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Calculation of the intrinsic thermal radiation of plane-parallel quasi-anisotropic multilayer plates with smooth boundaries

Mikhail G. Evtikhov, Galina V. Arzamastseva

Kotelnikov Institute of Radioengineering and Electronics of RAS, Fryazino Branch, http://fire.relarn.ru/
1,Vvedensky area, Fryazino 141190, Moscow region, Russian Federation

E-mail: emg20022002@mail.ru, arzamastseva@mail.ru

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Abstract: **The physical meaning of new algorithms for calculating the intensity of a plane homogeneous monochromatic wave of electromagnetic radiation after passing through a multilayer quasi-anisotropic plane-parallel plate is discussed, taking into account the thermal radiation of the layers. The formula connecting the brightness temperature obtained by a microwave radiometer and the effective temperature of the observed surface is used in remote sensing of the Earth's surface [16]. In this paper, we develop a mathematical apparatus that allows one to construct algorithms that generalize this formula to an arbitrary number of homogeneous quasi-anisotropic layers of a plane-parallel plate. The solution of the problem is complicated by the need to take into account coherent and incoherent effects in a multilayer plate, as well as by the need to construct an adequate method for identifying the waves and energy fluxes under consideration, by the need to clarify the concept of an ideal radiometer that records the observed microwave radiation. In order to test new algorithms and obtain the first results, the facts obtained earlier [19] by calculating the reflection and transmission coefficients for free ice sheets are reproduced using new algorithms for calculating intensities. For an isotropic ice plate 50 cm thick in the L-range, there is a "transparency window" in the area of observation angles of 30 degrees for both polarizations simultaneously. The influence of ice anisotropy on the effects of bleaching and anti-bleaching and related to the Brewster angle is considered. Additionally, the contribution of the ice's own radiation to the observed brightness temperature was estimated by new methods. The case of an anisotropic ice plate with the same parameters but floating in water is considered. It is shown that a change in the conditions of reflection at the ice-substrate interface can be partially compensated by a change in the ice thickness. To control and evaluate the theoretically possible accumulation of errors in calculations, physical quantities are discussed that are analogous to the components of the Poyting vector and remain constant at the boundaries of the layers. For the considered cases of ice, these values are conserved with high accuracy.**

Keywords: **remote sensing, microwave, anisotropy, fresnel equations, multi-layer interference, transparency,translucency, ice, water, brightness temperature**

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Contents

1. INTRODUCTION

The general theory of the propagation of linear electromagnetic waves [1,2,3] guarantees solvability, makes it possible to verify and interpret solutions to many problems of interference and diffraction. In rather difficult cases, it is necessary to use specialized approaches. A widely known specialized approach to the description of stationary electromagnetic waves in multilayer plane-parallel plates is the calculation of the Fresnel and Fresnel-Airy coefficients. The use of these coefficients is an effective method for solving a number of problems in optics [1,4,5], acoustics [6], ellipsometry [7], X-ray optics [8,9], remote sensing of the environment [10].

While the Fresnel approach is effective, it comes with a number of fairly stringent constraints. In the classical Fresnel formulas for electromagnetic waves [1,4-6], radiation is assumed to be stationary, monochromatic, the linear local Maxwell equations are valid. Layers of plane-parallel plates are considered transparent, isotropic, with smooth boundaries. The incident, reflected and refracted waves are coherent and homogeneous. In this case, the s- and p-polarization waves do not interact and propagate independently. Generalization of Fresnel's formulas to media with absorption was of great practical and methodological importance at the beginning of the 20th century [1-10]. If in Snell's laws and Fresnel's formulas it is purely formal to admit the possibility of complex refractive indices, then one can obtain results that are usually correct, but cause certain difficulties in interpreting the complex values of the angles. This approach is insufficient for solving Fresnel problems in amplified environments [11]. The principle of continuity of the tangent component of the complex wave vector makes it possible to correctly derive the Fresnel formulas for transparent and damped media [1]. This principle is used in this work. Currently, the development of methods that remove restrictions and adapt the application of Fresnel formulas to various ranges of electromagnetic waves remains relevant and promising. The development of experimental and theoretical approaches to the study of the propagation of electromagnetic waves in the microwave range was summarized in monographs [10,12].

In this work, attempts continue to generalize the Fresnel formulas to a limited, but rather wide class of anisotropic media, to quasi-anisotropic media. The concept of *p*-polarization waves was used in [13] to describe some surface and bulk electromagnetic waves in ferromagnetic films. The films considered were magnetized to saturation, i.e. the medium was essentially anisotropic. Under what general restrictions on the tensors of the magnetic and dielectric permittivity, *s*- and/or *p*-polarization waves propagate independently, but the media are anisotropic? The answer to this question is considered in [14-16], algorithms are obtained that generalize the Fresnel formulas for such "quasi-anisotropic media". The algorithms were used for the theoretical study of multilayer plates made of ferromagnets and dielectrics [15]. Further, using the mathematical apparatus of extremely sparse matrices [17,18], the algorithms were generalized to the case of an arbitrary number of layers of multilayer quasi-anisotropic plane-parallel plates, the concept of a quasi-anisotropic medium was refined [16]. Anisotropy in three orthogonal directions is admissible: orthogonal to the surface of the plate, along the projection of the wave vector of waves on the surface of the plate and along the surface of the plate, but orthogonal to the wave vector. For a multilayer plate, a system of linear equations was automatically formed, its solutions were Fresnel-Airy coefficients. The results of [16] were used in [19] for a theoretical study of the reflection and transmission coefficients of electromagnetic waves in the L-band (1-2 GHz) in free plates of isotropic and anisotropic ice. For the L-band, there are commercially available domestic [23,24] and foreign [25] radiometers used to monitor the Earth's ice cover using aircraft. It is known that the Fresnel approach is adequate for describing the propagation of L-band and lower frequency electromagnetic waves in ice. For higher frequencies, it is necessary to study and apply more complex theoretical models [26]. Therefore, the theoretical study of the properties of isotropic and anisotropic ice in the *L*-range and the testing of new theoretical approaches and algorithms seem promising for the subsequent development of the theory for higher frequencies.

Formula

$$T_{ya} = \kappa T_{ef} \tag{1}$$

is the theoretical basis for many works on remote sensing of the Earth's surface using aircraft [10]. This formula connects T_{ef} the effective surface temperature of the observed object and T_{ya} the brightness temperature observed with a microwave radiometer. The coefficient is calculated based on the Fresnel formulas, it depends on the polarization, viewing angle and frequency. It is possible to interpret the coefficient as the transmittance of microwave radiation by the surface of the body. In this case, the temperature in (1) can be interpreted as the radiation intensity in the microwave range, written in units of the Kelvin temperature. Generalizations of formula (1) to the cases of isotropic plates of several layers are known [10]. The possibility of generalizing formula (1) to multilayer quasi-anisotropic plates seems promising from the point of view of application in the field of remote sensing of both the earth's surface and other objects. In [19], the reflection and transmission coefficients of anisotropic free ice sheets at various thicknesses were discussed. The calculations were greatly simplified by the assumption that the plane-parallel plate is free, i.e. is in the air (vacuum). The use of the results for free wafers is limited to laboratory applications. It seemed promising to develop the ability to calculate the Fresnel and Fresnel-Airy coefficients for multilayer quasi-anisotropic plates for the cases where the sources of plane waves are located inside the layers of the plate. In addition, the question arose about the influence of the intrinsic thermal radio emission of the ice plate on the observed brightness temperature. The problems of calculating and observing the intrinsic thermal radiation of a multilayer plate are relevant for use in remote sensing, for example, on an ice plate floating in water. In [12], the assessment of the intensity of intrinsic thermal radiation of semi-infinite space in the microwave range was made on the basis of the Rayleigh-Jeans law and Kirchhoff's law for radiation. It was assumed that the radiation sources are completely incoherent. Based on the same laws, the formula for the intensity of intrinsic thermal radiation and the attenuation of the radiation intensity by a plate of finite thickness was discussed in [20].

