DOI: 10.17725/rensit.2019.11.337

Source detection in dispersive dnvironment

Venedikt M. Kuz'kin
Wave Research Center, A.M. Prokhorov General Physics Institute of RAS, http://www.gpi.ru/
Moscow 119991, Russian Federation
E-mail: kumiov@yandex.ru
Sergey A. Pereselkov, Sergey A. Tkachenko
Voronezh State University, http://www.vsu.ru/
Voronezh 394006, RussianFederation
E-mail: pereselkov@yandex.ru; sega-tk@mail.ru
Yuri V. Matvienko
Institute of Marine Technology Problems of Far Eastern Department of RAS, http://www.imtp.febras.ru/
Vladivostok 690091, Russian Federation
E-mail: ymat@marine.febras.ru

Received 10.10.2019, peer reviewed 01.11.2019, accepted 07.11.2019

Abstract. **The theory and implementation of a new information technology for passive source detection in a dispersive environment is presented. The method is based on the processing of the interference pattern formed by broadband noise source. The estimation of noise immunity of interferometric processing is received. The results of numerical analysis of noise source detection using the Neumann-Pearson criterion in oceanic waveguides are considered.**

Keywords: **dispersion, interferometry, hologram, noise source, Neumann-Pearson test, detection characteristics**

UDC 004.052.34

Acknowledgments: This work was financially supported by grants from the Russian Foundation for Basic Research (projects No. 19-08-00941, No. 19-29-06075) and the Presidium of the Russian Academy of Sciences No. 5 "Photonic Technologies in Sensing of Inhomogeneous Media and Bioobjects".

For citation: Venedikt M. Kuz'kin, Sergey A. Pereselkov, Yuri V. Matvienko, Sergey A. Tkachenko. Source detection in dispersive environment. *RENSIT*, 2019, 11(3):337-344; DOI: 10.17725/rensit.2019.11.337.

Contents

1. INTRODUCTION

In dispersing media, when multi-wave (multipath) propagation takes place, a mechanism of self-organization of the interference pattern (interferogram) of a moving (or stationary) broadband noise source is possible. The interference pattern formats the stable system of

localized bands in the frequency-time variables [1]. The slope of the bands is determined by the radial velocity of the source (the projection of the velocity towards the receiver). The frequency dispersion can be due to both the physical properties of the environment and the presence of the boundaries of the environment in which the waves propagate (waveguide propagation).

The information technology for signal processing of the source noise field (interferometric processing) is proposed by based on properties of interference pattern in waveguide. The offered information technology implements coherent accumulation of spectral intensity along localized bands by using 2D Fourier transform of interferogram. At the output of the 2D Fourier transformation of interferogram, the spectral density of the signal is localized in a narrow band in the form of focal spots caused by interference of modes (normal waves) of different numbers. In contrast of noise, the accumulation of interference along the interferogram bands is incoherent. The proposed interferometric signal processing makes it possible to solve the problem of localization of low-noise sources with high noise immunity: detection, direction, determination of radial velocity, distance and resolution [2-6].

The 2*D* Fourier transform of an interferogram is called a hologram, since it records and reconstructs the wave field of the source. Image reconstruction is achieved by filtering the 2*D* spectral density of the source on the hologram and application of the 2*D* Fourier inverse transform [2-6]. This clearing of the source signal from noise does not require knowledge of the nature of the signal, noise and transfer function.

The application of the proposed signal processing does not depend on the nature of the dispersion. At the same time, the quantitative parameters of the dispersion determine the effectiveness of its use. The proposed signal processing allows a new understanding of the statistical problem of passive detection of a noise signal at presence of noise background. This paper presents a solution of this problem based on the Neumann-Pearson criterion [7]. Its solution focuses on an advanced example of sound propagation in an oceanic waveguide (the case of waveguide dispersion) by using a single receiver.

