ELECTROMAGNETIC AND GRAVITATIONAL FIELDS IN THE 5-DIMENSIONAL EXTENDED SPACE MODEL, THEIR LOCALIZATION AND INTERACTION WITH MATTER

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Abstract. **Electromagnetic and gravitational fields are considered within the framework of the 5-dimensional Extended space model (ESM). The action is considered as the fifth coordinate. It is shown that in this model they are combined into a single electromagnetic-gravitational field. This field has 10 components: the vector field $\vec{G}$ and scalar field Q are added to the usual 6 fields $\vec{E}$ and $\vec{H}$. These fields satisfy the system of generalized Maxwell's equations. We find an expression for the Lorentz force, which determines the interaction of these fields with charged massive bodies. In the framework of this model, the question of the origin of the photon non-zero mass and its localization is studied.**

Keywords: **5-dimensional space, electromagnetic field, gravity, Maxwell's equations, photon, mass, localization**

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1. INTRODUCTION

The problem of combining electromagnetic and gravitational fields into a single field has been discussed since the late 19th century. It is characteristic that all these attempts have been made on the way of constructing geometric models of physical interactions and interpretation of physics as geometry in the spaces of a larger number of dimensions. In the late 19th century, German mathematician Felix Klein [1] constructed the Hamilton-Jacobi theory as optics in the space of the highest number of dimensions. However, at that time his ideas did not developed. A new surge of interest to the problem of geometrization of physics was stimulated by the creation of the General theory of relativity (GRT) [2]. The attempts to describe electromagnetism in geometric terms by analogy with gravity were made.

Their authors did not try to create a new model, but tried to expand the existing GRT scheme in one way or another. The most known were the models of T. Kaluza [3] and O. Klein [4]. Also noteworthy are the works of H. Mandel [5] and V. Fock [6]. It is characteristic that all they had to use a 5-dimensional space. The problem of physical interpretation of the fifth coordinate has not been solved satisfactorily. Hereinafter many scientists, including Einstein [7], de Broglie, Gamow, Rumer [8] tried to develop these approaches, but they failed to get any interesting results. In our opinion, the reason is that their works have been based on formal generalizations of existing models, without involving new physical ideas.

We should also mention the theory of gauge fields as one of the directions of geometrization of physical interactions [9]. Within this ideology, electromagnetism, gravity, and other interactions are considered from a single geometric point of view [10].

Later, in order to create the theory of elementary particles, another approach to combining gravity with other interactions was developed.

The need to take into account the gravitational field in the description of the interaction of elementary particles was indicated 50 years ago by K.P. Stanyukovich [11] and M.A. Markov [12]. They put forward the hypothesis of the existence of heavy particles – planckions and maximons.

The authors assumed that there are three fundamental constants in nature: the Planck constant $\hbar$, the speed of light c, and the gravitational constant G. These values can be used to construct expressions with dimensions of length, time, and mass. They are called Planck length l_{Pl}, Planck time t_{Pl} and Planck mass m_{Pl}.

$$l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33}\, sm,$$
$$t_{Pl} = \sqrt{\frac{\hbar G}{c^5}} \approx 10^{-43}\, \text{sec}, \qquad (1)$$
$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} \approx 10^{-5}\, gr.$$

In quantum theory a particle with a mass of m corresponds to the Compton wavelength

$$\lambda_{\tilde{n}} = \frac{\hbar}{mc}. \qquad (2)$$

This wavelength can be associated with the particle size, some of its "quantum radius". If the Planck mass m_{Pl} is put in the formula (2), it turns out that the Compton wavelength coincides with the Planck length l_{Pl}.

$$\lambda_{\tilde{n}} = l_{Pl}. \qquad (3)$$

But another linear parameter can be associated with mass m – the Schwarzschild gravitational radius

$$r_{gr} = \frac{Gm}{c^2}. \qquad (4)$$

According to the General theory of relativity, if a spherical-symmetric distribution of matter is compressed to such dimensions, it collapses, forming a black hole. Therefore, it is now considered that the value of m_{Pl} is the maximum value of the mass of an elementary particle. They are called maximons. Particles with large masses should turn into black holes. Accordingly, the corresponding gravitational radius r_{gr} can be considered as the minimum possible size of an elementary particle.

