha_1(S_m) - \alpha_2(S_m)]}. \quad (5)$$

The following notations were introduced in (5):

$$\begin{aligned}
 b_1(S_m) &= \beta[\Phi'_2(0)\Phi'_1(S_m) - \Phi'_2(S_m)\Phi'_1(0)], \\
 C(S_m) &= 1 - \beta\Phi_1(0)\Phi_2(S_m), \\
 \alpha_1(S_m) &= \beta\Phi'_2(S_m)\Phi_1(0) - \Phi'_1(S_m)/\Phi_1(S_m), \\
 \alpha_2(S_m) &= \beta\Phi'_1(0)\Phi_2(S_m) - \Phi'_2(0)/\Phi_2(0), \\
 \beta^{-1} &= \Phi_1(S_m)\Phi_2(0).
 \end{aligned}$$

Below we consider the properties of the average intensity of the wave field in layers with fluctuations in the permittivity $\varepsilon(x)$ of rectangular and triangular shapes. In these cases, the solution to Eq. (4) can be obtained analytically.

3. RESULTS OF NUMERICAL CALCULATIONS AND THEIR DISCUSSION

3.1. FIELD INTENSITY IN A LAYER WITH RECTANGULAR DIELECTRIC CONSTANT INHOMOGENEITIES

The function $f_m(x)$ in relation (1) for a layer with rectangular inhomogeneities is a constant $f_m(x) = A_m$. Then the solution of equation (4) has the form: $\Phi_{1,2} = \exp(\pm ik\sqrt{1+A_m}x)$. In this case, from (5) we obtain the following expression for the reflection coefficient: $R_m = (Z_m^2 - 1) / (Z_m^2 + 1 - 2iZ_m \operatorname{ctg} \theta_m)$, where $\theta_m = k\sqrt{1+A_m}S_m$, $Z_m = (1+A_m)^{-1/2}$ is the impedance of the inhomogeneity, normalized to the impedance of the unperturbed medium $Z^{(0)} = [\mu_0/(\varepsilon_0\varepsilon^{(0)})]^{(1/2)}$. Note that the obtained expression for R_m coincides with the coefficient of reflection from a homogeneous plane layer given in [15]. Using the relationship between κ and the average reflection coefficient $\langle R_m \rangle$ under small values of A , for the averaged parameter κ in (3) we have:

$$\kappa = (\sigma_A^2 + A^2 - \exp(-g)[(\sigma_A^2 - k^2 S^2 \sigma_A^4 + A^2) \cos \alpha - 2kAS\sigma_A^2 \sin \alpha]) / 8L, \tag{6}$$

where $g = 2k^2\sigma_s^2 + k^2S^2\sigma_A^2 / 2$, $\alpha = 2k\sqrt{1+AS}$.

Below, we present the results of a numerical analysis of the dependence of the

normalized average intensity $I(S)/I_0$ on the average width of the inhomogeneities S for different values of the x coordinate inside the layer. All calculations were carried out for a layer with a thickness of $L = 2 \cdot 10^4 \lambda$, the average distance between neighboring inhomogeneities $\langle L_m \rangle = 2\lambda$, and the standard of fluctuations of the average disturbance width $\sigma_s = 0.01\lambda$.

The character of the $I(S)$ dependence is determined by the properties of the parameter κL , which is confirmed by the results of numerical calculations. **Fig. 2** shows the $\kappa(S)L$ dependence (curves 1–3) at $A = 0.05, 0, -0.05$, respectively, and the fluctuation standard $\sigma_A = 0.07$. From expression (6) it is easy to obtain that $\kappa(S)L$ is an oscillating function with a period $\lambda / (2\sqrt{1+A})$ where $\lambda = \lambda_0\sqrt{\varepsilon^{(0)}}$, which is illustrated in Fig. 2.

Fig. 3 shows the dependence of the average intensity $I(S)$ (normalized to I_0) on the dimensionless width of the inhomogeneities S/λ at the fluctuation standard $\sigma_A = 0.07$. Curves 1–5 in Fig. 3a correspond to the values $x = 0.025L, 0.5L, 0.75L, L$ and $A = 0.05$; curves 1–6 in Fig. 3b correspond to parameters $A = 0.05, 0, -0.05$ at $x = 0$ (curves 1–3) and $x = L$ (curves 4–6).