2. NOISE IMMUNITY OF INTERFEROMETRIC SIGNAL PROCESSING

The efficiency of interferometric processing is characterized by the limit (minimum) input ratio – signal/noise (s/n): q_{lim}. When value $q_0 \geq q_{lim}$ coherent accumulation of spectral intensity along interference bands is realized [2]. At this condition the source detection is stable and estimates of the source parameters are close to real values. For the scalar component of the source noise field and isotropic noise $q_{\lim} \approx 1.5/J^2$ [3]. Here J – is the number of time intervals (samples)

at which coherent accumulations of spectral maxima of the wave field along the interference bands are realized

$$J = \frac{\Delta t}{T + \delta T}, \quad (1)$$

where Δt – time of observation, T – duration of noise realization of the source, δT the interval between samples. In order to noise realization to be independent, the duration δT must satisfy the condition $\delta T > \Delta f_1$, where Δf_1 noise source spectrum width.

Next, let's limit the area of input values $q_0 \geq 1.5/J$. That is practical interest. Signal and noise are considered independent Gaussian random processes with zero mathematical expectations. Let's put that in the band $\Delta\omega_1 = 2\pi\Delta f_1$ provides W localized bands with width $\Delta\omega_2 = 2\pi\Delta f_2$ and the contrast of the interferogram, i.e. the visibility of the bands, is equal to one. Along the interference bands, the intensity of the useful signal accumulates coherently, and the noise –incoherently. These provisions are not fulfilled accurately. But as it is shown by the results of computer modeling and nature experiments [2-6], the estimations remain quite correct.

Under the s/n ratio q_0 at the input of a single receiver at the initial time $t = 0$ refers to the value of

$$q_0 = q(0) = \frac{\overline{E}_s(0)}{\overline{E}_n(0)}, \quad (2)$$

where

$$\overline{E_s}(0) = \frac{1}{\pi}\int_0^\infty \overline{|s(0,\ \omega)|^2}\, d\omega = \frac{1}{\pi}\Delta\omega_1 \overline{|s(0,\ \omega'_s)|^2}, \quad (3)$$

$$\overline{E_n}(0) = \frac{1}{\pi}\int_0^\infty \overline{|n(0,\ \omega)|^2}\, d\omega = \frac{1}{\pi}\Delta\omega_1 \overline{|n(0,\ \omega'_n)|^2} \quad (4)$$

– average energy of useful signal and noise. Here ω'_s and ω'_n – selected signal frequencies and noise in the band $\Delta\omega_1$. The stroke above means averaging over the ensemble of realizations. In accordance with (3), (4), the input ratio s/n (2) is equal to

$$q_0 = \frac{\overline{|s(0,\ \omega'_s)|^2}}{\overline{|n(0,\ \omega'_n)|^2}}. \quad (5)$$

The values in the numerator and denominator (5) are equal to the mathematical expectations of the signal intensity and noise at the initial time

$$M_s(0) = \overline{|s(0,\ \omega'_s)|^2}, \quad M_n(0) = \overline{|n(0,\ \omega'_n)|^2}. \quad (6)$$

Signal dispersion and noise at the initial moment is [8]

$$D_s(0) = 2\left[\overline{|s(0,\ \omega'_s)|^2}\right]^2, \quad D_n(0) = 2\left[\overline{|n(0,\ \omega'_n)|^2}\right]^2. \quad (7)$$

The average useful signal energy and noise during time accumulation Δt can be written in the form of the addition of energies at time intervals of duration T along the interference bands

$$\overline{E_s}(\Delta t) = \frac{1}{\pi}\int_0^\infty \overline{\left|\sum_{j=1}^{J} Ws(t_j,\ \omega)\right|^2}\, d\omega, \quad (8)$$

$$\overline{E_n}(\Delta t) = \frac{1}{\pi} WJ \sum_{j=1}^{J}\int_0^\infty \overline{|n(t_j,\ \omega)|^2}\, d\omega. \quad (9)$$