In formula (1), one can see the following logical subtlety. In this formula, at the beginning, the total intensity of incoherent waves of thermal radiation of the half-space is estimated. This sum is then replaced by an “equivalent wave” with the same intensity. “Equivalent wave” is considered as a plane wave source in Fresnel formulas. The amplitude of the observed radiation is calculated using Fresnel's formulas, and at the end T_{ya} the intensity is estimated, expressed in units of temperature. This approach is convenient, it would be desirable to apply it for multilayer media, but it must be justified. Appendix contains mathematical calculations showing that, provided that thermal radiation sources are completely incoherent, then the "equivalent wave" approach is correct.

Microwave radiometers have certain limitations related to the properties of their antennas. It was shown in [19, 21] that with an ice thickness of 4.2 m, the presence of a 2 MHz radiometer passband in the *L*-band leads to a complete smoothing of the inhomogeneity in the polarization-angular dependences of the reflection and transmission coefficients. With an ice thickness of 50 cm, the presence of the radiometer passband can be neglected at observation angles less than 70 degrees and the radiometer can be considered ideal. The question arises: what other constraints should an ideal radiometer satisfy and how to use the idea of an ideal radiometer to take into account the bandwidth of real radiometers? In this work, the radiometer is considered ideal, i.e. we neglect its bandwidth. In addition, it is assumed that an ideal radiometer detects separately the *s*- and *p*-polarization waves and these plane waves are homogeneous. An ideal radiometer does not register inhomogeneous plane waves. In the considered range of parameters, these properties of an ideal radiometer are close to the properties of real radiometers used in [23-25]

The new algorithms being developed are very complex and it was supposed to check them by exact reproduction of the rather interesting result obtained earlier in [19] before studying more complex cases. In accordance with the results of [19], at an isotropic ice thickness of 50 cm and a temperature near $0°C$, there should be a transparency band at an angle of incidence of 30 degrees, for both polarizations simultaneously. The presence of ice anisotropy modifies the effects of bleaching and anti-bleaching near this transparency band. The question arose about assessing the influence of the intrinsic thermal radiation of the plate. In addition, the question also arises of controlling the theoretically possible accumulation of errors in calculations. In [12], the component of the Poynting vector, averaged over the oscillation period, orthogonal to the plate surface, is considered. It allows you to control the balance of energy flow between the layers. The energy fluxes along the surface of the layers are also not arbitrary; they must also obey certain balance relations. It is also useful to refine these relationships and use them to control calculations.

The purpose of this article is to discuss the physical meaning of new algorithms that take into account the intrinsic thermal microwave radiation of the layers and make it possible to calculate the intensity of a plane uniform monochromatic wave passing through a multilayer plane-parallel plate consisting of quasi-anisotropic layers with smooth plane boundaries.

This article discusses the registration of radiation with an ideal radiometer, which has a number of special properties that simplify the task. This radiometer is located in a homogeneous isotropic medium (in vacuum) outside the plate. The radiometer measures the intensities of monochromatic homogeneous waves and the directions of their propagation, and separately for *s*- and *p*-polarization waves.

The problem is to generalize formula (1). It includes the calculation of the Fresnel-Airy coefficients for plane-parallel plates made of quasi-anisotropic media. The assumption is made that the heat sources are completely incoherent. It is also possible to consider external incoherent sources of plane waves (incoherent with each other and with heat sources).

2. MATERIALS AND METHODS

2.1. Ideal radiometer and connected subsets of coherent waves

When deriving the Fresnel formulas, a plane monochromatic wave of *s*- or *p*-polarization of a given amplitude is considered, incident on a smooth plane interface between media. Wave vectors and complex amplitudes of the reflected and refracted waves are calculated [1-7].

It is assumed that *s*- and *p*-polarization waves propagate independently From the point of view of remote sensing problems, somewhat different problems are of interest, when the wave vector and the amplitude of the observed refracted wave are given, and the parameters of the incident wave are calculated [10]. Such problems are solvable using Fresnel coefficients. In a multilayer plate, the Fresnel-Airy coefficients play the same role. Coherent reflected and refracted waves, generated by one incident wave, form a connected set with the same property. If the complex amplitude and wave vector of one of the waves are known, then the parameters of all other waves can be calculated

In this paper, only an ideal radiometer is considered, which has a number of special properties that determine the properties of waves in the mathematical model. An ideal radiometer is located in a homogeneous isotropic medium (in a vacuum). The radiometer measures the intensities of monochromatic homogeneous waves and the directions of their propagation, separately for s- and p-polarization waves. It follows from these properties of an ideal radiometer that in a mathematical model one can restrict oneself to considering waves with the following properties. First, these are monochromatic waves of a given frequency. Second, all considered waves must have the same real projection of the complex wave vector onto the surface of the plate. The second property follows from the principle of continuity of the wave vector component, tangent to the surface of a plane-parallel multilayer plate. The imaginary part of this projection must be zero, i.e. the complex wave amplitude is constant parallel to the plate surface. This property is fulfilled provided that an ideal radiometer records homogeneous waves in an isotropic homogeneous medium.

Let's choose a coordinate system corresponding to the properties of the waves. Let the direction of the coordinate axis "*y*" coincide with the projection of the wave vector onto the surface of the plate. Let the direction of the "*x*" axis be normal to the surface of the multilayer plate under consideration. Then, in quasi-anisotropic media, the "*s*"-component of the wave vector should be equal to zero [14]. For possible projections of the complex wave vector of such waves on the *x*-axis, from Maxwell's equations for a given homogeneous layer, a quadratic equation is obtained. One solution to this equation corresponds to a refracted wave, the second – to a reflected wave in the same layer. Since there are only two solutions, the wave vector of a coherent incident wave from a wave source in this layer must coincide with the direction of the wave vector of the incident or reflected wave in this layer.

For each of the boundaries of the layers in a multilayer plate, it is possible to write down the conditions for the continuity of the tangential surfaces of the layers of the plate of the electric and magnetic field components. As a result, for a multilayer plate, separately for *s*- and *p*-polarization waves, it turns out to be possible to automatically form a nondegenerate, well-defined system of linear equations connecting the complex amplitudes of reflected and refracted waves with the complex amplitudes of wave sources. The solutions of this system of equations are the Fresnel-Airy coefficients for a multilayer plate, this was done in [16].

The concept of an ideal radiometer is implicitly used in measurements. All waves in a multilayer quasi-anisotropic plate are divided into disjoint connected subsets determined by the common value of k_y, frequency and polarization. From each measurement, the properties of the associated waves in each layer can be calculated. Theoretically, by increasing the number of measurements at different frequencies, k_y and polarizations, it is possible, in the limit, to obtain a complete picture of the properties of all waves of a given range. In this work, the radiometer is considered ideal. The properties of the mathematical model and issues related to the imperfection of the radiometer are discussed in more detail in the Discussion of Results section.

2.2. SQU- SPECIFICATION OF WAVES IN A MULTILAYER QUASI-ANISOTROPIC PLATE

In quasi-anisotropic media, *s*- and *p*-polarization waves do not interact [14-16]. In the formulas presented below, it is assumed that to take into account the change in polarization, it is sufficient to change the sign of the parameter *s*. The possibility of formally transforming formulas for *s*-polarization into formulas for *p*-polarization was described in [4] for isotropic plates. In [16], the realization of this possibility for quasi-anisotropic media was substantiated by a substitution that preserves Maxwell's equations for a medium without external currents and charges. The wave polarization will be identified by the parameter $s = 1$ for *s*-polarization, $s = -1$ for *p*-polarization.