At $x < L/2$, the intensity maxima $I(S)$ are achieved at the values of the average width of

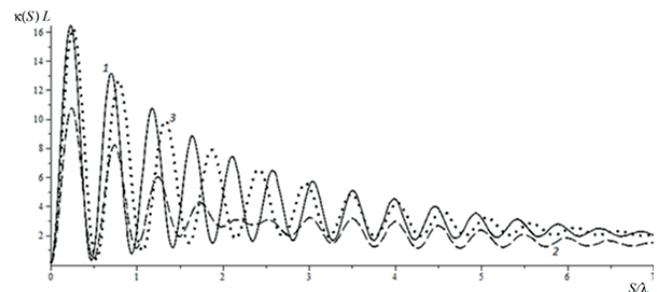


Fig. 2. Parameter $\kappa(S)L$ for a layer with rectangular inhomogeneities $\varepsilon(x)$ at $L = 2 \cdot 10^4 \lambda$, $\langle L_m \rangle = 2\lambda$, $\sigma_s = 0.01\lambda$, $\sigma_A = 0.07$. Curves 1–3 correspond to $A = 0.05, 0, -0.05$.

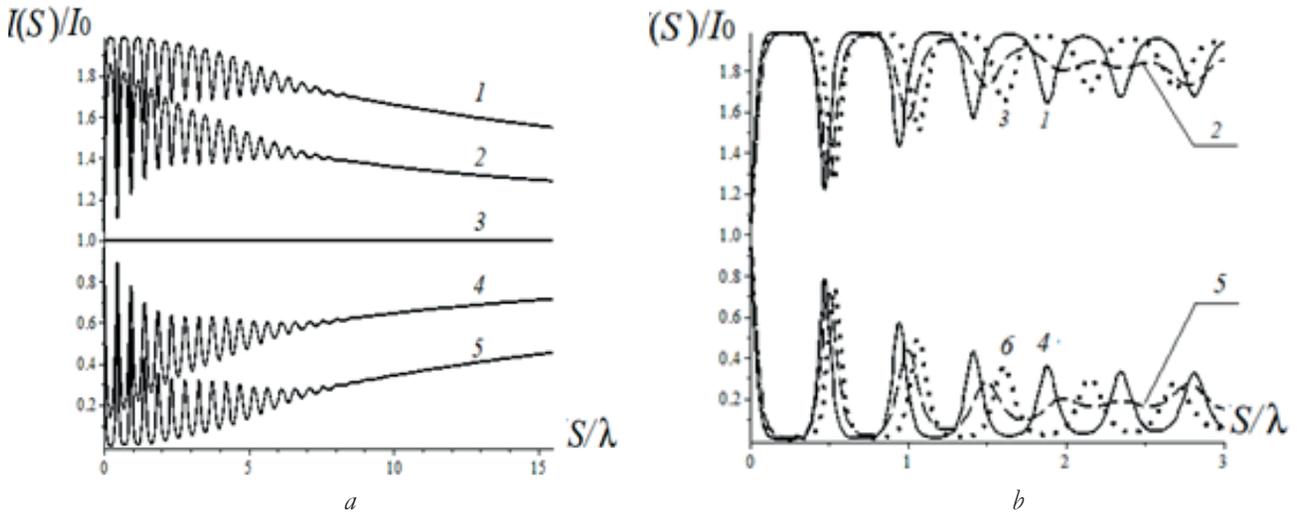


Fig. 3. Average intensity $I(S)/I_0$ for a layer with rectangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.07$ (a) $A = 0.05$; curves 1–5 correspond to the values $x = 0, 0.25L, 0.5L, 0.75L, L$; (b) $A = 0.05, 0, -0.05$; curves 1–3 at $x = 0$, curves 4–6 at $x = L$.

the disturbances $S_{\max} = (n + 0.5)\lambda / (2\sqrt{1 + A})$, and the minima – at $S_{\min} = n\lambda / (2\sqrt{1 + A})$, where $n = 0, 1, 2, 3, \dots$. For $x > L/2$, the oscillation phase of the function $I(S)$ is shifted by π in comparison with the case $x < L/2$ (curves 1 and 5, as well as curves 2 and 4 in Fig. 3a). For $x = L/2$, the ratio $I(S)/I_0 \equiv 1$ holds for all values of the average width of the inhomogeneities S . From Figures 2 and 3 and expressions (3), (6) it follows that the functions $\kappa(S)L$ and $I(S)$ have the same oscillation period. Figs. 2 and 3b show that the period of oscillations decreases with an increase in the value of A .

