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# Methods for Calculating Radio Holograms of Volumetric Objects

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**Abstract:** A method for calculating holograms for volumetric objects based on the representation of objects in the form of ensembles of virtual point sources distributed on a set of parallel planes has been proposed. The proposed method is the development of the well-known method in which objects are represented as ensemble of real point scatterers. The possibilities of the proposed method are demonstrated by calculating a hologram of a fragment of a sphere, on which 1000 points are randomly selected, at which radiation emanating from the center of the sphere is scattered. The choice of a fragment of a sphere as an object under study is due to the fact that when calculating its hologram, phase errors inherent in approximate calculations are most pronounced. The calculations were performed for the frequency range of 2...100 GHz, the sphere radius of 0.5 m, a two-dimensional hologram size of 0.65×0.65 m, and a pixel count of 512×512. It is shown that, in comparison with the known method, the proposed method makes it possible to calculate the amplitude of a hologram with satisfactory accuracy if virtual sources are placed on parallel planes in an amount of more than 64 pieces. In the case of objects that require representation in the form of an ensemble of point scatterers in the amount of more than 1000 pieces, the calculation of their holograms by the proposed method turns out to be much more efficient than the known method. *Keywords:* radio holograms, virtual point sources, parallel planes, sphere.

*Keywords:* computer modeling, radio holograms, volumetric objects, point scatterers, virtual point sources, parallel planes, sphere

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## 1. INTRODUCTION

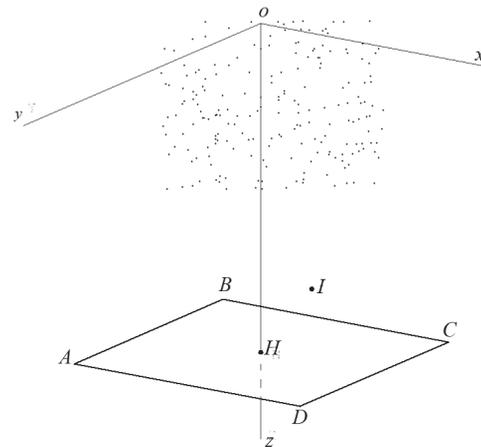
Radio holography is one of the methods for solving such radar problems as detection, determination of the coordinates of the location and shape of objects [1]. Obtaining radio holograms by experimental methods involves measuring the fields scattered by real objects, which is usually quite difficult. In some cases, an alternative method may be the calculation of radio holograms based on computer simulation of the scattering of electromagnetic waves by objects with desired properties.

Until now, there is no analytical solution of the problem of wave scattering by an object of arbitrary shape. In the general case, the field, scattered by an object, is calculated using the Kirchhoff theorem [2-5]. Another method for modeling radio holograms is a calculation based on the representation of an object as a set of point scatterers [6]. In the case, if these scatterers lie on the same plane, one can use the property of Fourier transform to convolve image and use the fast Fourier transform (FFT) to perform the calculations [1]. However, if the objects are not flat, then using the FFT is difficult.

The aim of this work is to develop a method for calculating radio hologram of volumetric objects, which is based on objects representation in the form of a set of point scatterers.

## 2. THE METHOD OF AN ENSEMBLE OF POINT SCATTERERS (THE METHOD 1)

Let us represent the volumetric object under investigation in the form of an ensemble of point scatterers and place the radiation source at point  $I$  as shown in **Fig. 1**. Let



**Fig. 1.** Scheme for measuring a hologram of an object presented as an ensemble of point scatterers.

the scattered field be measured within the rectangle ABCD with center  $H$  on the  $OZ$  axis.