Suppose that the mathematical expectations of the signal and interference do not depend on the moment of reference t_j, that is $\overline{|s(t_j,\ \omega)|^2} = \overline{|s(\omega)|^2}$ and $\overline{|n(t_j,\ \omega)|^2} = \overline{|n(\omega)|^2}$. Then expressions (8), (9) take the form

$$\overline{E_s}(\Delta t)=\frac{1}{\pi}W^2J^2\overline{\left|s(\omega_s'')\right|^2}\Delta\omega_2, \quad (10)$$

$$\overline{E_n}(\Delta t)=\frac{1}{\pi}WJ\overline{\left|n(\omega_n'')\right|^2}\Delta\omega_2, \quad (11)$$

so at the output of the time accumulation, the ratio s/n is equal to

$$q(\Delta t)=\frac{\overline{E_s}(\Delta t)}{\overline{E_n}(\Delta t)}=WJ\frac{\overline{\left|s(\omega_s'')\right|^2}}{\overline{\left|n(\omega_n'')\right|^2}}. \quad (12)$$

Here ω_s'' и ω_n'' – selected frequency of the signal and noise in the band $\Delta\omega_2$.

Using the input ratio s/n (5), the expression (12) is represented in the form

$$q(\Delta t)=WJ\frac{\overline{\left|s(\omega_s'')\right|^2}}{\overline{\left|n(\omega_n'')\right|^2}}\frac{\overline{\left|n(\omega_n')\right|^2}}{\overline{\left|s(\omega_s')\right|^2}}q_0. \quad (13)$$

At the initial time, the average useful signal energies (3) and noise (4) can also be expressed as

$$\overline{E_s}(0)=\frac{1}{\pi}W^2\overline{\left|s(\omega_s'')\right|^2}\Delta\omega_2, \quad (14)$$

$$\overline{E_n}(0)=\frac{1}{\pi}W\overline{\left|n(\omega_n'')\right|^2}\Delta\omega_2. \quad (15)$$

From the comparison of expressions (3), (4) and (14), (15) we find

$$\overline{\left|s(\omega_s')\right|^2}\Delta\omega_1=W^2\overline{\left|s(\omega_s'')\right|^2}\Delta\omega_2, \quad (16)$$

$$\overline{\left|n(\omega_n')\right|^2}\Delta\omega_1=W\overline{\left|n(\omega_n'')\right|^2}\Delta\omega_2 \quad (17)$$

and then the expression (13) takes the form

$$q(\Delta t)=Jq_0. \quad (18)$$

According to (10), (11) during the accumulation of mathematical expectations and dispersion of useful signal intensity and noise are equal

$$M_s(\Delta t)=W^2J^2\overline{\left|s(\omega_s'')\right|^2},\quad M_n(\Delta t)=WJ\overline{\left|n(\omega_n'')\right|^2}, \quad (19)$$

$$D_s(\Delta t)=2W^4J^4\left[\overline{\left|s(\omega_s'')\right|^2}\right]^2, D_n(\Delta t)=2W^2J^2\left[\overline{\left|n(\omega_n'')\right|^2}\right]^2. \quad (20)$$

Thus, the multiple coherent summation of the interference maxima of the noise source wave field along the localized bands increases the output s/n ratio $q(\Delta t)$ by J times with respect to the input value q_0. This increase becomes quite clear if we consider an analogy with the coherent spatial signal processing of a multi-element antenna containing Q receivers: with respect to a single receiver, the ratio of s/n increases Q-times.

Recording an interferogram onto a hologram and clearing the domain of spectral density localization from interference leads to an additional increase in the output ratio s/n compared to $q(\Delta t)$. The $2D$ Fourier transform of the interferogram localizes the $2D$ spectral density of the useful signal within a narrow hologram band, the area of which can be estimated as

$$S_s=\frac{\tau_{M-1}}{\Delta t}. \quad (21)$$

Here τ_{M-1} – position of the main maximum (M-1)-th focal spot on the time axis due to interference between extreme modes, where M – the number of modes forming the source field at the receiving point [2]. Outside this band, the spectral density is practically suppressed. The spectral density of the noise is distributed over the entire domain of the hologram, the area of which is equal to