The amplitude of the intensity oscillations decreases with increasing S/λ . Fig. 3b shows that as the average value of the disturbance amplitude (at $\sigma_A = \text{const}$) decreases, the amplitude of the intensity fluctuations $I(S)$ decreases. With an increase in S , the intensity $I(S)$ also tends to the unperturbed value I_0 , while the coefficient $\kappa(S) \rightarrow 0$.

As follows from (6), under the condition $\sin\alpha = 0$ (which for small values of $|A| \ll 1$ is equivalent to $S = \lambda n/4$, where $n = 0, 1, 2, 3, \dots$) the value $\kappa \sim A^2$; moreover, $I(A) = I(-A)$. Fig. 4 shows the dependence of the

parameter $\kappa(A)L$ at $\sigma_A = 0.02$ and $\sigma_A = 0.07$ (solid and dotted lines, respectively). Curves 1 and 3 correspond to the value $n = 4$, curve 2 – $n = 20$.

Fig. 5 shows the results of calculating $I(A)$ for the average width of inhomogeneities $S = \lambda$ ($n = 4$, Fig. 5a), $S = 5\lambda$ ($n = 20$, Fig. 5b) for $x = 0, 0.25L, 0.5L, 0.75L, L$ and the standard of fluctuations $\sigma_A = 0.02$ (solid curves 1–5), $\sigma_A = 0.07$ (dashed curves 1'–5'). Figs. 4 and 5 show that for integer values of

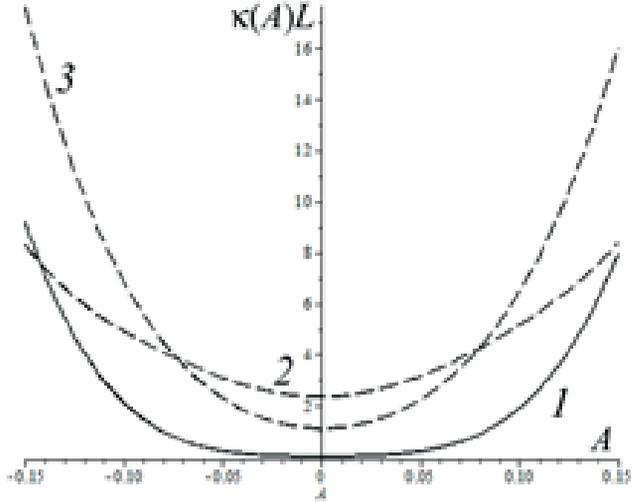


Fig. 4. Parameter $\kappa(A)L$ for a layer with rectangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.02$ (solid line), $\sigma_A = 0.07$ (dashed lines). Curves 1 and 3 correspond to $S = \lambda$, curve 2 – $S = 5\lambda$.

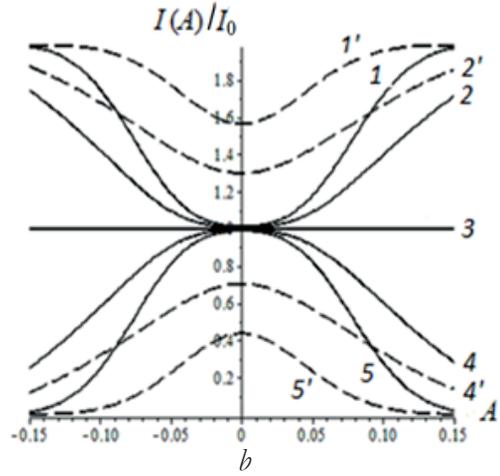
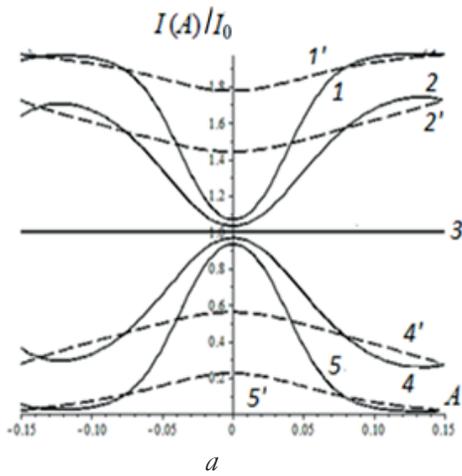


Fig. 5. Average intensity $I(A)/I_0$ for a layer with rectangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.02$ (solid curves), $\sigma_A = 0.07$ (dashed curves); $x = 0$ (curves 1 and 1'), $x = 0.25L$ (curves 2 and 2'), $x = 0.5L$ (curve 3), $x = 0.75L$ (curves 4 and 4'), $x = L$ (curves 5 and 5'); (a) $S = \lambda$, (b) $S = 5\lambda$.

n corresponding to the case of a symmetric dependence of the functions $\kappa(A)L$, the average field intensity is also independent of the sign of A .