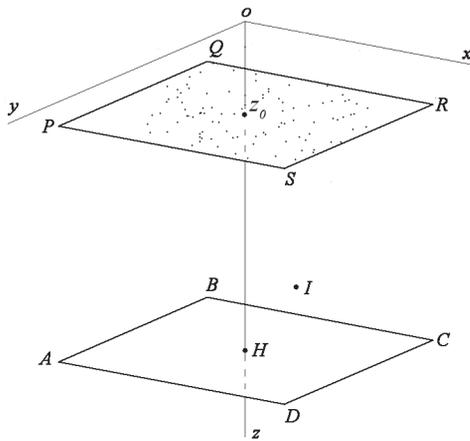
The field at the point  $(x_j, y_j, z_j)$  can be estimated using the Huygens-Fresnel approximation [1]:

$$E(\vec{r}_h) = E_0 \sum_{n=0}^{M-1} \beta(\vec{r}_n) \frac{\exp(ik|\vec{r}_l - \vec{r}_n|)}{|\vec{r}_l - \vec{r}_n|} \frac{\exp(ik|\vec{r}_h - \vec{r}_n|)}{|\vec{r}_h - \vec{r}_n|}, \quad (1)$$

where  $M$  is the number of point scatterers;  $\vec{r}_l, \vec{r}_n, \vec{r}_h$  – vectors of the coordinates of the position of the source, the  $n$ -th scatterer and receiver, accordingly;  $E_0$  – the amplitude of the source;  $\beta(\vec{r}_n)$  – coefficient of reflection (scattering) of the  $n$ -th scatterer [1].

## 3. THE METHOD OF VIRTUAL SOURCE ON THE SAME PLAIN (THE METHOD 2)

Let each point scatter of an object be correspond to a virtual source of a spherical wave in the  $PQRS$  plane as shown in **Fig. 2**. The  $x, y$  coordinates of the point scatter and the virtual source coincide, and the phase of the wave of the virtual source is adjusted taking into account its change in the distance from the point scatter to the virtual source.



**Fig. 2.** Scheme for measuring a hologram of an object in the form of an ensemble of virtual sources on the PQRS plane when they are irradiated from point I.

The field at the point  $(x_h, y_h, z_h)$  on the ABCD plane is determined taking into account the phase correction of the virtual sources with respect to the corresponding point scatterers using the following relation similar to relation (1):

$$E_1(\vec{r}_h) = E_0 \sum_{n=0}^{M-1} \beta(\vec{r}_n) \frac{\exp(ik|\vec{r}_I - \vec{r}_n|)}{|\vec{r}_I - \vec{r}_n|} \frac{\exp(ik|\vec{r}_h - \vec{r}_n|)}{|\vec{r}_h - \vec{r}_n|} \exp(i|\vec{r}'_n - \vec{r}_n|), \quad (2)$$

where  $\vec{r}_n$  is the coordinate vector of the  $n$ -th virtual source,  $|\vec{r}'_n - \vec{r}_n| = |z_o - z_n|$  is the distance from the plane ABCD to the  $n$ -th point scatterer,  $|\vec{r}_h - \vec{r}'_n| = \sqrt{(x_h - x_n)^2 + (y_h - y_n)^2 + (z_h - z_o)^2}$  is the distance from the receiver to the  $n$ -th virtual source.

Expression (2) can be calculated using the FFT.

**4. THE METHOD OF VIRTUAL SOURCES ON A SET OF PARALLEL PLANES (THE METHOD 3)**

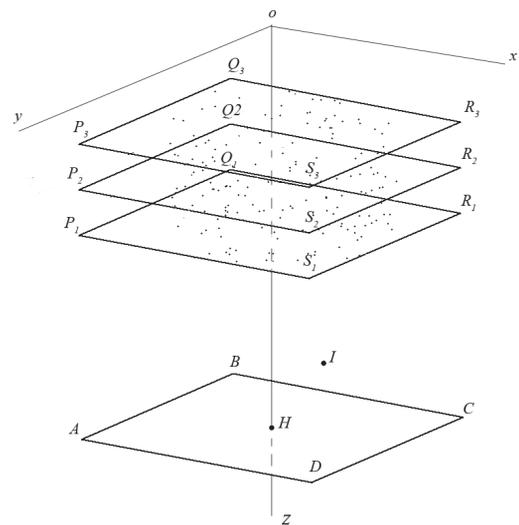
When a size of the object along the OZ axis is significant, the errors in the calculation of the hologram, as follows from (2), will increase in comparison with the method 1.

