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## The Decoding Algorithms For Error-Correcting Product-Codes Based On Project Geometry Low-Density Parity-Check Codes

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**Abstract:** The focus of this paper is directed towards the investigation of the characteristics of symbol-by-symbol iterative decoding algorithms for error-correcting block product-codes (block turbo-codes) which enable to reliable information transfer at relatively low received signal/noise and provide high power efficiency. Specific feature of investigated product codes is construction with usage of low-density parity-check codes (LDPC) and these code constructions are in the class of LDPC too. According to this fact the considered code constructions have symbol-by-symbol decoding algorithms developed for total class LDPC codes, namely BP (belief propagation) and its modification MIN\_SUM\_BP. The BP decoding algorithm is iterative and for implementation the signal/noise is required, for implementation of MIN\_SUM\_BP decoding algorithm the signal/noise is not required. The resulted characteristics of product codes constructed with usage of LDPC based on project geometry (length of code words, information volume, code rate, error performances) are presented in this paper. These component LDPC codes are cyclic and have encoding and decoding algorithms with low complexity implementation. The computer simulations for encoding and iterative symbol-by-symbol decoding algorithms for the number of turbo-codes with different code rate and information volumes are performed. The results of computer simulations have shown that MIN\_SUM\_BP decoding algorithm is more effective than BP decoding algorithm for channel with additive white gaussian noise concerning error-performances.

**Keywords:** noise-immune, product codes, error-correcting low density parity-check codes, signals, noise, iterative symbol-by-symbol decoding

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### CONTENTS

1. INTRODUCTION (399)
  2. FORMULATION OF THE PROBLEM (400)
  3. THE PROJECTIVE GEOMETRY LDPC AND PC BASED ON THESE (401)
  4. THE DECODING ALGORITHMS FOR LDPC CODES (401)
  5. THE SIMULATION RESULTS (404)
  6. CONCLUSION (405)
- REFERENCES (405)

### 1. INTRODUCTION

Error-correction codes are used to improve reliability of the information transmission through the channels with noise [1-3]. Code constructions, that in general named as product-codes (block turbo-codes) are considered to be ones of the most effective in terms of probabilistic characteristics ( $P_b$  – probability of error for information bit,  $P_{er}$  – probability of errors for decoding of

codewords) at the decoding phase [4-6]. These codes belong to certain groups of accepted standards (satellite communication systems (IESS-15, IESS-315), broadband wireless access IEEE 802.16, digital systems TV and radio broadcasting, optical systems transmission of information [5,7]).

The development of the theory of product-codes (PC) is an independent scientific field of investigations [4-9]. Specific algorithms of iterative decoding for PC have been invented [4,9] that are simpler in implementation than the algorithms of the optimal decoding with a mild degradation of the probabilistic characteristics. Recent studies demonstrate the probabilistic decoding characteristics can reach the theoretical limited conditions with augmentation of the PC information volumes for transmission channels with the additive white Gaussian noise (AWGN) [9].

In general, the product-codes are formed on the component block error-correcting codes usage [1]. Complexity of algorithms of the iterative PC decoding is determined by the complexity of the decoding algorithms for component block codes. The class of known component codes satisfying the condition of low complexity of decoding algorithms is constrained - block Hamming codes, block Hamming codes with the generalized parity check [4,6,9].

The open problem is expansion of the class product-codes with the variation of the parameters such as information volume and code rate. These parameters are employed for digital communication systems with adaptive mode noise-immune transmission. One of the solutions for this problem is the PC by shortening the information volume block of the source code [10].

The second direction is based on the low-density parity-check codes (LDPC) as the component error-correcting codes [11], in particular, the projective geometry LDPC [1].

Thus, the actual problem is investigation and simulation of algorithms for iterative decoding in terms of error-performances for the emerged family of product-codes.

