

DOI: 10.17725/rensit.2019.11.279

*Paper is devoted to 150-year of Mendeleev's Periodic Table***Numerical representation of fractals in physical chemistry of material sciences****Alexander D. Izotov**Kurnakov Institute of General and Inorganic Chemistry, Russian Academy of Sciences, <http://igic.ras.ru/>  
Moscow 119991, Russian Federation*E-mail: izotov@igic.ras.ru***Feodor I. Mavrikidi**Oil and Gas Research Institute, Russian Academy of Sciences, <http://www.ipng.ru/>  
Moscow 119333, Russian Federation*E-mail: mavrikidi@mail.ru**Received 05.11.2019, peer reviewed 18.11.2019, accepted 25.11.2019*

**Abstract.** The paper substantiates the promise of the model of numerical asymmetry of fractals, which allows, from a single point of view, to cover modern approaches to modeling in chemistry and physics. The basis of the model is the synthesis of two basic number systems of mathematics - real and p- adic numbers. A physical chemistry interpretation of p-adic numbers and their properties is given. The physical chemistry space of the model becomes hyperbolic. The conclusion is made about the need to take into account duality in chemistry. As an illustration serves a model of matter based on Stone duality, which formally represents a pair “substance - properties”. The identity of nonlinear models of the atom and the Universe is shown, which serves as a justification for the periodicity of properties and is consistent with the hyperbolicity of the space of the general theory of relativity. The existence of the “golden section” number as a unit of a new number model is theoretically substantiated. The natural connection between the model and quantum mechanics by means of quantum numbers extracted from the model is shown. This opens up possibilities for the synthesis and interaction of the natural sciences, physics and mathematics, which, in the future, can serve as a model for the co-evolution of nature and technology.

**Keywords:** material science, fractals, p-adic numbers, number asymmetry, “golden section”, duality of chemistry, Mendeleev's Periodic Table

**PACS:** 81.00.00; 05.45.Df; 02.10+p.

*For citation:* Alexander D. Izotov, Feodor I. Mavrikidi. Numerical representation of fractals in physical chemistry of material sciences. *RENSIT*, 2019, 11(3):279-290; DOI: 10.17725/rensit.2019.11.279

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**1. INTRODUCTION**

Modern materials science is a physical chemistry science. In this chemistry uses

mathematics mediated by physics [1], which strongly impoverishes specificity of chemistry. So, for example, the balance equations of chemical reactions do not say anything about the very essence of qualitative transformations of matter, only its mechanical face is modeled as the only available. Therefore, all the calculation schemes for parameters of chemical phenomena, elements, the periodic system, as well as their descriptions by linear language, are equivalent and, ultimately, to mechanics. Since the physical component of materials science is informative, the main focus of the work

will be on chemistry, in which the concept of fractality is also represented by physics. The physical side of the issue was highlighted in the works of the authors [2, 3] and others.

The linearity inherent in physical and mathematical models is, in general, foreign to chemistry, because, obviously, all substances are binary formations consisting of material atoms/molecules held by intangible/invisible, field bonds. Therefore, more adequately chemistry concept is not atomic-molecular, but *web-like* substance whose properties are equally defined by its material and immaterial parts. The mathematics of networks that are also subject to (chemical) transformations is very poorly developed. In particular, the rules of arithmetic, which, in fact, introduce unaccounted mechanistic linearity into the model, do not act on networks without distorting the essence of the problem. This problem should be discussed separately, the scope of the article does not allow the development of argumentation more fully.

In summary, the problem of "reduction of chemistry to physics" looks as follows. In the chemical – qualitative, transformation of matter, the substance participates in the interaction of its *binary internal* space with external forces, phenomena, factors. The external becomes internal and vice versa, the boundary of the body becomes conditional. While in physics the internal space of bodies does not take any part – the boundary is absolute. This is the meaning of heating and/or other energy penetrating effects on the body, dispersion and mixing of substances that enter into a reaction, etc. These mathematical flaws are complemented by many unsolved problems of nucleosynthesis and elementary particle theory. The issues of nucleosynthesis are closely related, on the one hand, to questions of the evolution of stars, and, on the other, to the properties of nuclear interactions.

[4]. This is a well-known problem in natural science of the connection between the local (atoms) and the global (Universe), also known as the problem of "part and whole". In this point converges the problem of uncertainty of logical consequence, the notion of truth and the existence in contradiction. For biology, chemistry, and other natural sciences, these generally accepted concepts of logic do not exist now. Moreover, duality and contradictions in them are (naturally) scientific facts. This incompleteness gives rise to the contradictory semantic variability of many concepts and, as a result, prevents the formation of a holistic picture of science and the completeness of the model – at least in some approximation, when the truth of the part means consistency with the truth of the whole. For example, as an alternative, a test suggests itself instead of the traditional abstraction of separation, the abstraction of interpenetration with its obvious chemical meaning.