To reduce the errors, we have designed point scatters that make up the object on a series of parallel planes, for example, on three planes  $P_1Q_1R_1S_1, P_2Q_2R_2S_2, P_3Q_3R_3S_3$ , as shown in Fig. 3, so that on each plane there are virtual sources corresponding to the nearby point scatterers.

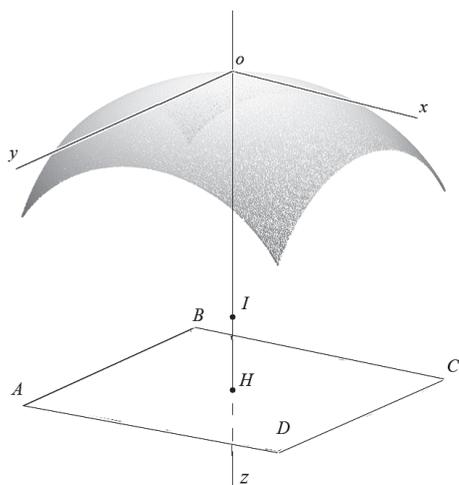
Then the field at the point  $(x_h, y_h, z_h)$  can be estimated taking into account the above phase correction as follows:

$$E_1(\vec{r}_h) = E_0 \sum_{p=0}^{N_p-1} \sum_{n=0}^{M_{np}-1} \beta(\vec{r}_n) \frac{\exp(ik|\vec{r}_I - \vec{r}_n|)}{|\vec{r}_I - \vec{r}_n|} \frac{\exp(ik|\vec{r}_h - \vec{r}'_{np}|)}{|\vec{r}_h - \vec{r}'_{np}|} \exp(i|\vec{r}'_{np} - \vec{r}_n|), \quad (3)$$

where  $|\vec{r}'_{np} - \vec{r}_n| = |z_{op} - z_n|$  is the distance from the  $P_pQ_pR_pS_p$  plane to the  $n$ -th point scatterer,  $|\vec{r}_h - \vec{r}'_{np}| = \sqrt{(x_h - x_{np})^2 + (y_h - y_{np})^2 + (z_h - z_{op})^2}$  is the distance from the receiver to the  $np$ -th virtual source, which is the projection of the  $n$ -th point source on the  $P_pQ_pR_pS_p$ ,  $N_p$  и  $M_{np}$  are the number of planes and the number of virtual sources on each plane, respectively.



**Fig. 3.** Scheme for measuring a hologram of an object in the form of an ensemble of virtual sources on the planes  $P_1Q_1R_1S_1, P_2Q_2R_2S_2, P_3Q_3R_3S_3$  which are irradiated from point I.



**Fig. 4.** Scheme for measuring a hologram on the ABCD plane when a fragment of a sphere is irradiated from its center I.