## 2. FORMULATION OF THE PROBLEM

Codewords of PC based on  $C_1, C_2$  component codes with parameters  $(n_1, k_1, d_1)$  and  $(n_2, k_2, d_2)$  are represented as a two-dimensional matrix, where rows are coded words of  $C_1$  code and columns are  $C_2$  coded words [1]. Here  $n, k, d$  are the length, information volume and minimum Hamming weight of code words correspondingly. For PC we have  $n = n_1 n_2, k = k_1 k_2, d = d_1 d_2$ , code rate  $R = k/n$ .

Optimal decoding algorithms for code  $(n, k, d)$  are based on the implementation of  $\min(2^k, 2^{(n-k)})$  correlators [3]. Under conditions  $\min(k, n-k) \gg 1$  execution of the optimal decoding algorithms in real time remains a challenging problem. For this purpose, the algorithms based on the principle of turbo-decoding have been developed [2,9]. These algorithms require significantly less complex implementation with insufficient energy losses in comparison with algorithms for optimal decoding. The basis of these algorithms are iterative procedures for decoding of codes  $C_1$  and  $C_2$ . Every iteration comprises of two steps. The first step computes a posteriori probabilities for the  $C_1$  code symbols using the input data and a priori probabilities of the code symbols [4]. Functionals derived from the calculated a posteriori probabilities are used

later as the a priori probabilities for the code symbols on the second step of the iteration when estimating a posteriori probabilities of the  $C_2$  code symbols. The decisions regarding the PC code symbols are made upon the fulfillment of the stop condition of the iterative processing

Product-codes based on the component block error-correcting LDPC codes are also belong to the class of the LDPC codes [11,12]. Therefore, the set of PC decoding algorithms can be applied for the iterative decoding specifically developed for the low-density check-parity codes. Modeling studies revealed that these algorithms are generally more efficient than those based on decoding PC based on the principle of turbo-decoding. The component LDPC codes based on the finite projective geometry are described below. These consider to be promising for the applications due to the mild complexity of algorithms of coded words encoding and decoding owing to their cyclic structure [1].

Herein, the essence of the considered problem is description and investigation of the iterative decoding algorithms for PC based on the component projective geometry LDPC codes.

**3. THE PROJECTIVE GEOMETRY LDPC AND PC BASED ON THESE**

Let us denote the class of LDPC codes on the projective geometry over the field  $GF(2^s)$  as  $PG(m, 2^s)$  ( $m, s$  are positive integer) [1]. Assume  $\alpha$  is a primitive element in the field  $GF(2^{(m+1)s})$ , that is in its turn, an extension of the field  $GF(2^s)$ . We consider an element  $\beta = \alpha^n$ ,  $n = (2^{(m+1)s} - 1)/(2^s - 1)$  with the order to be equal to  $(2^s - 1)$ . The set  $\{0, 1, \beta, \dots, \beta^{2^s-2}\}$  is denoted as the field  $GF(2^s)$ .

While examining row  $\{1, \alpha, \alpha^2, \dots, \alpha^{n-1}\}$  and partition of the  $GF(2^{(m+1)s})$  to the disjoint sets  $(\alpha^i) = \{\alpha^i, \beta\alpha^i, \dots, \beta^{2^s-2}\alpha^i\}$ ,  $0 \leq i < n$ , the geometry  $PG(m, 2^s)$  contains  $n$  points that are equivalent to the elements  $(\alpha^i)$  in terms of a vector with  $(m + 1)$  consisting of the components of the field  $GF(2^s)$ . Points  $(\eta_1\alpha^i + \eta_2\alpha^j)$  define a line passing through linearly independent points  $(\alpha^i), (\alpha^j)$ . Here  $\eta_1, \eta_2$  are the elements of the field  $GF(2^s)$  that not equal to zero at the same time. Geometry  $PG(m, 2^s)$  contains  $J = (2^{(m+1)s} - 1)(2^{ms} - 1)/(2^s + 1)(2^s - 1)^2$  lines summed up to  $2^s + 1$  points. Suppose  $H_{PG}(m, s)$  to be a matrix over the field  $GF(2)$  with rows that are equivalent to geometry lines  $PG(m, 2^s)$ . The matrix  $H_{PG}(m, s)$  is a check matrix of the LDPC code  $PG(m, 2^s)$  that contains  $J$  rows and  $n$  columns. Rows and columns of the  $H_{PG}(m, s)$  have Hamming weight of  $J_1 = 2^s + 1$  and  $J_r = (2^{ms} - 1)/(2^s - 1)$  respectively.