In the modeling of materials science, the main difficulty is its chemical part, which one usually tries to reduce to physics. During the existence and development of chemistry have been numerous attempts of its formalization by giving its basic law – the periodic system of Mendeleev mathematical form drawn via different sections of it – geometry, topology, group theory, etc. [5-14]. As far as the authors know, none of the approaches has given the fruitful development of mathematical methods for solving specific chemical (i.e., non-mathematical) problems and tasks. As recent work [15] showed, the form of the Periodic system changes depending on the choice of the properties of the elements, representing a hypergraph in which various types of periodicity and order can be detected. In other words, the Periodic system of elements is a *system* in the sense of the general systems theory, i.e. is special, distinct from physical

and mathematical, but closely connected with other sciences, space. The difference lies in the synthesis of many irreducible (i.e., mutually negating) formal languages that make up the model. Such objects lie at the intersection of all natural sciences, being their common part, which is consistent with a similar position in chemistry.

Today formed withdrawal line from standard models through engaging analysis and Fibonacci hyperbolic geometry, matrix structures [16-18]. In this direction, with general theoretical depth stands out book [19], in which the analysis of the periodic system is directly connected with the properties of numbers and the most general of the geometries – projective, i.e. the foundation of chemistry is associated with the fundamental concepts of mathematics, the closeness of the projective plane as a model of space, and the common constant of natural science – the number of the golden ratio. Therefore Fibonacci-like idea as closely related to the number of the golden section, looks very promising. In general, we can say that today there are numerous scattered model results related to the periodic table. The general physical and mathematical picture is not yet visible.

In this regard, it makes sense to pay attention to the following circumstance. Chemical elements make up the table as a whole – an *image*, i.e. *heterogeneous, integral in some sense*, formation. Such objects do not belong to the field of standard mathematics (except for tables, as auxiliary means of numerical methods). In contrast, *images* are typical for the fractal geometry of nature. A study conducted by the authors showed that empirical fractals are accompanied by an *already existing* logically connected line of mathematical constructions, which allows to look at the problems of materials science from a new point of view [2]. It arises when formalizing the idea of I.M.

Tananayev – to consider the particle size of the substance, i.e. the property of divisibility of matter (obviously also atoms – Auth.) by a separate degree of freedom. This formalization introduces into the arsenal of modeling the *second fundamental number system of mathematics* – *p*-adic numbers  $Q_p$ , which formalize this degree of freedom, thereby expanding the possibilities of modeling.

The aim of this work is to demonstrate that fractals via its formal analog – numerical asymmetry are promising candidate for the construction of adequate mathematical models of physico-chemical materials science. As new theoretical facts interpretation of *p*-adic numbers by S. Ulam, and their physico-chemical connotation by Tananaev, logical-linguistic idea of A.N. Parshin are introduced. These facts are described in detail in [20]. Mathematical modeling now operates in an expanded numerical space, making it possible to perceive the chemical subspace in it. This constitutes the content of the constructions.

**2. FRACTALS AND NUMBERS IN CHEMISTRY**

The periodic table as an *image* is semantically two-dimensional – along with an increase in the mass of elements, i.e. of their physical characteristics, it reflects a change in properties – qualitative, non-physical entities. One axis of the two-dimensionality is the mass axis or axis aggregation/divisibility of matter. It can be called the axis of nucleosynthesis. The second axis is the field axis – a set of electronic shells that determines the properties of elements. This is the axis of oxidation-reduction. This semantic two-dimensionality allows supplementing the physical types of duality – wave-particle, electromagnetic and thermodynamic ones by conjugated pairs having purely chemical specifics, such as “reaction-diffusion”, “oxidation-reduction”. These two pairs of oppositions correspond to

two basic numerical systems of mathematics – real  $R$  and  $p$ -adic numbers  $Q_p$  (here  $p$  is a prime number). Because of these numbers are the basis of the novelty of the method, here will be given the necessary information, equipped with physical and chemical interpretation. (more details in [20, part 1]).