### 5. COMPARATIVE ANALYSIS OF THE RESULTS OF CALCULATION OF RADIOGOLOGRAMS BY METHODS 1, 2, 3

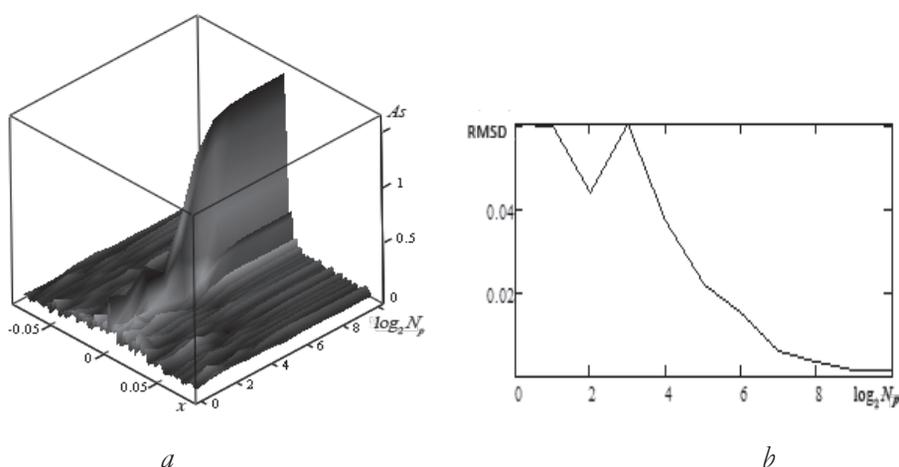
To compare the efficiency of calculations of radio holograms by the methods considered above, we choose a part of the metal sphere as a test object which, as shown in **Fig. 4**, is irradiated by a spherical wave from a point lying in the center of the sphere.

The choice of such a test object is due to the property of the sphere to focus the waves emitted from its center in its center. Therefore, as a criterion for the effectiveness of the application of the compared methods, one can take their ability, as a result of calculations, to provide high-quality focusing of the scattered radiation.

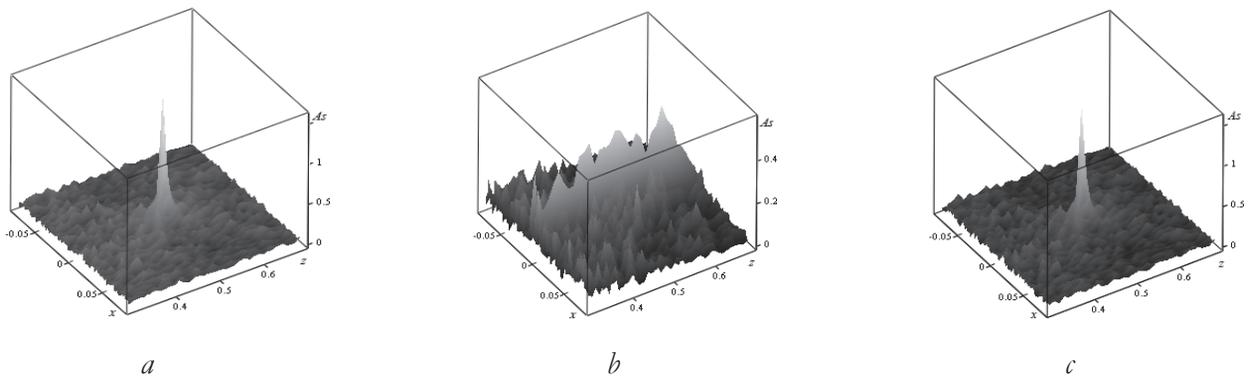
Next, we perform calculations for a radiation wavelength of  $\lambda = 0.0025$  m, the sphere radius  $R = 0.5$  m, a two-dimensional hologram size of  $0.65 \times 0.65$  m with a number of pixels  $512 \times 512$ . We choose the scattering fragment of the sphere in such a way that its projection onto the measurement plane of the hologram coincides with it in shape and size. Let 1000 point scatterers be randomly selected on the surface of a sphere fragment.

In the calculations, we used the transformation of the back projection algorithm to the form of a two-dimensional FFT developed in [4].

Let us compare the root mean square deviations (RMSD) between the amplitudes of the holograms calculated by methods 1 and 3, depending on the number of planes with virtual sources in method 3.



**Fig. 5.** Sections of the surface of the amplitude of the hologram along the OX axis at  $y = 0$  (a) and the graph of the dependence of the RMSD in the center of the hologram (b) depending on the number of planes with virtual sources.



**Fig. 6.** Sections of the hologram amplitude surface along the OX axis at  $y = 0$ , depending on the distance  $z$  from its plane to the surface of the fragment of the sphere, calculated by methods 1 (a), 2 (b), and 3 (c).