LDPC based on geometry  $PG(m, 2^s)$  with a check matrix  $H_{PG}(m, s)$  are cyclic and are defined by the generating polynomial  $g_{PG}(x)$  [1]. Assuming  $\alpha$  is a primitive element belonging to a field  $GF(2^{(m+1)s})$ . The element  $\alpha^h$  ( $0 < h < 2^{(m+1)s} - 1$ ,  $h$  is divided by of  $2^s - 1$ ) is a root of  $g_{PG}(x)$  when condition [1] is satisfied:

$$0 < \max_{0 \leq l < s} (W_{2^s}(h^{(l)})) \leq j(2^s - 1), \quad 0 \leq j \leq m - 1, \quad (1)$$

$$W_{2^s}(h) = \delta_0 + \delta_1 + \dots + \delta_{m-1}, \quad (2)$$

$$h = \delta_0 + \delta_1 2^s + \dots + \delta_{m-1} 2^{(m-1)s}. \quad (3)$$

Here  $0 \leq \delta_i < 2^s, 0 \leq i < m$  are parameters in equations (2), (3);  $h^{(l)}$  – a remainder of division of  $2^l h$  by  $(2^{(m+1)s} - 1)$  in condition (1).

**Table 1** contains the parameters  $n, k, d, J_r, J_s$ , as well as the degree of a primitive element  $\alpha$  of generating polynomials  $g_{PG}(x)$  for a set of codes  $PG(m, 2^s)$ . PC based on LDPC with

Table 1

Parameters of LDPC codes on the projective geometry  $PG(m, 2^s)$  ( $\alpha$  is a primitive element of the field  $GF(2^{(m+1)s})$ ).

$(m, 2^s)$	$n$	$k$	$d$	$J_l$	$J_r$	power $i$ for $d'$
$PG(3, 2^2)$	21	11	6	5	5	1, 3, 9
$PG(3, 2^3)$	73	45	10	9	9	7, 21, 35

parameters  $J_l$  and  $J_r$  belong to the class of LDPC with parameters  $J_l$  and  $2J_r$  [9]. **Table 2** reveals these parameters for the considered PC

#### 4. THE DECODING ALGORITHMS FOR LDPC CODES

We represent  $H = (h_{ir}; 0 \leq i < n - k; 0 \leq r < n)$  as a check matrix of the LDPC code  $(n, k)$ ,  $\vec{B} = (b_0, b_1, \dots, b_{n-1})$  – are the code words. Let us assume  $\vec{Y} = (y_0, y_1, \dots, y_{n-1})$  as a vector with samples  $y_i = s_i + n_i$  that are the signal demodulator output. Here  $s_i, n_i$  are signal and noise components,  $i = 0, 1, \dots, n-1$ . While introducing vector notation  $\vec{x} = (x_0, x_1, \dots, x_{n-1})$  we denote two “hard” solutions:  $x_i = 0$  if  $y_i \geq 0$  and  $x_i = 1$  otherwise.