### 3. p-ADIC NUMBERS IN PHYSICO-CHEMICAL INTERPRETATION

Real  $R$  and  $p$ -adic numbers  $Q_p$  are two basic number systems of mathematics. They are equally represented by formal power series in powers of the base equal to one of prime numbers  $p = 2, 3, 5, \dots, 13, \dots, 41, \dots$ , (which, however, to this day do not have a generally accepted natural-science interpretation):

$$\begin{aligned}
 x &= a_{-n}p^{-n} + a_{-n+1}p^{-n+1} + \dots + \\
 &+ a_{-1}p^{-1} + a_0 + a_1p + a_2p^2 + \dots + a_kp^k + \dots = \quad (1) \\
 &= \sum_{i=-n}^{\infty} a_i p^i, a_i \in A = \{0, 1, 2, \dots, p-1\}.
 \end{aligned}$$

In the same way, the positional record of both kinds of numbers is the same:

$$x = a_{-n}a_{-n+1} \dots a_0 a_1 \dots a_k \dots \quad (1^*)$$

This gives rise to measurable quantities of two kinds with the help of number metric functions, which are formal functional analogues of a universal pair of processes “convergence-divergence”. Real numbers are obtained from (1)-(1\*) by adding (“nucleosynthesis”) all the digits and then all the digits disappear,  $p$ -adic – by distinguishing the digits (“splitting”) in the positional notation. The quantities (metrics) of these numbers have the form

$$x \in R, |x|_{\infty} = |x|, \quad \xi \in Z_2, |\xi|_2 = p^{-(-n)}. \quad (2)$$

More precisely, for  $p$ -adic numbers there are two metrics – multiplicative and additive:

1. Additive one gives the coordinate on the  $n$  axis of divisibility (*nucleosynthesis*) of matter. Its physical interpretation is the energy of particle formation/decay:

$$\begin{aligned}
 v_p(\xi) &= ord_p(\xi) = -n = -\ln |\xi|_p^{\alpha} \Rightarrow \\
 &\Rightarrow v_p(\xi + \eta) \geq \min \{ v_p(\xi) + v_p(\eta) \}. \quad (2^*)
 \end{aligned}$$

2. Multiplicative one gives the size of the particles when dispersed, physically – their mass:

$$|\xi|_p^{\alpha} = p^{-\alpha n}, \quad \alpha > 0 \Rightarrow |\xi + \eta|_p^{\alpha} \leq \max \{ |\xi|_p^{\alpha}, |\eta|_p^{\alpha} \}. \quad (2^{**})$$

If one do not take into account the genesis, then on the material axis  $|x|_{\infty}$  and  $|\xi|_2$  are indistinguishable. It is easy to verify that these two quantities are connected by a hyperbolic relation, known as power laws, which allows them to be distinguished by behavior:

$$|x|_{\infty} = c \cdot |x|_2^D, \quad (2^{***})$$

where  $D$  is the fractal dimension.

Expansions (1)-(1\*) have a double meaning. When reading from right to left or from the reverse side of the sheet are converted into correct entry of real numbers. In other words, two kinds of numbers are connected by *negation*. In the proposed model (1)-(1\*), the numbers are considered partly as  $p$ -adic, partly as real, i.e. on both sides of the sheet. For example, with  $n > 0$ , the decomposition to the right is a  $p$ -adic number, and to the left is a real number. Forms (1)-(1\*) are also a numerical and coordinateless prototype of an iterated functions system, which is the main generator of fractals and the intersection of physics, language, biology, etc., where it is known as a hierarchy. Its action is the divisibility of matter, decomposability of systems, distinction and boundaries, disturbance of connectivity. For the first time such an interpretation of  $p$ -adic numbers  $S$ . Ulam suggested in the study of multiplicative processes arising in the fission chain reaction in 1955. Infinite division leads to zero dimensional sets or fractals. Therefore, like fractals,  $p$ -adic numbers are everywhere. Zero-dimensional sets do not have physical properties, they are invisible, they are numerical candidates for the role of fields of

various nature – physical (electromagnetic, gravitational, etc.), morphogenetic in biology, various linguistic ones. Moreover, all these heterogeneous fields coexist at every point in physical space, i.e.  $p$ -adic numbers are *multimodal*. In a recent work [21], the authors conclude – “The existence of a minimum length in quantum mechanics and a maximum speed of light are artifacts of the Archimedean basis of physics. Both of these assumptions are not supported by non-Archimedean (ie  $p$ -adic – Auth.) Physics.” Therefore, the number of mathematical systems makes up a pair of “material-ideal”. Ideal, symbolic, presents the properties of materials, often associated with human perception.