If one substitute the Planck mass m_{Pl} in the formula (4), it will take the form

$$r_{gr} = 2\sqrt{\frac{\hbar G}{c^3}} = 2 l_{Pl}. \qquad (5)$$

Thus, the gravitational radius of maximon coincides in order of magnitude with Planck length.

In Landau's work [13], estimates for the value of the "radius" of elementary particles were obtained, based on the limit of applicability of electrodynamics representations in quantum mechanics. Interestingly, the "radius" of the electron at the same time was equal to zero.

Such relations were discussed in an attempt to take into account the gravitational forces in the processes of interaction of elementary particles. This approach assumes the initial existence of particles with a large rest mass, and since we do not observe such objects, it is not clear how it can be used to describe the processes occurring in the laboratory.

Our approach is fundamentally different from all these and similar theories. The Extended space model (ESM) is based on the physical hypothesis that the mass (rest mass) and its conjugate action (interval) are dynamic variables, the value of which is determined by the interaction of fields and particles. In this respect, our model is a direct generalization of the special theory of relativity (SRT), in SRT the interval and rest mass of particles are invariants, in ESM they can change. In particular, a photon can acquire mass, both positive and negative. This mass can appear and change as a consequence of electromagnetic interaction and generate gravitational forces. It is

this circumstance that allows us to consider gravity and electromagnetism as a unit field.

Different aspects of the ESM are set out in articles [14-18]. In this paper, we give a systematic exposition of the formalism of the electromagnetic-gravitational field, introduce a generalized system of Maxwell's equations, which satisfy its tensions, and find an expression for the Lorentz force, which defines its interaction with matter.

2. CURRENTS AND POTENTIALS

The source of the electromagnetic field is a current. In the traditional formulation the electromagnetic current is described by (1+3) vector in Minkowski space $M(1,3)$ [18].

$$\tilde{\rho} = (\rho, \vec{j}) = \left(\frac{\rho_0 c}{\sqrt{1-\beta^2}}, \frac{\rho_0 \vec{v}}{\sqrt{1-\beta^2}} \right), \beta^2 = \frac{v^2}{c^2}, \tilde{\rho}^2 = c^2. \quad (6)$$

Here $\rho_0(t,x,y,z)$ is the density of electric charge at the point (t, x, y, z) in the space $M(1,3)$, and $v_x(t, x, y, z)$, $v_y(t, x, y, z)$, $v_z(t, x, y, z)$ – is local velocity of the charge density.

In transition to the extended space $G(1,4)$ the (1+3) current vector $\tilde{\rho}$ should be replaced by (1+4) vector $\bar{\rho}$. In accordance with the principles that form the basis of the developed model, the additional coordinate of the vector $\bar{\rho}$ is introduced in such a way that the resulting (1+4) vector is isotropic. In addition, we want our model to describe both the electromagnetic field and the gravitational field, so the fifth component of the current should be defined so that it serves as a source of the gravitational field.

We believe that the source of a single electromagnetic gravitational field is a particle that has both mass and charge. In this case, we assume that the mass may not have a charge, but the charge must always have a mass. In our model, we assume that the charge is a constant value, and does not change with transformations from the group of rotations $L(1,4)$ of the extended space $G(1,4)$. And the rest mass, which was a scalar with respect to the Lorentz group, is a component of the vector with respect to the group $L(1,4)$.

We want to obtain a 5-dimensional current vector $\bar{\rho}$ as a generalization of the 4-dimensional current vector $\tilde{\rho}$. To do this, it is necessary to assign another component.