The wave vector component orthogonal to the plane of the plate (k_x) is calculated for a given polarization based on Maxwell's equations and the values of the components of the magnetic and dielectric permittivity matrices for each layer. The corresponding equation has two complex solutions k_x: for refracted and reflected waves [14,15]. Let us introduce for them the parameter q, the sign of the real part k_x, which determines the direction of the phase velocity of the wave in the layer with respect to its boundaries, $q = \mathrm{sign}(Re(k_x))$. One solution has a positive real part ($q = 1$), the other has a negative one ($q = -1$). In the case of isotropic media, k_x should be the same for *s*- and *p*-polarized waves. In the general case for a quasi-anisotropic medium, all 4 values k_x may differ [16]. For each layer, with the number u and a given polarization s, two complex solutions $k_{x(s,q,u)}$ are obtained, with the parameters $q = -1$ and $q = +1$. The total of such waves in a plate with the number of layers U, for a given polarization s, is equal to $2U$ (counting also the layers of the half-spaces surrounding the plate). The waves are coherent, agree with each other at the layer boundaries and, as a result of multiple reflections, come to a stationary state with amplitudes $E_{(s,q,u)}$. $E_{(s=+1,q,u)}$ is the complex amplitude of the *s*-wave electric field. $E_{(s=-1,q,u)}$ is the complex amplitude of the p-wave magnetic field. $E_{(s,q,u)}$ – the amplitude of the wave with the direction of propagation q, in the layer u "at the boundary ($-q$)". The words "at the border ($-q$)" mean the following. The layer u, in the general case, has two boundaries. One − for a smaller, the second – for a larger, x-coordinate. Let us introduce the parameter $g = -1$ for the border with a smaller x, $g = +1$ − for the border with a large x. If the wave has a direction q, then it propagates in the medium u from the boundary $g = -q$ to the boundary with $g = +q$. The complex wave amplitude at the boundary $g = -q$ matters, $E_{(s,q,u)}$. The complex wave amplitude at the boundary $g = +q$ matters $E_{(s,q,u)}\exp(\mathrm{i}k_{x(s,q,u)}qd_u)$, where d_u is the layer thickness. The exceptions to this rule are the first and last layers, which are semi-infinite spaces. The amplitude of the reflected wave and the amplitude of the transmitted wave in an ordinary layer are set at the boundaries according to the general rule, but with the following exceptions. The border $g = -1$ for the first layer is moved to infinity, as well as the border $g = +1$ for the last layer. Therefore, as an exception, we assume that the amplitude $E_{(s,q=1,u=1)}$ is set at the boundary $g = 1$ of the first layer (and not at the boundary $g = -1$, which is absent). Similarly, on the last layer U, the amplitude $E_{(s,q=-1,u=\mathrm{U})}$ is set at the boundary $g = -1$. The wave observed by the measuring device belongs to the set of waves specified by integer parameters (s,q,u). The introduction of the parameters u, s, q, g makes it possible to separate very complex variants of the conditions for considering wave processes in a given problem and is a means of overcoming the logical problems that arise.

When deriving the Fresnel-Airy formulas in [1-7,16], it is assumed that an external radiation source creates an incident plane wave of a known complex amplitude. At given values of the frequency (1.41 Hz) and, as a result of multiple reflections, a stationary state of the amplitudes of the set of waves is formed, given by the parameters (s, q, u). The generalization of the problem to the case when the source of plane waves can be located not only outside the plate, but also in one of its inner layers is described by a linear equation with the same matrix as in [16], but with a modified column of free terms. The amplitudes of the set of waves (*s, q, u*) are represented as a 2*U* vector. The law of interaction of waves is represented as a matrix (2*U* by 2*U*) The direct action of the incident wave on the system of waves (*s, q, u*) gives a column of free terms of the linear system of equations. The matrix and the column are sparse, it is convenient to generate them automatically using the usual Fresnel coefficients for layer boundaries and extremely sparse matrices [17, 18]. Calculation of the Fresnel-Airy coefficients is reduced to solving an automatically generated linear system of equations [16].

When interpreting the solution in the case of sources of plane waves inside the plate, logical subtleties appear, which can be expressed by the following casuistic question. Is the wave created by the original plane wave source associated with the set of reflected and refracted waves? The answer to this question is negative, but with a caveat. Coherent sources of plane waves create waves with wave vectors that coincide with wave vectors from the set specified by the parameters (*s, q, u*). Let there be one such source, has a unit complex amplitude and is specified by parameters (s, q_0, u_0), i.e. has polarization *s*, is in the layer u_0 at its boundary $-q_0$ and shines in the direction q0. Then the complex amplitudes of the waves (*s, q, u*) in the multilayer plate will be equal to the corresponding Fresnel-Airy coefficients, with one exception. This exception is a wave with parameters (s, q_0, u_0). The complex amplitude of the source wave should be added to the Fresnel-Airy coefficient of this wave, i.e. unit, the result is a complex wave amplitude with parameters (s, q_0, u_0). The Fresnel problem is linear, therefore, if the source wave is not unitary, then all amplitudes should be multiplied by its complex amplitude. If there are several coherent sources of plane waves, then the effects of their complex amplitudes are summed up.

2.3. Fresnel-Airy coefficient specification

The relationship between the directions of wave propagation in a layer (parameter *q*) and layer boundaries (parameter *g*) should be taken into account when identifying the Fresnel-Airy coefficient. We will assume that the wave frequency and the *y*-component of the wave vector are given. The specification of the Fresnel-Airy coefficient can be represented as a function $f_{(s,q_0,u_0,q,u)}$, where *s* is the polarization, q_0, u_0 − the external incident wave is specified: in the layer u_0, it propagates in the direction q_0 near the boundary q_0. If the amplitude of the incident wave is unit, then the value of the function determines the amplitude of the wave in the layer u, near the boundary ($-q$) propagating in the direction (q). The amplitude of the wave in the *u* layer, with the direction *q* near the boundary ($-q$), is obtained as the product of the Fresnel-Airy coefficient by the amplitude of the original wave $f_{(s,q_0,u_0,q,u)}E_{(s,q_0,u_0)}$. To take into account the sources of plane waves at different boundaries in different layers, the complex amplitudes are summed; therefore, the function $f_{(s,q_0,u_0,q,u)}$ can be interpreted as an analogue of the Green's function. The representation of the Fresnel-Airy coefficients in the form of a function $f_{(s,q_0,u_0,q,u)}$ makes it possible to write down rather conveniently solutions to problems of many coherent sources in a multilayer quasi-anisotropic plate.

2.4. Averaging the Poyting vector over the oscillation period

The amplitude of the *s*-polarized wave (s = 1) is conveniently considered the amplitude of the electric field lying in the plane of the plate. The amplitude of *p*-polarization waves (s = -1) in [1] was considered the complex amplitude of the magnetic field lying in the plane of the plate. In [16,19] and in this article, the same approach is adopted.

Let $S = s_0EH$, where S, E, H are the complex amplitudes of the Poyting vector and the orthogonal electric and magnetic fields. Here $s_0 = c/4\pi$ is the coefficient in the Gaussian system, s_0 = 1 in *SI* system [1].

APPENDIX shows that P, the value of S averaged over the oscillation period, can berepresented in terms of the complex amplitudes E and H in the form

$$P = s_0 \frac{|E|^2}{2} \mathrm{Re}\left(\frac{H}{E}\right) = s_0 \frac{|H|^2}{2} \mathrm{Re}\left(\frac{E}{H}\right). \tag{2}$$

In accordance with [14, 15], for a wave in a quasi-anisotropic medium, we obtain for the period-averaged components of the Poyting vector at the boundary (–q) of the layer u:

$$\begin{aligned}
&P_x = s_0 \frac{|E_{(s,q,u)}|^2}{2} \mathrm{Re}\left(-\frac{D_{2,s}}{D_{3,s}}\right);\\
&P_y = s_0 \frac{|E_{(s,q,u)}|^2}{2} \mathrm{Re}\left(-\frac{D_{1,s}}{D_{3,s}}\right);\\
&D_{1,s} = \mu_{s,12}k_1 + \mu_{s,22}k_2;\\
&D_{2,s} = -(\mu_{s,11}k_1 + \mu_{s,21}k_2);\\
&D_{3,s} = (\mu_{s,11}\mu_{s,22} - \mu_{s,12}\mu_{s,21})k_0;\\
&k_0 = w/c;\\
&\mu_{1,ij} = \mu_{ij};\\
&\mu_{-1,ij} = \varepsilon_{ij}.
\end{aligned} \tag{3}$$