Geometrically real and  $p$ -adic numbers are represented by two oppositely directed trees. This mirror picture – numerical asymmetry  $U = R \times Z_p$ , is a formal analogue of the two-dimensional semantics of chemistry, which turns out to be consistent with all manifestations of duality in mathematics. Duality as the existence in a logical negation becomes one of the natural relationships and *mathematical operations*. It combines determinism with chance, gravity with repulsive force, compression with expansion, entropy processes are inextricable with energy. As a result, the resulting two trees – real and  $p$ -adic numbers are a condition for the existence of one another, they are connected as *real and possible, continuous and discrete*. Their combination provides another universal image of natural systems – the scale-invariant network, known in materials science as the atomic-molecular structure, Voronoi and Delaunay partitions.

The order of two numerical systems is reciprocal. They implement *two types of causality*, known as *compression, aggregation, materialization* and *expansion, dissipation, dematerialization* – synonyms of a universal pair of processes generated by a universal pair of *attractive*

*and repulsive forces*. Then, negation can be understood as complementarity – causality, the direction of time, topology, and the properties of members of the opposition are combined. The combination of two methods of coordination: physical and  $p$ -adic, provides the basis for adequate consideration of the geometry of the physico-chemical-symbolic space – it becomes locally hyperbolic, globally – projective (projective plane) Formally:

$U = R \times Z_2$ ,  $u = x \cdot \xi$ ,  $x \in R$ ,  $\xi \in Z_2$  – space and number;

$P^2(R) = R^2 \cup P^1(R)$  – projective plane; (3)

$R = inv Z_2 = \neg Z_2$  – "coordinate axes" of the  $U$  (inv – involution,  $\neg$  – negation). Any statement of the logic  $P$ , formulas, equations, makes sense in both numerical systems:

$R \leftarrow P \rightarrow Z_2$ , (3\*)

however, with different numerical results. This is the transfer principle, which includes the duality principle for networks.

$p$ -Adic numbers are spatially-geometric numerical system because it is isomorphic to the conventional iterated functions system (*IFS – iterated function system*) – the basic unit for obtaining fractal images of various shapes. This property allows to cover, from a single point of view, the various versions of the images of the periodic table proposed by researchers – spiral, pyramidal, disconnected, network, etc. [see site *meta-synthesis.com*]. In this case, a computer that implements such correspondences acts as a theoretical tool – an operator specific to chemical (and not only) science, the only one that can match *strings* and *images* [22], giving them numerical content using a *non-linear* language (see above). In other words, having received on the monitor screen a picture of a mixture of two/three reacting substances that differ in color (see **Fig. 1**), the computer as an operator assigns a certain number to each micro-region the corresponding color, thus turning the reaction region into a numerical

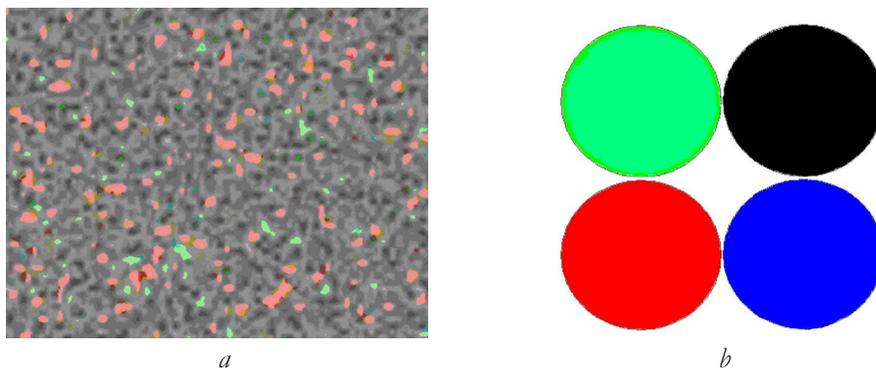


Fig. 1. The numerical image of a) the reaction as chemical addition; b) physical addition.

field, with the possibility of subsequent application of analytical/chemical calculation methods, the inclusion of motion, the contact of the valence orbits of atoms, etc.