$$S_n=\left|\nu_{M-1}\right|\tau_{M-1}, \quad (22)$$

where $\nu_{M-1} = -wh_{1M}(\omega_1)$ – position of the main maximum (M-1)-th focal spot on the axis of cyclic frequency [2]. Here $h_{1M} = h_1 - h_M$, h_m – horizontal wave number of m-th mode; ω_1 – average frequency of spectrum width $\Delta\omega_1$; w – radial velocity of the source. In the $2D$ Fourier transformation, the energy does not change. Therefore, assuming that the noise power is uniformly distributed in the hologram region, the s/n ratio in the localization band of the spectral density of the useful signal (at the output of interferometric processing) will be

$$q_{hol} = \frac{S_n}{S_s} q(\Delta t). \tag{23}$$

According to (18), (21), (22), expression (23) takes the form

$$q_{hol} = J\gamma q_0 \tag{24}$$

where is the concentration coefficient $\gamma = |\nu_{M-1}|\Delta t$. In the case of a unmoving source, the value $|\nu_{M-1}|$ replaced by the width of the spectrum $\Delta\nu$ in the hologram domain.

For shallow water in the low frequency range (several hundred hertz) and source velocities $w \approx$ 1-10 m/s value $|\nu_{M-1}| \approx$ 0.07-0.7 Gz [2−6]. Setting accumulation time $\Delta t = 100$ s, we obtain estimation γ = 7-70. The difference in the mechanism of accumulation of spectral intensity along the interference bands and the distribution of the spectral density of the wave field of the source and the noise on the hologram gives a high s/n ratio at the output of the interferometric processing.

3. DETECTION OF THE SOURCE BY THE NEUMAN-PIRSON CRITERION

Neumann-Pearson criterion optimizes the probability of correct detection p_1 (exceeding of given level at the output of the processing at the presence of useful signal) for a given probability of false alarm p_2, those. exceeding given level at the absence of useful signal due to noise [7].

Probability density $p_{sn,n}(x)$ of random value $I_{sn,n}(\Delta t)$ equal to temporary accumulation of spectral density

$$I_{sn,n}(\Delta t) = \sum_{j=1}^{J} \overline{\left|s_j(t,\omega) + n_j(t,\omega)\right|^2} \tag{25}$$

has a complex form defined by the composition of generalized χ^2-distributions and probability density determined by the Hankel function of zero order of the imaginary argument [8]. Here, the subscripts «*sn*» and «*n*» note the cases of presence and absence of a useful signal, respectively. In accordance with the central limit theorem for a large number of terms J probability density $p_{sn,n}(x)$ approximates for Gaussian distribution. Since value $I_{sn,n}(\Delta t) \geq 0$, so $p_{sn,n}(x) = 0$ at $x < 0$ and for probability density $p_{sn,n}(x)$, when $J >> 1$, obtain [8]

$$p_{sn,n}(x) = \frac{1}{\sqrt{2\pi D_{sn,n}(\Delta t)}} \left\{ \exp\left[-\frac{\left(x - M_{sn,n}(\Delta t)\right)^2}{2D_{sn,n}(\Delta t)} \right] + \exp\left[-\frac{\left(x + M_{sn,n}(\Delta t)\right)^2}{2D_{sn,n}(\Delta t)} \right] \right\}, x \geq 0, \tag{26}$$

Here $M_{sn,n}(\Delta t)$ and $D_{sn,n}(\Delta t)$ – expectation and variance of a random variable $I_{sn,n}$ at

time moment Δt. According to (19), (20), (25) we have

$$M_{sn}(\Delta t) = W^2 J^2 \overline{\left|s(\omega_s'')\right|^2} + WJ\overline{\left|n(\omega_n'')\right|^2}, \quad (27)$$

$$D_{sn}(\Delta t) = 2\left[W^2 J^2 \overline{\left|s(\omega_s'')\right|^2} + WJ\overline{\left|n(\omega_n'')\right|^2}\right]^2. \quad (28)$$