LDPC codes on the  $PG(m, 2^s)$  geometry possess the property of the orthogonal relations for symbols  $b_i$  of words  $\vec{B}$  [1,2]. Here we introduce parameters:  $N(m) = \{i: h_{mi} = 1\}$  is a set of numbers of code symbols of volume  $J_N(m)$  in the  $m$ -th check relation;  $N(m)/l$  - is a set  $N(m)$  without the  $l$ -th symbol;  $D(l) = \{m: h_{ml} = 1\}$  is a set of orthogonal relations for the code symbol  $b_l$  of volume  $J_D(l)$ ;  $D(l)/m$  is a set of orthogonal relations  $D(l)$  omitting  $m$ -th check. For LDPC codes on the projective

Table 2

Parameters of PC based on LDPC component codes (Table 1).

Component code	$n$	$k$	$d$	$J_l$	$J_r$	$R$
$PG(3, 2^2)$	441	121	36	5	10	0.27
$PG(3, 2^3)$	5329	2025	100	9	18	0.38

geometry  $PG(m, 2^s)$  the conditions  $J_N(m) = J_p, J_D(l) = J_r$  are satisfied.

Below we consider the results of employing the most effective algorithms for the iterative symbol-by-symbol decoding of LDPC codes in terms of probabilistic characteristics – the BP (belief propagation) algorithm and its modification MIN\_SUM\_BP [2,9].

When applying BP algorithm, the parameter  $(\gamma = 2A)/N_0$  is required, where  $A$  is an amplitude of the signal component and  $N_0$  is the spectral density (one-sided) of the AWGN. Before executing the iteration, the initialization  $z_{mi} = (\gamma y_i; m \in J_D(i), 0 \leq i < n)$  is required. Iterative decoding procedure includes the following steps [2].

Step 1. Calculation of the elements of the arrays  $T_{mi}$  and  $L_{mi} (m \in J_D(i), 0 \leq i < n)$

$$T_{mi} = \prod_{l \in N(m/i)} \tanh(z_{li} / 2), \tag{4}$$

$$L_{mi} = \ln((1 - T_{mi}) / (1 + T_{mi})). \tag{5}$$

Step 2. Calculation of the values

$$z_{mi} = y_i + \mu \sum_{l \in D(i)/m} L_{li} \text{ for the symbol } b_i.$$

Step 3. If the condition for stopping of the iterative decoding algorithm has not been satisfied, then processes (4) and (5) continue. When the stopping condition is fulfilled (for example, when the specified number of iterations is reached) the values  $z_i = y_i + \mu \sum_{l \in D(i)} L_{li}$  are calculated and decisions are made regarding the values of the code symbols:  $b_i = 0$  if  $z_i > 0$ , otherwise  $b_i = 1$ . Here  $\mu$  is a constant to be determined by the criteria upon modeling the BP algorithm. The criteria based on the achievement of the minimum values of the error probabilities  $P_p, P_{er}$ .

The decoding algorithm MIN\_SUM\_BP [2] is easier for implementation than BP while not requiring of parameter  $\gamma$ . Before executing

the values  $z_{mi} = y_i; m \in J_D(i), 0 \leq i < n$  are initialized. The MIN\_SUM\_BP algorithm includes the following steps [2].

Step 1. Computation of the "hard" solutions

$$\sigma_{mi} = \begin{cases} 1, z_{mi} > 0, \\ 0, z_{mi} \leq 0. \end{cases} \quad (6)$$

The values  $\sigma_m, L_{mi}$  are computed for each orthogonal verification  $m$

$$\sigma_m = \sum_{i \in N(m)} \sigma_{mi}, \quad (7)$$

$$L_{mi} = (-1)^{\sigma_m \oplus \sigma_{mi} \oplus 1} \min_{i' \in N(m)/i} (|z_{mi'}|). \quad (8)$$

Operand  $\oplus$  in (8) is identical to the addition in the field  $GF(2)$ .