*Measurements: definability of the “golden section”.* In the space of numerical asymmetry  $U = R \times Z_2$ , consisting of two subspaces with different topologies, the mathematical method of measurement – the axiom of Archimedes takes an unusual form. Since the number in  $U$  has the form  $\|u\| = |x|_\infty \cdot |\xi|_2$ , its measurement consists of the measurements of its two factors. In view of the fact that  $p$ -adic numbers are invariants of infinite divisibility, there is no definable unit in  $U$  (just as in ordinary mathematics, the unit is indefinable). And the axiom of Archimedes has a double meaning – as an additive and as a multiplicative process. Moreover, it can be written only recursively, in the sense of “the next result is equal to the sum of the previous ones and at the same time their product”. Formally:

$$u_{n+2} = u_{n+1} + u_n \text{ and } u_{n+2} = u_{n+1} \times u_n. \quad (4)$$

But these are the Vieta formulas of the quadratic equation  $x^2 - u_{n+2}x - u_{n+2} = 0$ . His decisions are known

$$x_{1,2} = u_{n+2} \cdot \frac{1 \pm \sqrt{5}}{2} = u_{n+2} \tau^\pm, \quad (5)$$

where the  $\tau^+ = \frac{1}{\tau}$  number of the golden ratio = 1.618. In (5), it  $u_{n+2}$  is a large-scale factor manifesting  $\tau$  on all floors of the natural hierarchy. Since a real number in  $U$  is defined as  $\|u\| = |x|_\infty \cdot |\xi|_2$ , due to the mutual

indeterminacy of two types of metrics, one of them is random in relation to the other. In physical space, random is ultrametric. If you write the "golden equation" in the form

$$\frac{x}{1-x} = \frac{1}{x}, \quad (6)$$

then, using the Logit-transform  $z = \ln\left(\frac{x}{1-x}\right)$  for  $\|u\|$  we obtain a distribution function  $P(z) = \frac{1}{\|u\| \cdot \frac{1}{\|u\|}}$ , that is very close to the Gaussian normal distribution, but with a finite support. Therefore, we find that the frequency of occurrence of “golden quantities” has a pronounced maximum and a clear dispersion [23].

Thus, this number is *determined* by the structure of the self-dual number system. Therefore, it  $\tau$  can be taken as the natural base of  $\tau$ -adic numerical system  $Q_2^\tau$ . And decomposition (1) will take the form analogous to the Bergman system [24], which, like the 2-adic one, has same alphabet/digits, but also has additional useful properties, such as redundancy – any number can be represented by different Bergman codes [25]:

$$\begin{aligned} x &= a_{-n} \cdot \tau^{-n} + a_{-n+1} \cdot \tau^{-n+1} + \dots + \\ &+ a_{-1} \tau^{-1} + a_0 + a_1 \cdot \tau + a_2 \cdot \tau^2 + \dots + a_k \cdot \tau^k + \dots = \quad (7) \\ &= \sum_{i=-n}^{\infty} a_i \cdot \tau^i \quad a_i \in A = \{0, 1\}. \end{aligned}$$

The unit is determined in two ways. Additively as  $1 = \tau^1 + \tau^2$ . And multiplicatively as  $1 = \tau^+ \times \tau$ . Fibonacci numbers  $F_n$  are related to the  $\tau$

by relations  $\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \tau$  and  $F_n = \frac{1}{\sqrt{5}}(\tau^n - (-\tau)^{-n})$  for  $n > 4$  with an error of less than 0.01. And also via the Binet formula:

$$F_n = \frac{\tau^n - (-\tau)^{-n}}{\tau - (-\tau)^{-1}} \tag{8}$$

A useful property of this numerical system is that as it follows from (5)  $\tau^+ \cdot \tau^- = 1$ . Comparing this relation with (2\*\*), which determines the relationship between two numerical metrics, we see that the two roots of the golden equation can be taken as real and ultrametrics, the constant  $c$  turns out to be computable and equal to the diameter of the scale level. And expansion (7) turns out to be an accurate representation of two numerical systems on a “golden” basis. In addition, the redundancy of the representation of an object by various codes of this system opens up the space for the existence of isotopes and isomers of chemical elements.

*Global periodicity.* Correspondingly, the expression for the ultrametrics (2\*) and (2\*\*) is corrected with the substitution of the bases 2 and  $p$  by  $\tau$ . Written together on the plane  $(|\cdot|_\infty, |\cdot|_\tau)$  isomorphic plane of complex numbers  $C$ , both metrics (2) and (2\*) in the form:

$$n \mapsto r = \tau^{\pm i \alpha \cdot n} \tag{9}$$

where  $r, n \in R$  give a famous expression for the “golden spiral”. Its peculiarity is that it does not spin on a plane, but around the axis of mass – nucleosynthesis. This axis is the axis of the 2-adic tree. Therefore, summing up, we arrive at a spatial hyperbolic model [16] that reproduces, according to Mendeleev, the bodily and discrete characters of the system (Fig. 2), reproducing the numerous chemical properties of elements. A similar model was proposed by the authors of [19], calling this spiral a global periodic function. Their work contains a link between local and global in chemistry. Reducing these poles - the atom