Let us introduce the parameters h, and the coefficients that allow us to calculate the components of the Poyting vector for a given wave , averaged over the oscillation period, and its absolute value at the boundary (-q), if the wave amplitude is specified.

$$\begin{aligned}
&V_{(s,q,u,h=1)} = \mathrm{Re}\left(-\frac{D_{2,s,q,u}}{D_{3,s,q,u}}\right);\\
&V_{(s,q,u,h=2)} = \mathrm{Re}\left(-\frac{D_{1,s,q,u}}{D_{3,s,q,u}}\right);\\
&V_{(s,q,u,h=0)} = \sqrt{(V_{(s,q,u,h=1)})^2 + (V_{(s,q,u,h=2)})^2};\\
&P_{(s,q,u,h)} = s_0 V_{(s,q,u,h)} \frac{|E_{s,q,u}|^2}{2}.
\end{aligned} \tag{4}$$

For $s = +1$, algorithms are obtained that make it possible to calculate the intensities for s-polarized waves.

For $s = -1$, algorithms for calculating the intensities for p-polarized waves are obtained.

The parameter $h = 1$ allows one to obtain the period-averaged component of the Poynting vector directed orthogonal to the plate surface for a wave in the u layer propagating in the q direction.

The parameter $h = 2$ corresponds to the period-averaged component of the Poynting vector directed tangentially to the plate surface.

The parameter $h = 0$ − corresponds to the period-averaged absolute value of the Poynting vector.

2.5. POYNTING VECTOR OF WAVES AT THE INTERFACES OF LAYERS

In the Fresnel problem, the incident, reflected and refracted waves are coherent. In superposition, their complex amplitudes are added, but the intensities (energy fluxes) must be calculated in a more complex way.

The energy flux (i.e., the Pointing vector) near the boundary q of the u layer is determined by the vector product of the total electric field and the total magnetic field. At the interfaces between the layers, the tangential components of the magnetic and electric fields are preserved. Therefore, at the interfaces between layers must be constant S_x, S_y, in general, it is not preserved at the interface between layers. Nevertheless, to check the calculations, it would be desirable to have values similar to the energies that should be stored at the boundaries in the directions tangential to the interface. Such quantities can be constructed, since the field inductions, orthogonal to the surface, must be constant at the interfaces. For s-polarization, after averaging over the oscillation period, the vector products of the electric field and the x-component of the magnetic induction and the vector product of the x-component of the magnetic induction and the y-component of the magnetic field must be preserved. For p-polarization, respectively, is the vector product of the magnetic field and the x-component of the electric induction and the vector product of the x-component of the electric induction and the *y*-component of the electric field.

Let's note an interesting detail. For non-magnetic media, the induction of the magnetic field in the *x*-direction coincides with the magnetic field; therefore, for the *s*-polarization, not only the *x*-component, but also the *y*-component should be preserved, i.e. the entire Pointing vector. *P*-polarized waves do not possess this property.

For the problems of estimating energy fluxes, the $V_{(s,q,u,h)}$ coefficients are convenient, but their application is limited, since the intensities of coherent waves do not add up, but their amplitudes add up. The amplitude of the total electric field of an *s*-polarized wave near the boundary (*q*) in the *u* layer is not the amplitude of only one wave. To the complex amplitude of a wave propagating in the (-*q*) direction, add the complex amplitude of a wave traveling in the (+*q*) direction, taking into account its phase shift. If the wave source is in the *u* layer and radiates in the *q* direction, then the source wave amplitude must also be added to the sum (but without the phase shift). The total complementary field (magnetic field in the case of *s*-polarization) is also obtained as the sum of the complementary fields of these three waves. The vector product of the corresponding fields, averaged over the oscillation period, should be calculated by formula (2). It is proportional to half the square of the modulus of the incident wave amplitude. The aspect ratio is calculated using the Fresnel-Airy coefficients. Let us write it as a function with the following specification for waves generated by an incident wave of unit intensity from a source of a given polarization located in a layer u_0 that shines in the direction q_0 and gives a $F_{(s,q_0,u_0,g,u,h)}$ function in the u layer at the *g* boundary.

For $s = 1$

For $h = 1$, the function F is determined by the vector product E_3 by H_2 and gives the coefficient for calculating the *x*-component of the Poyting vector averaged over the oscillation period.

For $h = 2$, the function F is determined by the vector product E_3 by H_1 and gives the coefficient for calculating the *y*-component of the Poyting vector averaged over the oscillation period.

For $h = 0$, the function F gives the absolute value of the Poyting vector averaged over the oscillation period

For $h = -2$, the function F is determined by the vector product E_3 by B_1 and gives the value that must be preserved at the interface between the media.

For $h = -3$, the function F is determined by the vector product H_2 by B_1 and gives the value that must be preserved at the interface between the media.

When $s = -1$, values for the *p*-polarization are obtained, the fields E and H and their inductions change their roles.

If the amplitude of the radiation source were known, then the product $s_0 F_{(s,q_0,u_0,q,u,h)} \frac{|E_{(s,q_0,u_0)}|^2}{2}$ would be an energy flux in the *h* direction in the *u* layer at the *g* boundary. For applications in remote sensing, a slightly different interpretation of the observed radiation intensity seems promising. In the region of radio waves and in the microwave region of the spectrum, the radiation intensity can be interpreted as the brightness temperature and related to the effective temperature of the observed object [10].

2.6. Summation of the energy of incoherent waves. Brightness temperature

In [20], based on the Kirchhoff law of radiation, a formula was obtained for the intensity I of radio waves emitted by a uniform layer $I_v = B(v,T)(1 - \exp(-\tau))$. A similar formula was used to estimate the brightness temperature of a cloud formation in the atmosphere in the isothermal approximation in the spectral region where scattering can be neglected [12]. $B(v,T)$ − Planck function, depending on the radiation frequency and layer temperature, τ − optical layer thickness.

For the radio and microwave ranges, the Planck formula degenerates into the Rayleigh-Jeans formula, in which the radiation intensity is proportional to the temperature. In our case, Im $(k_y) = 0$, $k_z = 0$, therefore, absorption and emission are associated only with k_x. Therefore

$$I_{(s,q=g,u)} = A_{(s,q=g,u)} B(\nu, T_u),$$
$$A_{(s,q=g,u)} = 1 - \exp(2\,\mathrm{Im}(k_{x(s,q,u)} q d_u)). \tag{5}$$

$B(\nu,T) = \dfrac{2k_B\nu^2}{c^2}T$, c - peed of light, k_B is the Boltzmann constant, ν is the frequency, T is the temperature in degrees Kelvin.

For an infinitely thick homogeneous layer, the integrating coefficient A degenerates into unity. The intensity at the layer boundary becomes proportional to the layer temperature. The coefficient $\dfrac{2k_B\nu^2}{c^2}$ allows you to convert the observed intensity into the brightness temperature. This coefficient is reduced in the ratio (1). Relation (1) is considered in [10] as a theoretical basis for many methods of remote sensing of the earth's surface using radiometers.

In the case of multilayer media, the observed brightness temperature should be the sum of the effect of the intrinsic thermal radiation of all layers. $T_{ya} = \sum_{u_0} \kappa_{u_0} T_{u_0}$. (In all formulas in the article - the number of the layer where the radiation source is located). The κ_{u_0} coefficients are not independent. Changing, for example, the thickness of one layer affects the coefficients of all layers. All κ_{u_0} coefficients depend on the polarization s. The problem of calculating these coefficients is not easy, but in a number of isotropic cases its solutions are known from the literature [10]. The algorithms developed in this work make it possible for quasi-anisotropic media to take into account the anisotropy of the media in calculations, to evaluate the effect of each layer, and to ensure the comparability of the results in the formation of complex dependences on the observation angle at different polarizations.