The current vector (6) is structurally similar to the energy-momentum vector of a particle, having a rest mass. The difference between them is, that in the vector (6) instead of a rest mass m_0 there is a local density of a charge ρ_0. In the transition to the extended space $G(1,4)$ we pass from the energy-momentum vector to the energy-momentum-mass vector, and in the transition from mechanics to electrodynamics the mass changes to a charge. But, because we want to get a current that will simultaneously serve as a source of both electromagnetic and gravitational field, we multiply each component of the energy-momentum-mass vector on the density of a charge ρ_0, while maintaining the mass density m. For brevity, we will denote the charge density by the letter e. Thus, the 5-dimensional current vector generating a single electromagnetic-gravitational field has the form

$$\bar{\rho} = (j_0, \vec{j}, j_4) = \left(\frac{emc}{\sqrt{1-\beta^2}}, \frac{em\vec{v}}{\sqrt{1-\beta^2}}, emc \right). \quad (7)$$

This is an isotropic vector

$\bar{\rho}^2 = 0.$

The continuity equation, as in the usual case, is expressed as zero 5-divergence of 5-current

$$\sum_{i=0}^{4} \frac{\partial j_i}{\partial x_i} = 0. \quad (8)$$

If the charge is at rest, the continuity equation takes the form

$$\frac{\partial m}{\partial t} + \frac{\partial m}{\partial x_4} = 0. \quad (9)$$

The ratio (9) can be interpreted as the law of change in the rest mass of a particle due to changes in the properties of the environment.

In ordinary electrodynamics the law of charge conservation follows from the continuity equation

$$\frac{\partial}{\partial t} \int j_0 dV = -\int \vec{j} d\vec{n}. \quad (10)$$

In the left part of this relation there is an integral on volume, and on the right – an integral on a surface limiting this volume.

In electrogravity, there is a law of conservation of the value, which is the product of the charge to the mass of the particle, which has this charge. This law has the form

$$\frac{\partial}{\partial t}\int j_0 dV = -\int \vec{j} d\vec{n} - \int \frac{\partial}{\partial x_4} j_4 dV. \quad (11)$$

In this case the change of the value of the product of *em* charge per mass inside a certain volume is determined both by the flow of charged particles through the surface of this volume and by change of the mass of particles inside the volume due to their dependence on the coordinate x_4. Thus, we do not violate the law of charge conservation, since the mass changes in the product of *em*, and the charge remains constant.

The current (7) generates an electro-gravitational field in the extended space $G(1,4)$. This field is given by a 5-vector-potential A.

$$A = (A_t, A_x, A_y, A_z, A_s) = (A_0, A_1, A_2, A_3, A_4). \quad (12)$$

Here and below we use the notation $t = x_0$, $x = x_1$, $y = x_2$, $z = x_3$, $s = x_4$.

The components of this vector-potential are determined by the equations

$$\Pi_{(5)} A_0 = -4\pi\rho, \quad (13)$$

$$\Pi_{(5)} \vec{A} = -\frac{4\pi}{c}\vec{j}, \quad (14)$$

$$\Pi_{(5)} A_s = -\frac{4\pi}{c} j_s. \quad (15)$$

Here

$$\Pi_{(5)} = \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}. \quad (16)$$

Note that in the case when the dependence on the coordinate s is absent and the mass m, which is included in the current components (7), is a constant, the system of equations (13)-(15) splits into two independent parts. The equations (13), (14) define the usual potentials of the electromagnetic field, and the equation (15) defines the potential of the scalar gravitational field. In this case these fields exist independently of each other. They are combined into one field only when the mass of *m* becomes a variable and there is a dependence on the coordinate *s*.