Numerical calculations of the dependences $\kappa(A)L$ and $I(A)$ plotted for $S = 1.9\lambda$ and $S = 2.1\lambda$ and for the same other parameter values as in Fig. 5, confirm the conclusion about the asymmetry of the coefficient $\kappa(A)$ and intensity $I(A)$ at non-integer values of S/λ ($\sin\alpha \neq 0$).

3.2. FIELD INTENSITY IN A LAYER WITH TRIANGULAR DIELECTRIC CONSTANT INHOMOGENEITIES

For a layer with inhomogeneities of the dielectric constant in the form of right-angled triangles (Fig. 1), the functions $f_m(x)$ in (1) are written as $f_m(x) = p_m(x - x_m)$, where $p_m = A_m/S_m$. In this case, the solution to equation (4) has the form [16]:

$$\Phi_{1,2}(\zeta) = (1 + p_m \zeta)^{0.5} H_{1/3}^{(1,2)} [(2/3)kp_m^{-1}(1 + p_m \zeta)^{1.5}]. \quad (7)$$

Here $H_{1/3}^{(1,2)}(\zeta)$ are the Hankel functions of the first and second kind with index $1/3$, $\zeta = x - x_m$. In this case, the reflection coefficient R_m from one triangular inhomogeneity is determined by expression (5), in which the functions $\Phi_{1,2}(\zeta)$ are given by relation (7).

Fig. 6 shows the dependence of the parameter $\kappa(S)L$ on the normalized average width of inhomogeneities S/λ for the considered case $\varepsilon(x)$ at $\sigma_A = 0.07$ and $A = 0, 0.1, -0.1$ (curves 1–3, Fig. 6). Numerical calculations show that the $\kappa(S)L$ dependence for different values of the parameter A has a similar form as in a layer with rectangular inhomogeneities $\varepsilon(x)$ (i.e., the oscillation period decreases with increasing A).

Fig. 7 illustrates the dependence $\kappa(A)L$ at $S = 2.1\lambda$, $\sigma_A = 0.02$ and $\sigma_A = 0.07$ (curves 1 and 2); the values of the parameters $L, \langle L_m \rangle$ and σ_s are the same as in Section 3.1. It follows from the calculation results that, in the case of triangular perturbations $\varepsilon(x)$, the

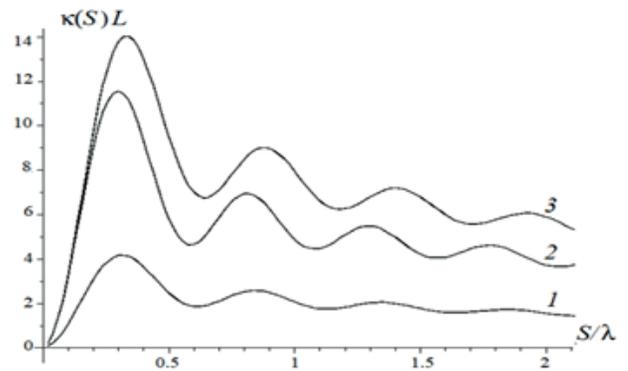


Fig. 6. Parameter $\kappa(S)L$ for a layer with triangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.07$; $A = 0$ (curve 1), $A = 0.1$ (curve 2), $A = -0.1$ (curve 3).

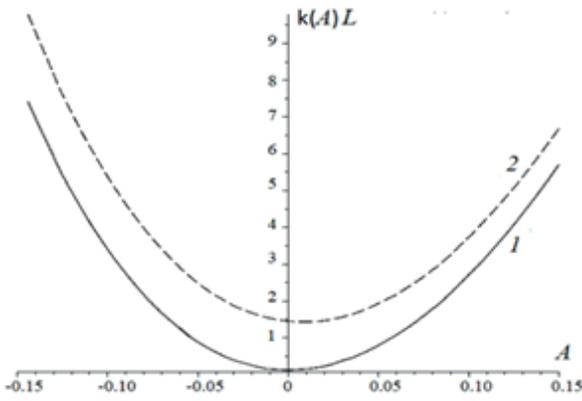


Fig. 7. Parameter $\kappa(A)L$ for a layer with triangular inhomogeneities $\varepsilon(x)$ at $S = 2.1\lambda$; $\sigma_A = 0.02$ (curve 1), $\sigma_A = 0.07$ (curve 2).

function $\kappa(A)L$ depends on the sign of A for any values of the normalized average width of the inhomogeneities S/λ .