If measured with a radiometer in the first layer, then

$$T_{ya,s,h} = \sum_{u_0} \kappa_{s,u_0,h} T_{u_0},$$
$$\kappa_{s,u_0,h} = \sum_{q_0=\{-1,1\}} V_{(s,q=-1,u=1,h)} \,|\, f_{(s,q_0,u_0,q=-1,u=1)} \,|^2 \, \frac{A_{(s,q_0,u_0)}}{V_{(s,q_0,u_0,h=0)}}. \tag{6}$$

In this formula, the coefficient A provides the summation of the intensities of heat sources over the thickness of the layer with the number u_0. Dividing by a factor $s_0 V_{(s,q_0,u_0,h=0)}$ converts the radiation intensity into the equivalent amplitude at the layer boundary. Multiplication by $s_0 V_{(s,-1,1,h=0)}$ performs the inverse transformation in the radiometer layer. The coefficients $\dfrac{2k_B\nu^2}{c^2}$ associated with the units of measure of intensity are abbreviated and are not present in the formula for the observed brightness temperature. For s = 1, formulas for the s-polarization are obtained. When s = -1 - for p-polarization.

General formula for calculating energy fluxes averaged over the period taken at the boundaries of the layers:

$$T_{(s,q,u,h)} = \sum_{u_0} \kappa_{(s,u_0,g,u,h)} T_{u_0},$$
$$\kappa_{(s,u_0,g,u,h)} = \sum_{q_0=\{-1,1\}} F_{(s,q_0,u_0,g,u,h)} \frac{A_{(s,q_0,u_0)}}{V_{(s,q_0,u_0,h=0)}}. \tag{7}$$

For h = 1, the y-period-averaged energy flux is obtained orthogonal to the plate plane, for h = 2 - parallel to the plate plane, for h = 0 - the absolute value of the Poiting vector averaged over the oscillation period.

The introduction of parameters s, q_0, u_0, q, u, g made it possible to reveal and express logical subtleties in the description of waves in multilayer media. There is a certain subtlety in the fact that the parameters of the quantities V and f include the direction of propagation of the wave q, while the parameters of the quantities F include the boundary number g.

The question of the legitimacy of introducing the concept of equivalent amplitude at the interface between media instead of the sum of the intensities of many incoherent waves is considered in more detail in Appendix 1.2.7.

2.7. Electrical properties of ice and water

In the calculations presented in the next section, as in [19], the value of the relative complex dielectric constant of ice near 0°C at a frequency of 1.41 GHz is used, which is equal to $\varepsilon = 3.18 + 0.0007i$ [12,27–29]. The relative error in measuring the imaginary part is usually large and amounts to tens of percent. In [30,31], ice models based on extrapolation and averaging of various literature data are considered. The structure of water and models for calculating its complex dielectric constant were considered in [27-29]. We use the value of the relative complex dielectric constant of water near 0°C at a frequency of 1.41 GHz equal $\varepsilon = 85.79 + 12.72i$.

3. CALCULATION RESULTS

As the first application of the algorithms developed above, polarization-angular diagrams of radiation of a free ice plate 50 cm thick were obtained under illumination by radiation with a brightness temperature with a frequency of 1.41 GHz *s*- and *p*-polarization taking into account the intrinsic thermal radiation of the ice plate at this temperature (**Fig. 1**). The solid line is the curve of the absolute value of the Poyting vector (intensity) averaged over the oscillation period for *s*-polarization. The dotted line corresponds to *p*-polarization. The dashed lines show the results of calculations when the intrinsic thermal radiation of the ice plate is not taken into account. This version reproduces the results of [19] obtained by another method: the reflection and transmission coefficients of free ice plates were considered under the given conditions. In the region of observation angles of 30-40 degrees, there is a transparency region, both for *s*-polarization waves and for *p*-polarization waves (the total maximum of all curves). Around 60 degrees, the second maximum of the *p*-wave curve corresponds to the Brewster angle for ice under the conditions under consideration. The dotted curves were obtained without taking into account the intrinsic radiation of ice and correspond to the results of [19].

At viewing angles of 0-70 degrees, the contribution of the intrinsic thermal radiation is no more than 2-5 Colvin degrees. As the observation angles approach 90 degrees (more than 70 degrees), the behavior of the curves is significantly different and requires additional study. It is necessary to take into account the bandwidth of the radiometer, the roughness of the surface, separate observation and the calculation of the Poynting vector components.

How will the polarization-angle diagrams change if the same plate floats in water and is illuminated not by external radiation, but by its own radiation. **Fig. 2** shows the case of a 50 cm thick plate floating in water. Solid curve - *s*-polarization, dots represent the curve for *p*-polarization. Changes in the conditions of reflection and transmission at

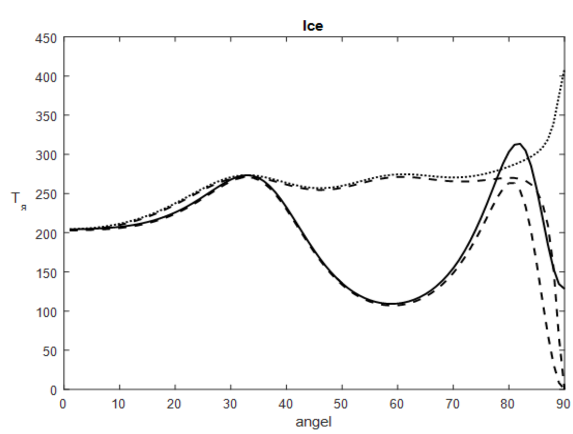

Fig. 1. *Polarization-angular diagrams of the brightness temperature for a free plate of ice 50 cm thick, under illumination with radiation with a brightness temperature 273.15 K.*

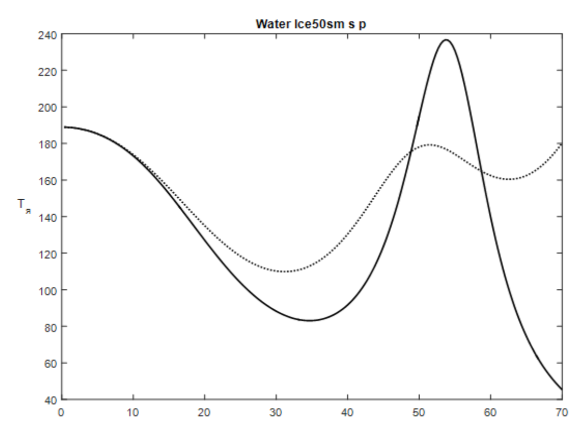

Fig. 2. *Polarization-angle diagrams of the brightness temperature in the direction orthogonal to the surface of the ice plate floating in water at. Plate thickness 50 cm. Solid curve - s-polarization. The dotted curve is p-polarization.*

the ice-substrate interface when replacing air with ice qualitatively changes the course of the curves in Figs. 1 and 2.

The wavelength for isotropic ice at the considered frequency is about 12 cm. **Fig. 3**, as well as in Fig. 2, shows the case of a plate floating in water at. Plate thickness 47 cm, i.e. the thickness of the plate in Fig. 3 differs by a quarter wave from Fig. 2. Comparing Fig. 1, Fig. 2, and Fig. 3, it can be concluded that replacing the substrate layer from air to water with a simultaneous change in the wafer thickness by a quarter of the wavelength leads to the restoration of the bleaching region in the region of 30 degrees for both polarizations. According to the *p*-polarization, one can note the restoration of the bleaching conditions in the region of 60 degrees (Brewster's angle for ice). Such transformations of the curves can be associated with the mutual substitution of the conditions of anti-reflection and anti-reflection of plane-parallel plates, which is known in the theory of optics of multilayer coatings [5].

In [19], the influence of ice anisotropy on polarization-angle diagrams for the selected frequency 1.41 GHz and thickness 50 cm was noted. **Fig. 4** shows the polarization-angle diagrams of *p*-polarization waves for the case when the anisotropy axis is directed orthogonally to the plate surface (solid curve). Anisotropy coefficient - 15%. The dotted line represents isotropic ice.