**Fig. 5a** shows sections of the hologram amplitude surface along the OX axis at  $y = 0$ , and **Fig. 5b** shows the RMSD diagram at  $x = y = 0$  depending on the  $N_p$  number of planes with virtual sources. Note that the center of the hologram and the center of the sphere have the same coordinates  $x = y = 0$  for different values of  $z$ .

When  $\log_2 N_p > 6$  (i.e.,  $N_p > 64$ ), the amplitude is stabilized as shown in Fig. 5a, and the standard deviation becomes less than 0.01 (i.e. it is negligible) as shown in Fig. 5b. In this regard, in the future, applying method 3, we will use 64 planes with virtual sources.

Next, we consider the dependences of the amplitude calculated by methods 1, 2, 3 at the center of the hologram (i.e., at  $x = y = 0$ ) on the distance to the object along the OZ axis. **Fig. 6** shows sections of the surface of the hologram amplitude along the OX axis at  $y = 0$ , depending on the distance  $z$  from the hologram plane to the surface of the sphere.

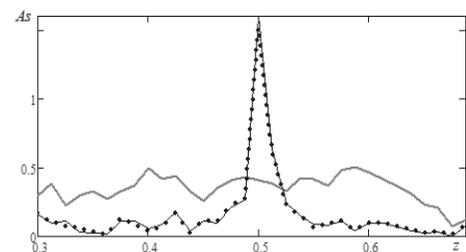
It can be seen that the holograms calculated by both methods 1 and 3 have pronounced peaks in its center that are not observed in the hologram calculated by the method 2. More accurate quantitative data of

these dependences are presented in **Fig. 7**, which shows graphs of the dependences of the hologram amplitudes at the central point  $x = y = 0$  on the distance  $z$ .

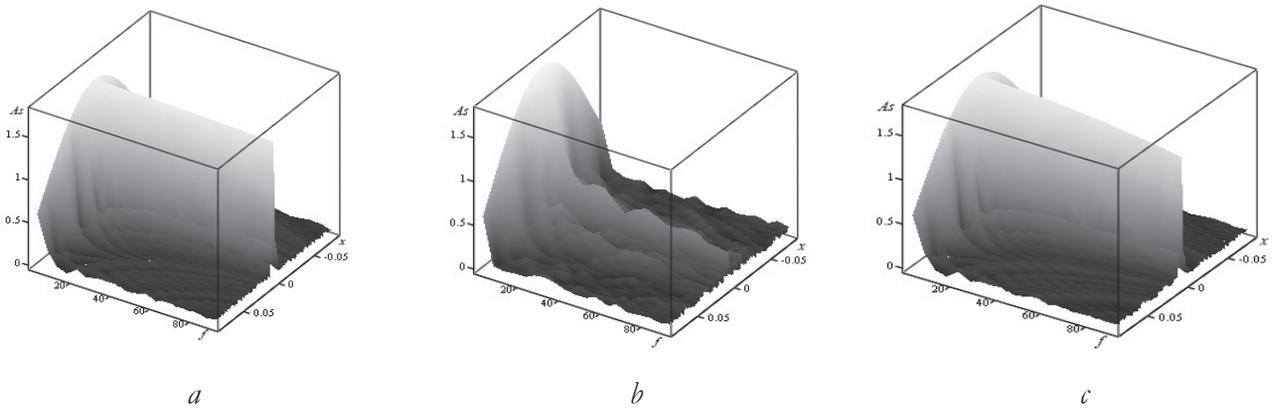
From a comparison of these graphs, it follows that, firstly, the results calculated by both the methods 1 and 3 practically coincide, and, secondly, the focus of the hologram calculated by the method 2 is absent.

Next, we consider the dependence of the hologram amplitude on the radiation frequency.