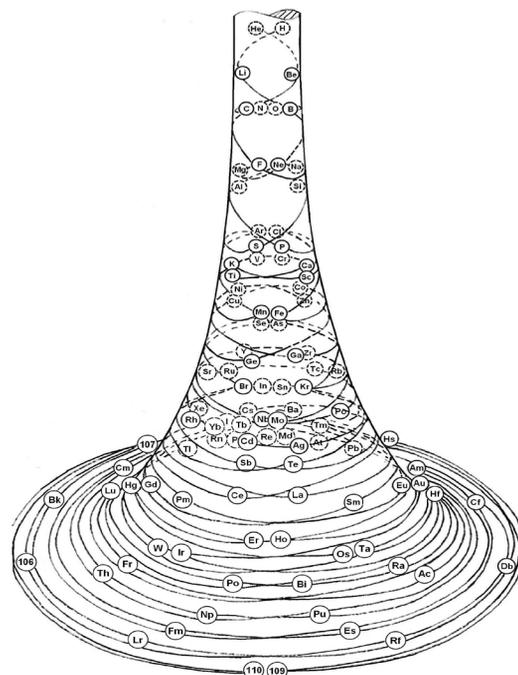


Fig. 2. The solid and discrete (according to Mendeleev) image of the periodic system.

and the Universe, through fractal geometry (it’s 2-adic!), they thus come to a projective model of the *chemical* universe. Clarification is required here. The authors obtain a projective plane by identifying infinity  $+\infty = -\infty$  on the real axis. But by virtue of what was said above about the indeterminacy of unity, and, therefore, zero, the minus sign ( $-$ ) has a conditional character, introduced for convenience, and often serving as an “operator” of time reversal, without a noticeable natural-science meaning. The Fibonacci series and 2-adic numbers do not contain it. Instead, the projective plane is obtained by identifications  $|\cdot|_\infty \rightarrow \infty = |\cdot|_2 \rightarrow 0$  and  $|\cdot|_2 \rightarrow \infty = |\cdot|_\infty \rightarrow 0$ . In this case, the global periodicity of matter becomes a consequence of the closeness of space (see (3)). The work mentioned above [15] gives a scaled invariant network (hypergraph) as a model of the chemical Universe, and curvature can also be determined on it [26]. As a result, we get a variant of the Universe in which there is a place of duality-complementarity, i.e. chemistry and other natural sciences.

The general expression of a real number leads to a power law, which is similar to flicker noise – “amplitude-frequency”. Here, the additive variable is represented by mass  $m$ , and the multiplicative by frequency  $f$ . Since the golden section number that has a *Logit* distribution is included in this ultrametric, the obtained law of distribution of elements by mass (amplitudes) of nuclei has the form of a distorted hyperbola [4, Introduction, Fig. 8]. It is symptomatic that all these power-law dependencies have a completely different form in the sense of proximity of functions – from a smooth hyperbola (E. Fermi) to a “sawtooth” one (Ishkhanov B.I. et al.). Such power laws are typical for evolving systems and, in principle, can serve as an entry point for introducing  $p$ -adic numbers into the model. In this case, the periodic table is really seen “... as if by the reflection of a stopped some unknown yet dynamic process of development of inorganic matter” [30]. Thus, the model includes the idea of evolution of the composition of chemical elements.

#### 4. NONLINEARITY OF CHEMICAL SPACE

The arguments of the previous section allow us to formalize the chemical Universe (for the developed theory, see [31]) as the resultant of the forces of aggregation (synthesis) and scattering (dispersion), formally represented by numerical asymmetry,  $U = R \times Z_2$ ,  $u = x \cdot \xi$ , where  $R$  is the axis of nucleosynthesis, and  $Z_2$  is the field measurement, the axis of oxidation-reduction, the axis of the electron shells. The motion along the field axis changes the properties of the elements, which is expressed by a change in the electronic composition of atoms. Stone duality allows us to imagine the volume of matter in two ways – as a material entity and as a formal language. From theoretical informatics it is known that language and property are one and the same thing (text and graphic editors of modern computers are

an example of this). Here, the properties of a substance are understood widely, including phase transitions between aggregate states. An expanded form of Stone duality is given in [20] ( $C$  is a Cantor perfect set, a classic example of a fractal  $C \cong Z$ , (not to be confused with  $C$  is complex numbers)):

$$C \cong C_{\text{matter}} \cong \exp(C) \cong 2^C \cong Z_\tau \cong [IFS \equiv \{0,1\}^N] \cong \\ \cong [Z_\tau \rightarrow Z_\tau] \cong C(Z_\tau, Z_\tau) \cong H \cong C_{\text{Bool}} \cong C. \quad (10)$$