3. STRESS FIELDS AND THE GENERALIZED SYSTEM OF MAXWELL'S EQUATIONS

The stress tensor can be constructed from potentials $(A_0, A_x, A_y, A_z, A_s)$

$$F_{ik} = \frac{\partial A_i}{\partial x_k} - \frac{\partial A_k}{\partial x_i}; i,k = 0,1,2,3,4. \quad (17)$$

$$\|F_{ik}\| = \begin{pmatrix} 0 & -E_x & -E_y & -E_z & -Q \\ E_x & 0 & -H_z & H_y & -G_x \\ E_y & H_z & 0 & -H & -G_y \\ E_z & -H_y & H_x & 0 & -G_z \\ Q & G_x & G_y & G_z & 0 \end{pmatrix}. \quad (18)$$

Here

$$Q = F_{40} = \frac{\partial A_4}{\partial x_0} - \frac{\partial A_0}{\partial x_4} = \frac{\partial A_s}{c\partial t} - \frac{\partial \varphi}{\partial s}. \quad (19)$$

$$G_x = F_{41} = \frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4} = \frac{\partial A_s}{\partial x} - \frac{\partial A_x}{\partial s},$$

$$G_y = F_{42} = \frac{\partial A_4}{\partial x_2} - \frac{\partial A_2}{\partial x_4} = \frac{\partial A_s}{\partial y} - \frac{\partial A_y}{\partial s}, \quad (20)$$

$$G_z = F_{43} = \frac{\partial A_4}{\partial x_3} - \frac{\partial A_3}{\partial x_4} = \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}.$$

We give equations which satisfy the tensions F_{ik}. We call them the Maxwell extended system.

The usual system of Maxwell's equations consists of two pairs of equations that have a fundamentally different structure. They are usually called, the first and second pair of Maxwell's equations. The extended system of Maxwell equations also consists of two types of equations of fundamentally different structure, but now they are not two, but more, so we will call them not the first and second pair, but the equations of the first and second types.

The equations of the first type are formal consequence of the formula (17) expressing tensions through potentials. It directly follows from its form that for any three indices (i, j, k) the relation is executed

$$\frac{\partial F_{ij}}{\partial x_k} + \frac{\partial F_{ki}}{\partial x_j} + \frac{\partial F_{jk}}{\partial x_i} = 0. \quad (21)$$

The validity of the equation (21) is checked by direct substitution. Indeed, substituting in the equation (21) the expression (17) for stresses via potentials, we have

$$\frac{\partial^2 A_i}{\partial x_k \partial x_j} - \frac{\partial^2 A_j}{\partial x_k \partial x_i} + \frac{\partial^2 A_k}{\partial x_j \partial x_i} - \frac{\partial^2 A_i}{\partial x_j \partial x_k} + \frac{\partial^2 A_j}{\partial x_i \partial x_k} - \frac{\partial^2 A_k}{\partial x_i \partial x_j} = 0.$$

We have as many such equations as there are different sets of indices (*i, j, k*), i.e. the number of combinations from 5 to 3, which is equal to 10. Let us now consider the specific forms of these equations, using the stress tensor (18).

If we limit ourselves to sets of indices that take values (0, 1, 2, 3), then the corresponding 4 equations are just the first pair of Maxwell's equations

$$div\vec{H} = 0, \ \sim indexes\ (1, 2, 3) \tag{22}$$

This is one equation, the other 3 corresponding to the index sets (0, 1, 2), (0, 1, 3), (0, 2, 3), combined into a single vector equation

$$rot\vec{E} + \frac{1}{c}\frac{\partial \vec{H}}{\partial t} = 0. \tag{23}$$

Thus, the first pair of Maxwell's equations retains its form. In the extended space $G(1,4)$ six more equations are added to them. Three of them, responding sets (1, 2, 4), (1, 3, 4), (2, 3, 4), can be combined into one vector equation

$$rot\vec{G} + \frac{\partial \vec{H}}{\partial s} = 0. \tag{24}$$

Three other threes (0, 1, 4), (0, 2, 4), (0, 3, 4) give three remaining equations of the first class. They also can be combined into a single vector equation

$$\frac{\partial \vec{E}}{\partial s} + \frac{1}{c}\frac{\partial \vec{G}}{\partial t} + gradQ = 0. \tag{25}$$

Thus, the equations of the first type included of the extended Maxwell system equations in space $G(1,4)$ read (22)-(25). These 10 equations are combined in a vector 3 equations and one scalar. Note that the vector operators *div, rot, grad* have the usual 3-dimensional form.