Step 2. Computation of the values  $z_{mi}$  based on  $L_{mi}$

$$z_{mi} = y_i + \mu \sum_{m \in D(i)/m} L_{mi}. \quad (9)$$

Step 3. Computation of step 1 and step 2 of the next iteration if the stop condition has not been met. If the stop condition is gratified one should follow an IF condition regarding the symbols  $b_i$  values using the expression  $z_i = y_i + \mu \sum_{m \in D(i)} L_{mi} : b_i = 0$  if  $z_i \geq 0$ , otherwise  $b_i = 1$ .

To increase the reliability of the resulted decisions, the considered algorithms of iterative decoding can be supplemented with the orthogonal conditions  $H\vec{B}^T = 0$ : when this condition is executed a decision regarding the calculated codeword  $\vec{B}$  is made. In the opposite case an error is detected as a result. The basic characteristics of such verification reside on the probability of error detection  $P_{det}$ . When receiving codewords, the error probability  $P_{(p,er)}$  can be described as  $P_{(p,er)} = P_{er} - P_{det}$ . Current approach is used in information transmission systems with Automatic Repeat Request (ARQ) [2].

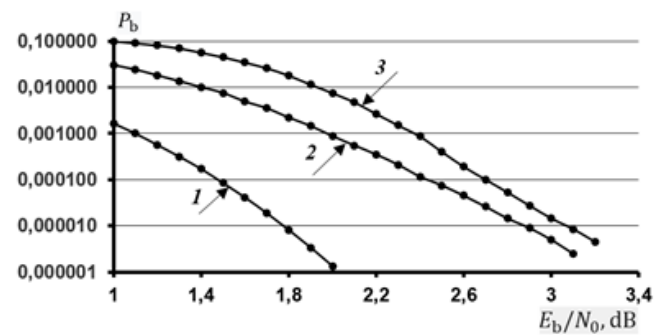
## 5. SIMULATION RESULTS

The results of the BP and MIN\_SUM\_BP simulations for the PC as well as the model parameters are shown in Table 2. The obtained curves revealed relationships of the probabilities  $P_b$  and signal-to-noise ratios  $E_b/N_0$  employing signals with binary phase shift keying for the AWGN channel. Here  $E_b$  is the signal energy per bit. The probabilities  $P_b$  and  $P_{er}$  for an error-correcting code with parameters  $(n, k, d)$  are related by the approximate relationship  $P_b \approx dP_{er}/n$  [1].

The interval estimates of the erroneous decoding probabilities were calculated during the simulations using the probability estimate  $x/u$ , where  $x$  is the number of erroneous decisions among the sequence of transmitted codewords  $u$ . The required volume  $u$  was determined by the size of the confidence interval  $[0.5P_{er}, 1.5P_{er}]$  with a confidence probability  $P_{"cp"} = 0.95$  [13].

**Fig. 1** exhibits the values  $P_b$  for the PC based on the PG code (21,11) (PC parameters:  $n = 441, k = 121, d = 36$ , code rate  $R = 0.27$ ). Curve 1 corresponds to the upper bound  $P_b$  of random coding for AWGN which justifies the existence of a code with parameter code rate of the considered PC [4].

Curve 2 corresponds to the MIN\_SUM\_BP decoding algorithm (parameter  $\mu = 0.2$ ),



**Fig. 1.** Values  $P_b$  for the PC (441,121) based on the component PG code (21,11): curve 1 - upper bound of random coding; curve 2 - MIN\_SUM\_BP decoding algorithm; curve 3 - BP decoding algorithm.

whereas curve 3 indicates the BP decoding algorithm (parameter  $\mu = 0.19$ ) (the number of iterations does not exceed 20). MIN\_SUM\_BP and the BP algorithms determine the energy loss compared to the theoretical curve 1 to be equal to 1.3 dB under the value  $P_b = 10^{-5}$ . In addition, the MIN\_SUM\_BP algorithm reveals to be more efficient than the BP algorithm. Thus, the energy gain when using MIN\_SUM\_BP reaches 0.2 dB for  $P_b > 10^{-5}$ . It should be also noted that the energy gain with examined PC for values  $P_b > 10^{-5}$  reaches 0.8 dB with respect to the known convolutional code with a code rate of 1/3 [3].