At the absence of a noise, the mathematical expectation $M_n(\Delta t)$ and variance $D_{sn}(\Delta t)$ are determined by expressions (19), (20). The linear $2D$ Fourier transform has the Gaussian probability density as well. Therefore, the probability density of the localized energy on the hologram has form similar to (26) by replacing expressions (27) and (28) by the relations

$$M_{sn(hol)}(\Delta t) = \gamma W^2 J^2 \overline{\left|s(\omega_s'')\right|^2} + WJ\overline{\left|n(\omega_n'')\right|^2}, \quad (29)$$

$$D_{sn(hol)}(\Delta t) = 2\left[\gamma W^2 J^2 \overline{\left|s(\omega_s'')\right|^2} + WJ\overline{\left|n(\omega_n'')\right|^2}\right]^2. \quad (30)$$

Probability of false alarm p_2 at the output of interferometric signal processing equals to the probability that the noise level is more than given level g. So, probability of false alarm equals to

$$p_2 = \frac{1}{\sqrt{2\pi D_n(\Delta t)}} \int_g^\infty \left\{ \exp\left[-\frac{\left(x - M_n(\Delta t)\right)^2}{2D_n(\Delta t)}\right] + \exp\left[-\frac{\left(x + M_n(\Delta t)\right)^2}{2D_n(\Delta t)}\right]\right\} dx. \quad (31)$$

After simple transformations, taking into account the relationship (19), (20) between the quantities $M_n(\Delta t)$ and $D_n(\Delta t)$, expression (31) is reduced to the following form

$$p_2 = 1 - 0.5\left[\Phi(\kappa_n - 0.5) + \Phi(\kappa_n + 0.5)\right], \quad (32)$$

where $\kappa_n = g/2M_n(\Delta t)$, $\Phi(x)$ – error integral

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

The probability of a false alarm is determined by the parameter κ_n. Expectation $M_n(\Delta t)$, according to (4), (17), can be represented as

$$M_n(\Delta t) = \frac{\pi}{\Delta\omega_2} J\overline{E_n}(0) = JM_n(0)\frac{\Delta\omega_1}{\Delta\omega_2}, \quad (33)$$

The noise energy density at the output of the temporary accumulation (for fixed input power and samples) is higher, than it is greater the ratio of the bands of the signal spectrum and the interferogram.

Probability of correct detection p_1 at the output of interferometric signal processing

$$p_1 = \frac{1}{\sqrt{2\pi D_{sn(hol)}(\Delta t)}} \int_g^\infty \left\{ \exp\left[-\frac{\left(x - M_{sn(hol)}(\Delta t)\right)^2}{2D_{sn(hol)}(\Delta t)}\right] + \exp\left[-\frac{\left(x + M_{sn(hol)}(\Delta t)\right)^2}{2D_{sn(hol)}(\Delta t)}\right]\right\} dx, \quad (34)$$

By using the ratios (12), (18), (24), (29), (30), can be represented as

$$p_1 = 1 - 0.5\left[\Phi(\eta_1) + \Phi(\eta_2)\right], \quad (35)$$

where

$$\eta_1 = \frac{2\kappa_n - 1 - q_{hol}}{2(1 + q_{hol})}, \quad (36)$$

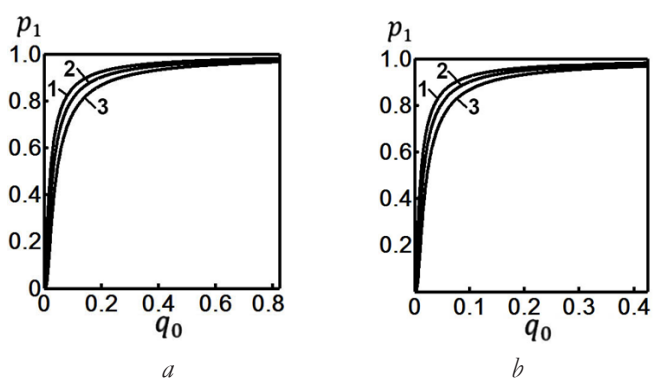
$$\eta_2 = \frac{2\kappa_n + 1 + q_{hol}}{2(1 + q_{hol})}. \quad (37)$$

One can see the probability of correct detection is determined by the parameter κ_n and the s/n ratio q_{hol} (24) at the output of interferometric processing.