Here:  $Z_\tau$  is the set of numbers of the form (7), and the signs of equivalence (isomorphism) “ $\cong$ ” mean in order from left to right:

- (1)  $C_{\text{matter}}$  is a model of fractal matter,
- (2)  $C$  is exponentially complete, i.e. mechanical transformations of matter do not change its numerical nature,  $\tau$ -adic numbers  $Z_\tau$  are an example of a spatially geometric numerical system, i.e. can encode objects of various shapes.
- (3) Such a distribution of matter is a spectrum of truth functions of Boolean algebra.
- (4) This is matter with numerical properties; there is  $Z_\tau$  a topological algebra.
- (5) Such a structure of matter (zero-dimensional, discountual, fractal) coincides with the formal languages of theoretical informatics, is a domain in theoretical informatics (iterated functions system, is the central technique for generating fractals). This is a symbolic space, the scope of symbolic dynamics.
- (6) As a lattice, it coincides with the space of continuous functions above itself.
- (7) Such a numerical or algebraic image of matter can be represented by its field of continuous functions by Stone duality theorem.
- (8) The set of 2-adic numbers is a Hilbert space. This means that its elements can be interpreted in the form of vectors of the Hilbert space and in the form of functions. Thus,  $p$ -adic numbers can be regarded as a Banach algebra with involution, that is, a  $C^*$ -algebra.  $Z_\tau$  – a discontinued version of the Hilbert space. In this form, it  $Z_\tau$  is also Boolean algebra – the

basis of the symbolic technique (according to the same Stone theorem).

(9) The fractal distribution of *matteris*, according to the statement of M. Stone, the flip side of Boolean algebra, i.e. perfect, symbolic object.

Then a substance can be defined as a system of its properties using the reflective properties of  $p$ -adic numbers:

$$\forall n \in N, \forall p = \tau,$$

$$Q_\tau \cong Q_\tau \times Q_\tau \times \dots \times Q_\tau = Q_\tau^n, \tag{11}$$

$$Z_\tau \cong Z_\tau \times Z_\tau \times \dots \times Z_\tau = Z_\tau^n.$$

Here, each factor corresponds to one of the property fields. Since they are zero-dimensional formations, (11) answers the question – how can different fields coexist at one point in space, not only physical, but also linguistic and morphogenetic fields of biology. Using (10) and (11), it is possible to solve the problems of “material equivalent of function and properties” of material science and a similar “structure-function” of biology. As shown by the authors of [32], the stationary orbit of an electron is the resultant of the forces of Coulomb attraction and centrifugal repulsion. The electron orbit in this case, according to Bertrand’s theorem, has a two-loop form. The same image has numerical asymmetry, i.e. globally chemical universe  $U$ .

The multilayered image of the Universe as applied to an atom means many of its electronic orbits (**Fig. 3**). The image of the nucleus in the first figure, in comparison with its absence of  $m$  in the second, means just the presence of the axis of nucleosynthesis ( $2^{**}$ ), while the second figure is simple  $U = R \times Z_2$ , i.e. image of electronic orbits. Attractors of nonlinear

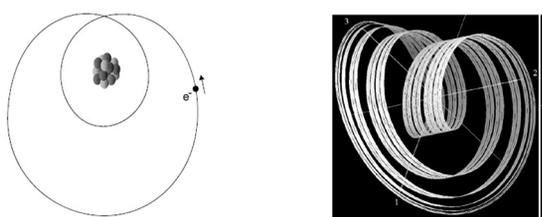
systems have the same two-loop images (see [20, Appendix to Chapter 7]). The authors’ conclusion [19] on the identity of the micro- and macrocosms of the chemical universe has a formal justification.

*Quantum numbers.* This section outlines the reasons for using the  $\tau$ -adic numerical system  $Z_\tau$  as a model of the chemical properties. As a first step, the interpretation of  $Z_\tau$  as a coordinateless functional space of quantum mechanics and the relationship of quantum numbers with it is proposed. According to the arguments of S. Ulam, at subnuclear distances, space cannot be smooth, it demonstrates a strong violation of local behavior, which entails the possibility of irregular topologies similar to  $p$ -adic ones [33].

The general structure of chemical elements is a dense core surrounded by an electron cloud, similar to the binary structure of chemistry. Then the physical meaning of the base  $p = \tau$  of the numerical system of  $\tau$ -adic numbers is a pair of “*matter (core) – field (e-cloud)*” or, mathematically, “*connectivity-discreteness*”. Then  $Z_\tau$  is a Hilbert space consisting of wave functions as a zero-dimensional space; it is capable of generating field and matter in addition (i.e., the known wave nature of matter). The quantum mechanical properties are preserved under such an interpretation.