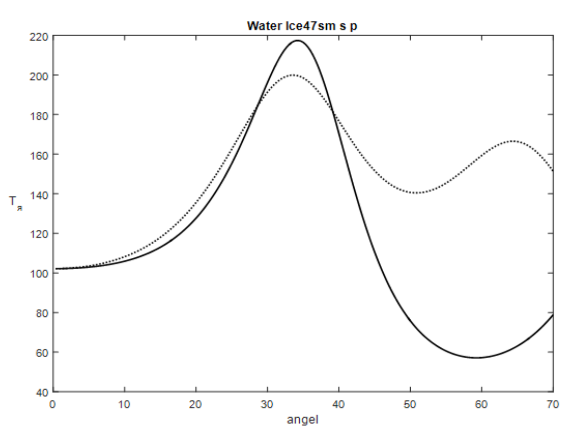

Fig. 3. *Curves for the case as in Fig. 2, but the plate thickness is reduced by a quarter of the wavelength and became 47 cm. The conditions of the plate antireflection were restored*

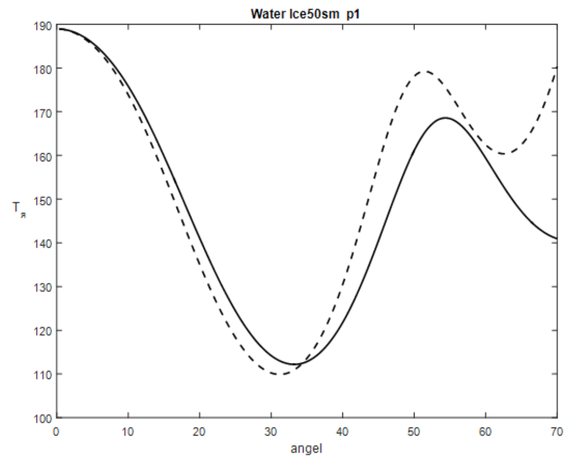

Fig. 4. *Polarization-angle diagram of the brightness temperature of an ice plate floating in water for p-polarization waves. The dotted curve is an isotropic plate. The solid curve is anisotropic ice with a coefficient of 15% in the direction orthogonal to the plane of the plate.*

If the anisotropy axis is parallel to the plane of the plate, then the effect of anisotropy on the polarization-angle diagrams increases significantly. **Fig. 5** shows the curves for the case of an anisotropic surface, 50 cm thick, floating in water.

Controlling the values stored at interfaces (see Section 2.5) gives relative error 10^{-13} in double precision calculations. There is practically no accumulation of errors in calculations.

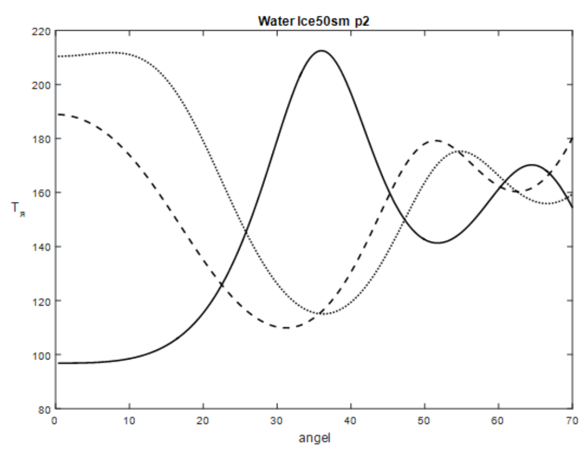

Fig. 5. *Angular diagram of the brightness temperature for p-polarization waves of anisotropic ice floating in water. The point curve is an isotropic plate. The dashed curve corresponds to the anisotropy coefficient of 3% in the direction ε_{22}. The solid curve corresponds to an anisotropy coefficient of 15% in direction ε_{22}.*

4. DISCUSSION OF THE RESULTS

The use of the Fresnel and Fresnel-Airy coefficients is an effective approach to solving many physical problems [1-7], but an adequate application of the Fresnel approach is associated with a number of limitations mentioned in the introduction. Generalization of the Fresnel approach to media with absorption [1,5-7] had an important methodological and practical importance at the beginning of the 20th century, but it is not universal, for example, it is not applicable to media with amplification [11]. Attempts to go beyond the Fresnel approach are being made in our time and remain relevant. For turbid media and media with high internal scattering (for example, for snow), methods based on the theory of radiation transfer [30–32] are used; the methods described here are inapplicable to such media. In this work, only linear waves (waves of small amplitude) are considered, described by local Maxwell equations for homogeneous media with smooth boundaries without external currents and charges. Quantum effects are not considered. The ability to consider *s*- and *p*-polarization waves propagating independently in homogeneous isotropic media simplifies the problem. If a number of constraints are imposed on tensors in Maxwell's equations, then independent propagation of *s*- and *p*-polarization waves in some anisotropic media is obtained [14,15]. In [16], a definition of quasi-anisotropic media is given and algorithms generalizing Fresnel's formulas are discussed.

In this article, it is noted that the concept of an ideal radiometer makes it possible to split the set of plane electromagnetic waves in a quasi-anisotropic multilayer plane-parallel plate into non-intersecting coupled subsets. These subsets are identified by frequency, polarization, projection of the wave vector onto the surface of the plate. Waves from different subsets are (completely) incoherent, i.e. their intensities add up. This circumstance makes it possible to take into account the presence of a finite bandwidth in real radiometers in the calculation of intensities by means of integration [16,19,21]. In this paper, we discuss algorithms for an ideal radiometer, assuming further generalization to radiometers with complex bandwidth. In [19], the effects of smoothing of inhomogeneities in the ballization-angle diagrams, related to the bandwidth of a real radiometer, were discussed. For ice thicknesses of the order of 50 cm and viewing angles less than 70 degrees discussed in this article, the *L*-band radiometer can be considered close to ideal. In [23-25] the parameters of *L*-range radiometers are given, which record separately the intensities of orthogonal polarizations of radiation. When studying thicker ice sheets and considering viewing angles greater than 70 degrees, it is necessary to take into account the bandwidth of the radiometer.

In this article, the term "homogeneous" is applied to different objects and its meaning is somewhat different. The layers of the plates are “homogeneous”; do not have any defects and special parameters depending on coordinates. An ideal radiometer should be located in an isotropic homogeneous medium without absorption and register only homogeneous waves, while it (ideally) should not register inhomogeneous ones. It is not entirely obvious fact that inhomogeneous plane waves can exist in an isotropic homogeneous medium without absorption. Inhomogeneous waves exist in such media, for example, spherical waves. The decomposition of a spherical wave into plane waves includes inhomogeneous plane waves [34]. The requirement that an ideal radiometer fixes only homogeneous waves is essential; it is determined by the radiometer's antenna. Moreover, this radiometer must be located outside the multilayer plate in a homogeneous transparent medium. Homogeneous waves recorded by such a radiometer will have real components of wave vectors tangent to the surface of the plate. All waves included in the mathematical model will have the same property, i.e. the amplitude of such waves will remain constant along the surface of the plate. Such waves are necessary $\mathrm{Im}(k_y) = 0$. In the layers of the plate in a direction orthogonal to its surface, the waves turn out to be inhomogeneous in layers with damping.