Fig. 8 shows the average intensity $I(S)/I_0$; in Fig. 8a, curves 1–5 corresponding to $x = 0, 0.25L, 0.5L, 0.75L, L$ correspond to the values of $\sigma_A = 0.07, A = 0.1$; curves 1–6 in Fig. 8b correspond to $\sigma_A = 0.07, A = 0; 0.1; -0.1$ at $x = 0$ (curves 1–3) and $x = L$ (curves 4–6). Comparing Fig. 8 and Fig. 3, we come to the conclusion that the periods of oscillations of the function $I(S)$ for fluctuations $\varepsilon(x)$ of triangular and rectangular shapes practically

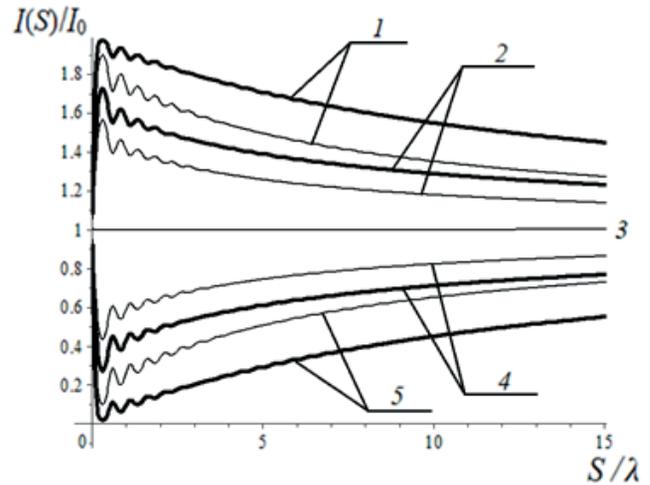


Fig. 9. Average intensity $I(S)/I_0$ for a layer with triangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.07, A = 0$ (thin lines), $A = 0.1$ (bold lines); curves 1–5 correspond to $x = 0, 0.25L, 0.5L, 0.75L, L$.

coincide, while the amplitude of oscillations for triangular inhomogeneities turns out to be significantly smaller (Figs. 4 and 7).

Fig. 9 shows the normalized average intensity $I(S)/I_0$ for a layer with triangular inhomogeneities $\varepsilon(x)$ at $x = 0, 0.25L, 0.5L, 0.75L, L$ (curves 1–5). The curves in the figure correspond to the $I(S)/I_0$ dependence at $\sigma_A = 0.07, A = 0$ (thin lines), and $A = 0.1$ (bold lines). It should be noted that the difference between the intensities $|I(S) - I_0|$ increases with increasing $|A|$ and σ_A .

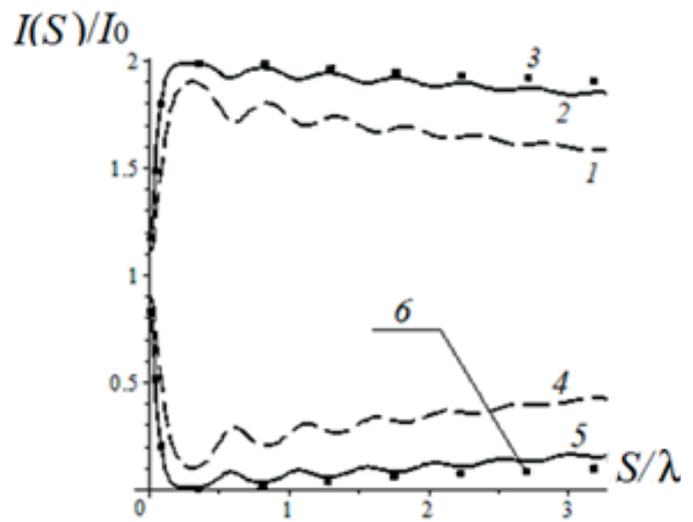
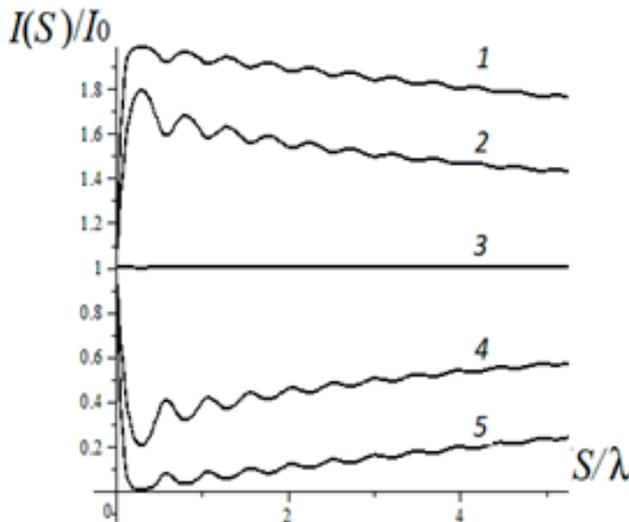