**Fig. 2** shows the values  $P_b$  for the PC based on the PG code (73,45,10) (PC parameters:  $n = 5329$ ,  $k = 2025$ ,  $d = 100$ , code rate  $R = 0.38$ ). Curve 1 corresponds to the upper bound  $P_b$  of the random coding for AWGN [4]. Curve 2 and curve 3 display results of the MIN\_SUM\_BP algorithm (parameter  $\mu = 0.17$ ) and the BP algorithm (parameter  $\mu = 0.226$ ) (the number of iterations does not exceed 20) correspondingly. The MIN\_SUM\_BP and the BP algorithms reveal the energy loss in relation to the theoretical curve 1 to be 1.6 dB for the value  $P_b = 10^{-5}$ . Based on the

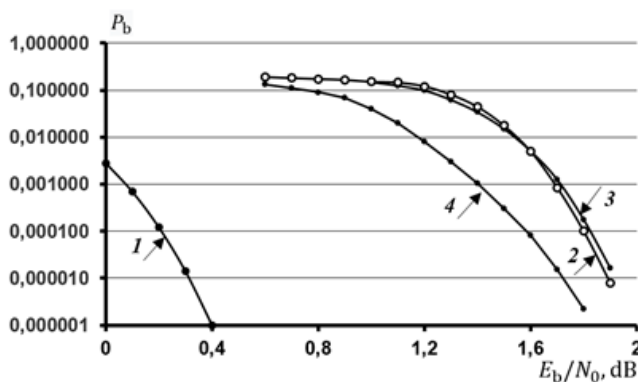
graphical representation the MIN\_SUM\_BP and BP algorithms are practically equivalent with respect to the probabilistic characteristics for  $P_b > 10^{-5}$ . It should be noted that the considered PC is practically equivalent in terms of probability characteristics to the most effective LDPC code AR4J with a code rate of 1/2 and parameters (4096,2048), i.e. the differences do not exceed of 0.2 dB for  $P_b < 10^{-5}$ . The code AR4J is recommended for use in satellite communication systems [14].

Simulations of iterative decoding algorithms in combination with the solution rejection strategy for the considered PC demonstrated the fulfillment of the conditions  $P_{det} = P_{er}$  and  $P_{(p,er)} = 0$ . This way, the information transmission was carried out in the error-free manner.

## 6. CONCLUSION

The investigations of probabilistic characteristics for error-correcting product codes on the component LDPC codes have been performed. These code constructions appear to be low-density parity-check codes. Specific feature of investigated PC that these are in the class of LDPC too. These PC codes have symbol-by-symbol decoding algorithms developed for total class LDPC codes, namely BP (belief propagation) and its modification MIN\_SUM\_BP. The parameters of the considered PC codes (the length of coded words, the information volume, minimum Hamming weight and the code rate) are represented, while being formed using a class of component LDPC codes on finite projective geometry.

Generative and check matrices of the projective geometry codes possess the cyclicity property, which determines the low



**Fig. 2.** Values  $P_b$  for the PC (5329,2025) based on the component PG code (73,45): curve 1 - upper bound of random coding; curve 2 - MIN\_SUM\_BP decoding algorithm; curve 3 - BP decoding algorithm; curve 4 - LDPC code AR4J with code rate 1/2 [14].

complexity of the encoding and iterative decoding algorithms.

Simulations of the iterative decoding algorithms for the AWGN channel was carried out for two PC codes. MIN\_SUM\_BP algorithm has been shown to be the most efficient in terms of probabilistic characteristics, in this case an estimate of the energy parameter has not been required. The probabilistic curves were demonstrated to be close to the probabilistic curves of the theoretical upper bounds of random coding. This determines the perspectives of usage such code constructions for noise-immune transmission of information.

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