When reproducing the results for free plates, an external source of electromagnetic waves is considered at the effective temperature . It is assumed that this is a large black screen, at a sufficiently large distance from the ice plate. Heat sources inside the plates are assumed to be completely incoherent, and their effect on the intensity of the observed radiation can be taken into account by summation [12]. According to the Rayleigh-Jeans law, the intensity of thermal radiation in the microwave range is proportional to the Kelvin temperature; therefore, it is convenient to use the Kelvin temperature as a measure of radiation intensity [10]. The temperature dependences of the complex permittivities of water and ice were considered in [27–31]. It is assumed that further research will consider multilayer systems at different temperatures of the layers. The Fresnel-Airy coefficients are the proportionality coefficients between the amplitudes of the linear waves. The intensity of external radiation can be measured in brightness temperature. The intensity of the thermal radiation of a layer is determined by its effective (energy) temperature. Therefore, it is possible to estimate the contribution of the intrinsic thermal radiation of the plate layer to the observed brightness temperature as corrections to the results obtained in [19]. Fig. 1 shows the polarization-angular dependences of the brightness temperature for a free plate of ice, which is illuminated by an external radiation source with a brightness temperature of 273.15 K (°*C*). The solid curve corresponds to the *s*-polarization. The dotted curve corresponds to *p*-polarization. Dotted curves - dependences calculated without taking into account the contribution of thermal sources of ice. For the observation angles range of 0-70 degrees, the contribution to the brightness temperature of the intrinsic radiation of the considered ice plate is insignificant: 2-5 degrees. The dotted lines represent the transmittances of a 50 cm thick ice sheet multiplied by 273.15 K (°*C*). These curves correspond exactly to the results of [19]. Inherent thermal electromagnetic radiation exists in all frequency ranges. For a black body, the intensity must obey the Planck formula, in the microwave region, the Rayleigh-Jeans law [12]. The intensity of real bodies is always less than the intensity of an absolutely black body. If we interpret the effective temperature in formulas (1,6-7) as thermodynamic temperature, then the calculations will give slightly overestimated values. "The radio-thermal radiation of homogeneous solids and liquids in most cases does not have intense selective components and the brightness of the emitters varies little over the spectrum" [12]. This means that a spectral emissivity correction can be entered for water and ice. For a vacuum, such a correction factor is 0, for an absolutely black body it is equal to 1. It would be very interesting to obtain corrections for "blackness" for water and ice and their dependences on ice anisotropy. According to data from [33], for water and ice, this coefficient is close to 0.9 and depends on the type of ice (annual, long-term). Data on the anisotropy of the permittivity for various types of Arctic ice are discussed in [34].

In the optics of thin-layer coatings [5], there is a theorem that the conditions for anti-bleaching bleaching are reversed if the monotonic change in the refractive indices in the substrate, film, and environment changes to a non-monotonic change. Strictly speaking, the theorem is formulated for real refractive indices and for small angles of incidence, but it can serve as an explanation for the transition from Fig. 1 to Fig. 2. For a free plate (Fig. 1), the refractive indices change non-monotonically, and for a plate floating in water (Fig. 2), these changes can be considered monotonic. The reason for the rearrangement of the polarization-angle diagrams is the change in the phase of the reflected wave by a value close to. Changing the thickness of the plate changes the phase of the reflected wave, therefore, changing the thickness of the plate floating in water by ¼ of the wavelength restores the conditions of bleaching in the region of 30 degrees for both polarizations (Fig. 3).

For quasi-anisotropic media, the amplitudes of the *s*-polarized waves depend only on ε_{33}. The amplitudes of *p*-polarized waves depend on ε_{11}, ε_{22} (and also on ε_{12},ε_{21}) [16]. The curves for *p*-polarization depend on different fusion of the anisotropy axis and are shown in Figs. 4, 5.

With the help of the proposed mathematical apparatus, it is possible to calculate the energy fluxes of monochromatic waves in any layer of a multilayer plate. In this work, it is assumed that thermal sources of microwave radiation are incoherent. The difficulty in solving the problem of the intrinsic radiation of the layers lies in the need to separate and adequately take into account coherent and incoherent effects. Conventionally speaking, a "wave from one heat source" generates many reflected and transmitted waves in multilayer plates, they are coherent and obey the Airy equations based on the Fresnel coefficients. Their amplitudes can be added, and the energy is calculated in a more complex way, through the Poiting vectors. At the same time, the thermal sources of electromagnetic waves themselves are incoherent and their energies can be added. Based on the Fresnel-Airy coefficients (functions *f*), functions *F* are constructed to calculate the Poyting vectors averaged over the oscillation period of coherent waves at a given boundary of a multilayer plate. Appendix substantiates the possibility of using the concept of the effective amplitude of radiation sources at the layer boundary. At the function specification level, there is a subtle but significant difference between *f* and *F*. Among the parameters of the function *f* there is a parameter (*q*) that fixes the direction of propagation of the calculated wave. Instead of this parameter, the function *F* contains a parameter (*g*) that fixes the number of the boundary around which the energy flux is calculated. This subtlety is taken into account in formulas (6,7).

The developed mathematical apparatus is rather complicated, therefore, for the automated control of calculations, it is proposed to use a number of quantities that must be stored at the interfaces between layers. It is checked for a 20-layer medium of ice, air and water layers, that there is no accumulation of errors in calculations, Poynting vectors and other auxiliary values are stored at the boundaries of media sections with a relative error of no more than 10^{-13}, when calculating with double precision.

It is convenient to write solutions of linear inhomogeneous non-degenerate systems of equations using the inverse operator. The kernel of such an integral operator in physics is called the Green's function. The use of Green's functions turns out to be an effective and productive mathematical apparatus, for example, in the equations of mathematical physics [35]. Is it possible to construct an analogue of the Green's function to solve the Fresnel problems in multilayer quasi-anisotropic plates? The set of Fresnel-Airy coefficients, in essence, are such a discrete analogue of the Green's function; it remains only to write it in the appropriate form, this is the function *f* (see Section 2.3). In the event that the radiation sources are completely incoherent, with the superposition of waves, their intensities can be added. The problem of calculating the intensities at the boundaries of the layers at given intensities of incoherent sources turns out to be linear. To solve it, there must also be a discrete analogue of the Green's function. These are the coefficients κ in formula (1) and in its generalization (7) (see Section 2.6). The derivation of these coefficients is complicated by the need to take into account coherent and incoherent effects in a multilayer plate, as well as by the need to construct an adequate method for identifying the waves and energy fluxes under consideration. The introduction of the parameters (*s, q, u, g*) turned out to be sufficient for adequate identification of waves and energy fluxes in a multilayer plate. (section 2.2). The *V* coefficients relate the amplitudes and intensities of the waves (Section 2.4). The coefficients *F* have the structure of Green's functions and allow calculating the components of the Poynting vectors averaged over the oscillation period taken at the boundaries of the layers (Section 2.5). The products of the functions $V_{(s,q,u,h)}\,|\,f_{(s,q_0,u_0,q,u)}\,|^2$, which give the intensities of the observed waves, have the same property. This mathematical apparatus allows you to calculate the coefficients κ, change the context of problems using the parameters *s* and *h*.

This article introduced the specifications for the function *f*. The functions *F* have been extended over the functions *f* to calculate the radiation intensities. They were implemented solely for research purposes: to reveal the physical essence and the possibilities of numerical solution of the problem in the most general possible form. For narrower and more utilitarian purposes, more efficient implementations are possible and necessary. The transition to more developed methods of using computing resources, described, for example, in the textbook [36].

5. CONCLUSION

Using the algorithms developed in the article, the polarization-angular dependences of the brightness temperatures of isotropic and anisotropic ice sheets 50 cm thick, at 0°C, for a microwave radiation frequency of 1.41 GHz, were theoretically obtained. In the case of free ice sheets, they are illuminated by radiation with a brightness temperature of 273.15 K. The curves have exactly the same characteristic features as the transmission coefficients obtained earlier in [19]. In addition to the results of [19], the influence of the self-radiation of the ice plate on the intensity recorded by the radiometer was estimated. For angles 0-70 degrees, it does not exceed 2%. For large angles, the influence of intrinsic radiation increases and requires more detailed study.

Additionally, the case is considered when there is no illumination, but the plate floats in water. In this case, the sign of the amplitude Fresnel reflection coefficient between the ice-substrate layers changes. Therefore, on the curves, the highs and lows are reversed. Changing the plate thickness can also change the sign of the Fresnel coefficient. With a plate thickness of 47 cm, the curves for a plate floating in water are similar to those for free plates of 50 cm. The obtained calculations of the polarization-angular dependences of the brightness temperature correspond to the estimates and concepts developed in [16,19] for the reflection and transmission coefficients of free ice plates. For thicker ice plates, it is necessary to take into account the bandwidth of the radiometer [19,21].