Let us now proceed to construction second type Maxwell equations of the. These equations follow from the equations for potentials (13)-(15). However, it is necessary first to impose a Lorentz gauge condition that must be satisfied by the potential (12). In the space $G(1,4)$ it reads

$$\frac{1}{c}\frac{\partial A_0}{\partial t} + \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} + \frac{\partial A_s}{\partial s} = 0. \tag{26}$$

The second type equations from the extended Maxwell system has the form

$$\sum_{k=0}^{4} \frac{\partial F_{ik}}{\partial x_k} = -\frac{4\pi}{c} j_i; \ i = 0,1,2,3,4. \tag{27}$$

Substituting the stress tensor elements (18) in (27) and considering the Lorentz gauge condition (26), we obtain 5 equations. In vector form they take the form

$$div\vec{E} + \frac{\partial Q}{\partial s} = 4\pi\rho, \ (i = 0), \tag{28}$$

$$rot\vec{H} - \frac{\partial \vec{G}}{\partial s} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}, \ (i = 1,2,3), \tag{29}$$

$$div\vec{G} + \frac{1}{c}\frac{\partial Q}{\partial t} = 4\pi j_4, \ (i = 4). \tag{30}$$

The stress tensor (18) contains, in addition to components that are analogous to conventional electric and magnetic fields, additional components that describe the gravitational field. More specifically, in the case where the components of the 5-current (7) depend on the coordinate x_4, all components of the tensor (18) describe the unified electromagnetic-gravitational field, if the current does not depend on the coordinate x_4, then the system of equations (22)-(25), (28)-(30) splits into two parts. At the system of Maxwell equations and the Laplace equation for the scalar gravitational field.

Thus, according to our model, in the empty space gravitational and electromagnetic fields exist separately, and in the area where external forces act at particles and fields, they are combined into one field.

We discuss briefly the physical meaning of the equations of extended Maxwell system (22)-(25), (28)-(30).

Equation (22) shows that there are no magnetic charges does not appear in this model and its magnetic lines of force still remain closed.

It follows from equation (23) that, as before, the circulation of the electric field in a closed loop is determined only by the change in the

magnetic field inside this contour. In this case, the fact that the fields $\vec{E}$ and $\vec{H}$ can now depend on the variable s, not felt by the equations (22), (23), because they include only derivatives of the usual spatial and temporal variable.

The equation (24) shows that the circulation of the new field $\vec{G}$ by an arbitrary spatial closed loop is defined by a change of the field $\vec{H}$ covered by this contour over the new variable s.

The physical meaning of the equation (25) is that it relates the change the old field $\vec{E}$ by the new variable s with change of the new fields Q and $\vec{G}$ by old variables. If the field $\vec{E}$ is changed by the s variable, then an inhomogeneous field Q and a nonstationary field $\vec{G}$ should exist in space.

The equation (28) shows that the electric charges of density ρ and the change of the field of Q by the variable s can be considered as the source of the electric field is $\vec{E}$. The electric field lines can start and end not only on electric charges, but also at those points where there is a change of the field Q on the variable s.

It follows from the equation (29) that the circulation of the magnetic field $\vec{H}$ by a closed loop is defined not only by the current flowing inside the loop and by change of the electric field $\vec{E}$ covered by it, but also by change of the field $\vec{G}$, located inside the contour on the variable s.

The equation (30) shows that the source of the field $\vec{G}$ can be not only the charges forming the current component j_4, but also change in time of the field Q. The lines of the field $\vec{G}$ lines can start and end not only at electric charges, but also at those points where the derivative $\partial Q/\partial t$ is different from zero.