The hypothesis of a  $\tau$ -adic relation of quantum numbers is as follows. Consider  $Z_\tau$  as the union of four self-similar “circles”. Visually, these are 4 binary trees with a common root (**Fig. 4**).

On this tree, we consider the trajectories, i.e. functions – paths connecting vertices. For example, the vertices of the upper right branch with the vertices of the lower right. Then, if we put the principal quantum number  $n$  equal to the number of the level of the division hierarchy, then the orbital quantum number  $l = (0, 1, 2, \dots, n-1)$  coincides with the set of levels of the hierarchy. In Fig. 4  $n = 4, l = (0, 1, 2, 3)$ . These



**Fig. 3.** The nonlinearity of the atom and space.

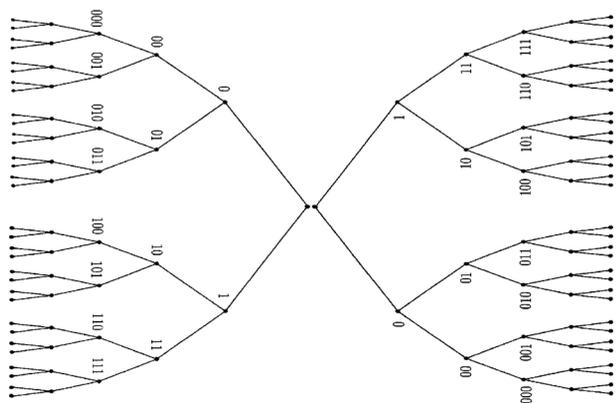


Fig. 4. 2-adic space of waves and particles.

two numbers are common to all four branches. The magnetic quantum number  $m_l$  coincides with the segment  $(-l, l)$ . When moving to the root, the path is deterministic, from the root it is non-deterministic. The left and right half of the figure are associated with a spin quantum number  $m_s = (1/2, -1/2) \cong (\alpha, \beta)$ .

Thus, the considered trajectories  $\xi^- = a_0 a_1 \dots a_n \in Z_2$  as wave functions that have a double – deterministic and nondeterministic meaning, and as elements of the numerical system  $Z_2$  naturally connect all four quantum numbers. Based on the foregoing, the closest seems to be the interpretation of the periodic law as an infinite-dimensional functional space [11]. The stated hypothesis concerns the periodic system of free atoms, which, however, is different from the periodic system of chemical elements [34].

## 5. CONCLUSIONS

Based on the analysis of mathematical models of materials science, the introduction of a new degree of freedom is substantiated – the divisibility of matter and its digitization by  $p$ -adic numbers. Physicochemical and general scientific interpretations of  $p$ -adic numbers are given.

Together with real numbers, they form a physic-chemical space that allows the existence of the properties of substances, as illustrated by the expanded form of Stone duality. In this space, the axis of nucleosynthesis and the axis

of oxidation-reduction are distinguished, which makes it possible to introduce, in addition to the known physical pairs of oppositions, pairs inherent in chemistry. The existence of two types of causality, expressed by the pairs “attraction-repulsion”, “synthesis-decay”, “convergence-divergence”, “aggregation-dissipation”, and others synonymous with them, is shown. The formal model of material science as a physical and chemical science is a self-dual numerical system that expands the modeling arsenal in the direction of increasing its adequacy.

As an alternative to disparate theories, an approach based on Fibonacci numbers as the common constant of natural sciences is chosen. It has been formally proved that the number of the “golden ratio” is the base of the numerical system of the physico-chemical space, which is largely analogous to 2-adic numbers. Thus, all disparate works on the models of the periodic table of Mendeleev gain a single framework and reliable formal justification.

The periodicity of matter as a consequence of the identity of nonlinear models of the atom and the Universe is substantiated. Geometrically, it follows from the projective model of space. The consistency of the proposed model with quantum representations by extracting quantum numbers from the structure of the model without additional assumptions and calculations is shown.

In general, it has been demonstrated that the proposed numerical version of physico-chemical materials science is consistent with the known results, providing their empirical nature with mathematical validity.

Special mention should be made of the nonphysical properties of materials. Properties are described in formal languages. By the Rice-Uspensky theorem, any non-trivial property of a language is undesirable, i.e. it is impossible to single out any specific property of a material

in a formally familiar language. The question remains whether this solution is possible within the framework of the proposed approach.

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