For this purpose, dependences of the hologram amplitude at  $y = 0$  were computed on both  $x$  and the frequency in the range 2-100 GHz using methods 1, 2,



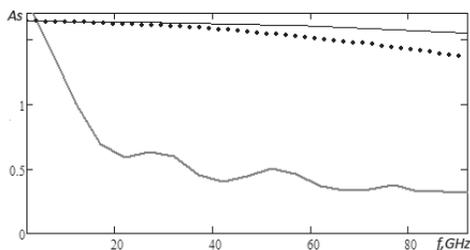
**Fig. 7.** Amplitudes of holograms at  $x = y = 0$ , depending on the distance to the surface of the sphere fragment, calculated by methods 1 (solid line), 2 (dashed line), and 3 (dotted line).



**Fig. 8.** Sections of the amplitude surface of the holograms along the OX axis at  $y = 0$  as a function of  $x$  and frequency  $f$  computed by methods 1 (a), 2 (b) and 3 (c).

3. The computed data are shown in **Fig. 8a, 8b, 8c.**

Comparing the data in Fig. 8a, 8b, 8c, it follows that the results computed by method 2 at high frequencies differ significantly from those computed by methods 1 and 3. To compare method 1 with method 3, graphs of the frequency dependence of the amplitudes at the center point of the hologram coinciding with the center of the sphere are shown in **Fig. 9.** It is seen that method 3, with a total number of planes equal to 64, makes it possible to calculate the amplitudes of holograms of volumetric objects with an acceptable error in comparison with the



**Fig. 9.** The frequency dependences of the amplitude in the center of the hologram computed by methods 1 (dotted line), 2 (dotted line) and 3 (solid line).

calculation results obtained by the known method 1.

### 6. COMPARATIVE ANALYSIS OF THE COMPUTED TIME OF HOLOGRAMS BY VARIOUS METHODS

From (1) it follows that the time of direct calculation  $t_1$  of a hologram of size  $N \times N$  and an object consisting of  $M$  scatterers is estimated as

$$t_1 \sim M \times N \times N \tag{4}$$

that is, it is proportional to the number of scatterers. Since the total number of scatterers in the object under study is 1000, and the number of pixels in the hologram  $512 \times 512 = 262\,144$ , then, for example, when using a computer with a Pentium E5400 processor, calculations by method 1 will take several tens of hours.

Since expression (2), as noted above, can be calculated using the FFT, so method 2 requires less the computation time  $t_2$  which does not independent on the number of scatterers  $M$  and is determined as follows

$$t_2 \sim N \times N \times \log_2 N. \tag{5}$$

Method 3 requires computation time  $t_3$ , which is  $N_p$  longer than method 2:

$$t_3 \sim N \times N \times N_p \times \log_2 N. \quad (6)$$

If the number of scatterers making up the volumetric object under study is equal to  $M \gg (N_p \times \log_2 N)$ , then it follows from a comparison of (4) with (6) that method 3, which is a method of virtual sources on a set of parallel planes, significantly reduces the computation time as compared to the known method 1.

## 7. CONCLUSION

In this paper, a method for calculating holograms for volumetric objects based on the representation of objects in the form of ensembles of virtual point sources distributed on a set of parallel planes has been proposed. The proposed method is the development of the known method in which objects are represented as ensemble of real point scatterers.

The possibilities of the proposed method are demonstrated by the example of calculating a hologram of a fragment of a sphere, on which 1000 points are randomly selected and those scatter radiation emanating from the center of the sphere. The choice of a fragment of a sphere as an object under study is due to the fact that when calculating its hologram, phase errors inherent in approximate calculations, are most pronounced.

The calculations were performed for the frequency range of 2...100 GHz, the sphere radius of 0.5 m, a two-dimensional hologram size of 0.65×0.65 m, and a pixel count of 512×512. It is shown that, in comparison with the known method, the proposed method makes it

possible to calculate the amplitude of a hologram with satisfactory accuracy if virtual sources are placed on parallel planes in an amount of more than 64 pieces. In the case of objects that require representation in the form of an ensemble of point scatterers in the amount of more than 1000 pieces, the calculation of their holograms by the proposed method turns out to be much more efficient than the known method.

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