4. GENERALIZED LORENTZ FORCE

Now we find the forces acting on the point charged particle located in the field (18), and the equations of motion of such a particle. Let us write Lagrangian of the system particle+field for a particle of mass of m and charge e and a field given by the potential (12). Choose it as

$$L = -mc^2\sqrt{1-\beta^2} + mcv_s - e\varphi + \frac{e}{c}(\vec{A}\vec{v} + A_s v_s). \quad (31)$$

Here $v_s = ds/dt$.

The Lagrangian (31) differs from the usual Lagrangian describing motion of charged particle in an external field by a member $(e/c)A_s v_s$. And it is assumed in addition that all 5 components of the potentials (12) depend on the variable s. This leads to the fact that mass m of the particle depends on time $m = m(t)$. This result is quite consistent with the original postulates of the model. Indeed, according to our assumptions, only mass of free particle is constant. But if it interacts with other particles, its mass may vary.

Interaction constant e/c, with which the product $A_s v_s$ is included in the Lagrangian, is the same as at the products of spatial component of the potential and velocity of the particle $\vec{A}\vec{v}$. This is necessary for that the equations of motion of the particle include only the field strength (18), not its potentials (12). As in the usual classic electrodynamics, we believe that the observed values are intensities of the field, not its potentials.

The momentum of the particle is determined by the formula

$$P_i = \frac{\partial L}{\partial v_i}. \quad (32)$$

From the Lagrangian (31) we obtain

$$\vec{P} = \vec{p} + \frac{e}{c}\vec{A} = \frac{m\vec{v}}{\sqrt{1-\beta^2}} + \frac{e}{c}\vec{A}. \quad (33)$$

$$P_s = p_s + \frac{e}{c}A_s = mc + \frac{e}{c}A_s. \quad (34)$$

By Lagrangian one can construct Euler's equations of motion, which in general case have the form [19, 20]

$$\frac{d}{dt}\frac{\partial L}{\partial v_i} = \frac{\partial L}{\partial x_i}. \quad (35)$$

These are four equations. First three of them read

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}[\vec{v}, \vec{H}]. \quad (36)$$

In form they coincide with the usual equations of motion of charged particle. The only difference

is that now the particle's mass m depends on time. This dependence is determined by the equation

$$\frac{dp_s}{dt} = eQ + \frac{e}{c}[\vec{v}, \vec{G}]. \tag{37}$$

Given that $p_s = mc$ and the speed of light c is constant, we obtain the equation evolution of mass m

$$\frac{dm}{dt} = \frac{e}{c}Q + \frac{e}{c^2}(\vec{v}, \vec{G}). \tag{38}$$

Thus, four equations (37)-(38}) describe the evolution of four values $v_x(t)$, $v_y(t)$, $v_z(t)$, $m(t)$.

The equation (38) shows that in presence of an external field a mass of a particle changes. Below we will look at specific examples and find exact solutions equations (36)-(38) and interpret them in terms of rotations in the extended space $G(1,4)$.

5. LOCALIZATION OF FIELDS AND PARTICLES

In the framework of the ESM, it is possible to establish in natural way connection between photon mass and some linear parameter, which we will call the localization parameter. In some sense, it can be considered as the size of a photon. The starting point for us is analogy between dispersion relation of a free particle

$$E^2 = (c\vec{p})^2 + m^2c^4 \tag{39}$$

and dispersion relation of a wave mode in a hollow metal waveguide

$$\omega^2 = \omega_{kr}^2 + (c\xi)^2. \tag{40}$$

Here ω_{kr} – is the critical frequency of the waveguide mode, and ξ – is the wave propagation constant.

The similarity of the relations (39) and (40) was noticed by de Broglie [21], Feynman [22] and other scientists. The essence of the problem is that with the critical frequency ω_{kr} is associated the parameter

$$m = \frac{\hbar\omega_{kr}}{c^2}, \tag{41}$$

which has a dimension of mass, and the question arises whether this value can be interpreted as the real mass? The mass that the electromagnetic field acquires when it enters the waveguide. In the works of Rivlin, this problem has been studied in a systematic manner [23, 24].