The generalized Fresnel-Airy coefficients obtained in [16] are generalized below. They are reduced to a form $f_{(s,q_0,u_0,q,u)}$ that can be considered as a method for constructing a solution to the linear problem of interference of coherent plane waves in multilayer plates with quasi-anisotropic layers. It is assumed that the frequency and projection of the wave vector of waves on the surface of the plate are given. The parameter sign (s) identifies the s- or p-polarization of the wave. The parameters set the location of the plane wave source, which can be located not only outside the plate, but also inside one of the layers.

Using functions $f_{(s,q_0,u_0,q,u)}$, functions $F_{(s,q_0,u_0,q,u,h)}$ are constructed which can also be considered as a way to construct a solution to the linear problem of interference of completely incoherent plane waves in multilayer plates with quasi-anisotropic layers. The coefficients $F_{(s,q_0,u_0,q,u,h)}$ relate the intensity of the plane wave source to the intensity of reflected and refracted waves in all layers of the plate. With the help of the parameter h, it is possible to obtain the components of the Poynting vector and quantities similar to the energy, which are constants at the interfaces of the quasi-anisotropic layers.

It has been verified that in the case of 20 layers of ice, water and air, the control values at the boundaries of the layers are preserved with an accuracy 10^{-13}, i.e. there is practically no accumulation of computational errors.

The work was carried out at the expense of budget funding within the framework of a state assignment.

APPENDIX. Explanation of the effective wave concept use.

The following theorems on averaging over the oscillation period are useful.

Let $X(t) = |X|\exp(iwt + \varphi_X)$, $Y(\mathrm{t}) = |Y|\exp(iwt + \varphi_Y)$ – complex oscillating functions with frequency w and phase shifts.

$M_t(X(t),Y(\mathrm{t}))$ – the average over the period from the product of these functions.

Then $M_t(X(t),Y(\mathrm{t})) = (|\mathrm{X}||\mathrm{Y}|/2)\cos(\varphi_X - \varphi_Y)$.

This theorem is well known in electrical engineering, for example [37]. Corollary of this theorem:

$$\mathrm{Re}\left(\frac{X}{Y}\right) = \frac{|X|}{|Y|}\cos(\varphi_X - \varphi_Y) \Rightarrow M_t(X(t), Y(t)) = \\ = \mathrm{Re}\left(\frac{Y}{X}\right)\frac{|X|^2}{2} \equiv \mathrm{Re}\left(\frac{X}{Y}\right)\frac{|Y|^2}{2}. \tag{A1}$$

The following statements are also valid: if (*a*) and (*b*) are complex numbers, (*a**) and (*b**) are their complex conjugates, then

$$M_t(aX(t), bX(t)) = \frac{|X|^2}{2}\mathrm{Re}(ab^*) \equiv \frac{|X|^2}{2}\mathrm{Re}(a^*b),$$

$$M_t\left(\sum_i X_i(t), \sum_j Y_j(t)\right) = \sum_i \sum_j M_t(X_i(t), Y_j(t)).$$

There are coherent electromagnetic waves, partially coherent and incoherent waves (completely incoherent waves) [4]. When considering a superposition of linear coherent waves, the amplitudes of the waves must add up. When considering the superposition of completely incoherent waves, their intensities must add up. The intermediate case of the superposition of partially coherent waves is not considered in the article.

Heat sources of waves in the microwave range are completely incoherent with each other [12]. The ability to restrict consideration of completely incoherent and coherent waves greatly simplifies the algorithms for calculating the intensities. I would like to calculate the contribution to the radiation intensity of thermal sources of a given layer as follows, implicitly used in formula (1).

1. Calculate the intensity of waves from heat sources of a given layer with a given wave vector and polarization at the boundaries of the layer.
2. To convert the intensity into the amplitude of the corresponding effective wave.
3. The effective wave is refracted and reflected in the multilayer plate and reaches a given place in the form of a superposition of coherent waves. Convert this superposition of waves to intensity and get the desired contribution to the calculated intensity.

The superposition of waves: waves of radiation sources, waves reflected from the boundaries and waves going to the opposite boundaries of this layer – transfers energy, but its calculation is not trivial. Waves received from one source are coherent, so their intensities cannot be added. However, you cannot add their amplitudes either, since they have different wave vectors. In order to correctly obtain the energy flow, it is necessary to add the electric fields of these waves, the magnetic fields of these waves, obtain the Poynting vector from these sums and average over the oscillation period. It turns out that all these non-obvious operations can be folded into F coefficients (Section 2.5) if the radiation sources are statistically independent. Further, we use the coefficients F and V in a simplified form; in the text of the article, the arguments of these functions are discussed in more detail.

We consider only monochromatic waves with a given frequency and wave vector k_y. Let's choose layer (*m*) in a multilayer plate, some point in it and calculate the intensity in it. We will consider the contribution to this intensity of the sources in the layer with the number *n* (then going to sum up the contributions of the layers). Let us narrow down the task even more. Let us choose one boundary (*g*) of the (*n*) layer and consider the waves of intrinsic thermal radiation going towards this boundary. The patterns established for such a contribution to intensity can be carried over to their sum. The source waves in layer (*n*) are incoherent, but have the same wave vector. Let them be numbered (*i*). The amplitude of each wave will be multiplied by a complex coefficient when it reaches the border *g* and gives L_iE_i. Further, $E(t) = F_E(m,n,h)L_iE_i$, $H(t) = F_H(m,n,h)L_iE_i$ the products will give the contribution of the source (i) to factors for the component (h) of the energy variable P(m, h) at the selected place of the layer (m).

The intensity of each source, according to Planck's formula, is related to a certain small volume of the medium (v). With a decrease in this volume, the number of sources increases, their intensities proportionally decrease. Random variables can be considered independent (uncorrelated) therefore in the limit (v → 0):

$$P(m,h) = \sum_{i,j} M_t(F_E(m,n,h)L_iE_i, F_H(m,n,h)L_jE_j) \rightarrow$$
$$\rightarrow \sum_i M_t(F_E(m,n,h)L_iE_i, F_H(m,n,h)L_iE_i). \qquad \text{(A2)}$$

The wave intensity in layer n with amplitude E is given by the product $V(n)\frac{|E|^2}{2}$.

$I(n,g)$ – radiation intensity of sources in the layer (n) at the boundary (g).

Then

$$P(m,h) = \mathrm{Re}(F_E(m,n,h)(F_D(m,n,h))^*)\frac{\sum_i (|L_i||E_i|)^2}{2} =$$
$$= \mathrm{Re}(F_E(m,n,h)(F_D(m,n,h))^*)\frac{\sum_i (|L_i||E_i|)^2 V(n)}{2V(n)} =$$
$$= \mathrm{Re}(F_E(m,n,h)(F_D(m,n,h))^*)\frac{I(n,g)}{V(n)} = F(m,n,h)\frac{E_{ef}^2}{2}.$$

These equalities make it possible to quite reasonably introduce the definitions: $I(n,g)$ - the intensity of sources at the boundary (g) of the layer (n); E_{ef} - the amplitude of the effective wave and the function $F(m,n,h)$.

$$E_{ef} = \sqrt{\frac{2I(n,g)}{V(n)}}; \quad I(n,g) = \sum_i \frac{1}{2}(|L_i||E_i|)^2 V(n);$$
$$F(m,n,h) = \mathrm{Re}(F_E(m,n,h)(F_D(m,n,h))^*).$$

Note that the concept of an effective wave is correct only under a number of restrictions. First, heat sources are considered completely incoherent (formula (A2)). Second, the behavior of the waves is described quite well by the complex wave vector, i.e. in the layer environment, internal reflections can be neglected. Third, the quasi-anisotropic medium of the layer is transparent or a medium with absorption. Media with amplification and internal reflections are not considered.

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