Fig. 8. Average intensity $I(S)/I_0$ for the layer with triangular inhomogeneities $\varepsilon(x)$ at $\sigma_A = 0.07$ (a) $A = 0.1$, curves 1–5 correspond to $x = 0, 0.25L, 0.5L, 0.75L, L$; (b) $A = 0, 0.1, -0.1$; $x = 0$ (curves 1–3), $x = L$ (curves 4–6).

Thus, it is shown that for a layer with triangular inhomogeneities $\varepsilon(x)$, the intensity $I(\mathcal{A})$ depends on the sign of \mathcal{A} for any values of the average width of inhomogeneities, in contrast to the case of dielectric constant with rectangular inhomogeneities. This is explained by the fact that the reflection coefficient from a single inhomogeneity with a linear dependence $\varepsilon(x)$ is a function of the parameter \mathcal{A} , in contrast to the case of constant $\varepsilon(x)$ inside the inhomogeneity. Note that despite of the asymmetric shape of the inhomogeneities $\varepsilon(x)$ of a triangular shape, the layer with inhomogeneities of this shape, as well as the layer with rectangular fluctuations $\varepsilon(x)$, does not possess anisotropy properties, i.e. the distribution of the average intensity inside the layer with one-dimensional discrete fluctuations $\varepsilon(x)$ does not depend on the direction of incidence of the wave on the layer. This is due to the multiple reflection of the wave from a large number of random irregularities within the layer.

4. CONCLUSION

The problem of scattering of a plane electromagnetic wave by a dielectric layer with one-dimensional random discrete inhomogeneities of arbitrary width is considered. When calculating the intensity of the scattered field, it was assumed that the coordinates of the points of origin of inhomogeneities, as well as their width and amplitude, are independent random variables distributed over Gaussian. The cases of layers with perturbations of the permittivity of rectangular and triangular shapes are investigated. One of the results of this work is the lack of localization of mean intensity. The localization of plane waves in chaotically layered media are discussed in [17-21]. In particular, it was shown in [17] that dynamic localization takes place for individual realizations of the

field, while statistical energy localization expresses the properties of the entire statistical ensemble of realizations. The main conclusion of the authors of [17] is that, despite the presence of field localization in individual realizations, it may be absent for the average wave intensity for the entire statistical ensemble of realizations. The calculation method used in our work is based on averaging the field intensity over all random realizations.

It is shown in this work that the amplitude of intensity oscillations $I(S)$ for a layer with rectangular inhomogeneities $\varepsilon(x)$ is significantly larger than for fluctuations with a triangular profile. With an increase in the average width of inhomogeneities, the amplitude of oscillations of the average intensity at a nonzero average value of the amplitude of fluctuations decreases faster than at $\mathcal{A} = 0$. The period of oscillations for both types of disturbances is the same and is approximately equal. The deviation of the average intensity $I(S)$ from the unperturbed value turns out to be proportional to the amplitude $|\mathcal{A}|$ and the standard of fluctuations $\sigma_{\mathcal{A}}$ of inhomogeneities and decreases with an increase in the average width S . Differences in the behavior of the average intensity when the wave is scattered by layers with inhomogeneities of the dielectric constant with a constant and linear dependence on the x coordinate inside the inhomogeneities are explained by the features of the reflection coefficients of the wave from individual fluctuations of rectangular and triangular shapes [12]. For a layer with rectangular inhomogeneities, the $I(\mathcal{A})$ dependence is a symmetric function of the average value of the parameter \mathcal{A} at integer values of S/λ for small fluctuation amplitudes. For a layer with fluctuations

in the permittivity of a triangular shape, the dependences $\kappa(\mathcal{A})$ and $I(\mathcal{A})$ are not symmetric functions of \mathcal{A} at any width of the inhomogeneities, as well as the modulus of the reflection coefficient from an individual inhomogeneity. The noted fact can be used in the diagnostics of natural environments.

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