The given false alarm probability p_2 determines the level of g. This level g determines the probability of correct

detection p_1. Thus, it is possible to calculate the useful signal detection curves, representing the dependence of the probability of correct detection on the output (or input) s/n ratio at a fixed probability of false alarm p_2. The range of input relations of s/n $q_0 = 1.5/J^2$ corresponds to the range of output values of s/n $q_{bol} \geq 1.5\gamma/J$. One can see, that the possibility of correct detecting of useful signal in interferometric signal processing for given probabilities p_1 and p_2 is not depend upon signal form and type of interference. The possibility of correct detecting is determined only by the energy density of the interference $M_n(\Delta t)$, the number of samples J and the concentration coefficient γ.

The **Fig. 1** shows the curves for correct detecting of useful signal as dependence of input s/n ratio q_0 for concentration coefficient γ = 10. Probability of false alarm $p_2 = 10^{-3}, 10^{-6}, 10^{-9}$. Number of samples J = 15, 30. These values corresponds to the limiting input relations of s/n $q_{\lim}$ = 0.00667, 0.00167. The decreasing of the probability of false alarm leads to the probability of correct detection decreasing for a given input s/n ratio. At the samples increasing, the source detection efficiency increases.



a *b*

Fig. 1. *The dependence of the probability of correct detection p_1 upon the input relations s/n q_0 for different values of the probability of false alarm p_2: (a) J = 15; (b) J = 30. Concentration coefficient γ = 10. Curve 1 – $p_2 = 10^{-3}$; curve 2 – $p_2 = 10^{-6}$; curve 3 – $p_2 = 10^{-9}$.*

4. CONCLUSION

In dispersing environment, the information technology for processing of signal interferograms of broadband sources is proposed. This information technology allows to solve the problem of localization of low-noise sources with high noise immunity. Of course, for each type of dispersing environment, the preliminary analysis of the efficiency of the offered signal processing is required.

The offered signal processing effectiveness is demonstrated by the example of waveguide dispersion of oceanic waveguides for the problem of noise source detection by the Neumann-Pearson criterion by using a single receiver. Noise immunity of signal processing is estimated. Analytical dependences of detection curves are obtained. The results of numerical calculations are presnted.

Acknowledgments: *This work was financially supported by grants from the Russian Foundation for Basic Research (projects No. 19-08-00941, No. 19-29-06075) and the Presidium of the Russian Academy of Sciences No. 5 "Photonic Technologies in Sensing of Inhomogeneous Media and Bioobjects".*

REFERENCES

1. Chuprov SD. The interference structure of the sound field in a layered ocean. In: *Acoustics of Ocean. The current situation.* Moscow, Nauka Publ., 1982, p. 71-82.

2. Kuznetsov G.N., Kuzkin V.M., Pereselkov S.A. Spectrogram and localization of the sound source in the shallow sea. *Akust. zhurn.*, 2017, 63(4):406-418 (in Russ.).
3. Kaznacheev I.V., Kuznetsov G.N., Kuzkin V.M., Pereselkov S.A. Interferometric method for detecting a moving sound source by a vector-scalar receiver. *Akust. zhurn.*, 2018, 64(1):33-45 (in Russ.).
4. Kuz'kin VM, Pereselkov SA, Kuznetsov GN, Kaznacheev IA. Interferometric direction finding by a vector-scalar receiver. *Phys. Wave Phenom.*, 2018, 26(1):63-73.
5. Kuz'kin VM, Kuznetsov GN, Pereselkov SA, Grigor'ev VA. Resolving power of the interferometric method of source localization. *Phys. Wave Phenom.*, 2018, 26(2):150-159.
6. Kuznetsov GN, Kuz'kin VM, Lyakhov GA, Pereselkov SA, Prosovetskiy DYu. Direction finding of a noise sound source. *Phys. Wave Phenom.*, 2019, 27(3):237-241.
7. Tikhonov VI. *Optimum signal reception.* Moscow, Radio i svyaz' Publ., 1983, 320 pp.
8. Tikhonov VI. *Statistical radio engineering.* Moscow, Radio i svyaz' Publ., 1982, 624 p.