But here we will not go into this question, and only note the fact that the mass m is related with geometry and size of a waveguide. In particular, if the waveguide has a square shape with a side size a, then this connection has the form

$$a = \frac{\sqrt{2}\pi\hbar}{mc}. \tag{42}$$

It is this value that we propose to consider as a linear parameter which is associated with a massless particle when it acquires the mass m. We believe that at the same time, when a massless particle enters the external field and acquires a nonzero mass, the corresponding infinite plane wave shrinks to finite size. And this finite size is characterized by localization parameter

$$l = \frac{2\pi\hbar}{mc}. \tag{43}$$

In form, the value (43) resembles the Compton wavelength of the electron, however, its physical meaning is quite different. In the formula for the Compton wavelength of the electron parameter m – is the rest mass of the electron, but in the formula (43) m – this is the mass, which acquires a photon, when it is exposed to external influences.

In the ESM, the external action is described by rotations in the extended space $G(1,4)$. Above we have considered such rotations from the group $O(1,4)$, and found how the photon mass changes at these rotations. Since the linear parameter l is expressed by the formula (43) through the photon mass, it can be used to find the dependence of this parameter from the values that define these rotations.

Thus, in the case of rotations in the plane (TS), the dependence of the photon mass on the rotation angle θ is determined by the formula $mc^2 = \hbar\omega\sinh\psi$ [14, 17].

Substituting this expression into a formula (43), we obtain the expression for the parameter l through the angle θ.

$$l = \frac{2\pi c}{\omega\sinh\vartheta}. \tag{44}$$

In the case of rotations in the plane (XS), the dependence of the photon mass on the rotation angle ψ is determined by the formula

$mc^2 = \hbar\omega \sinh\psi$ [14, 17]. With its help one can obtain an expression for the parameter l through the angle ψ.

$$l = \frac{2\pi c}{\omega \sin\psi}. \tag{45}$$

We now compare the formulas for the linear parameter (44) with the formula for the photon mass. If we exclude from these formulas the value sinhθ, and as the mass m substitute Planck mass m_{Pl}, then we get the expression

$$l = 2\pi\sqrt{\frac{\hbar G}{c^3}}. \tag{46}$$

for the length l.

The value (46) actually coincides with the Planck length l_{Pl}. Since the Planck mass m_{Pl} and the Planck length l_{Pl} are considered to be the limit values of length and mass, we will also consider the corresponding rotation angle ϑ_{Pl} to be the limit value of such angle. Its value is

$$\sinh\vartheta_{Pl} = \frac{m_{Pl}c^2}{\hbar\omega}. \tag{47}$$

We see that the photons of different frequencies correspond to different angles of limiting rotation ϑ_{Pl}. But the corresponding values m_{Pl} and l_{Pl} are the same and do not depend on the frequency ω.

Within the ESM there is another mechanism of appearing a photon mass. More precisely, not for one photon, but a group of photons. Consider two free photos on. Let them move in Minkowski space in the directions given by the unit vectors $\vec{k}_1$ and $\vec{k}_2$. Then in ESM they correspond to two 5-vector energy-momentum-mass.

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}_1, 0\right). \tag{48}$$

and

$$\left(\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}\vec{k}_2, 0\right). \tag{49}$$

We emphasize that in this case we are not talking about the interaction of these photons with each other, their scattering on each other, etc. Here we are talking only about the fact that in 3-dimensional spaces there are two photons. We consider a system consisting of these photons and try to find a 5-vector that corresponds to it in the ESM. Since these photons do not interact, the energy of the system is equal to the sum of their energies.

Similarly, the momentum of the system is equal to the vector sum of their pulses. Since this is an isolated system, which is not affected by any external forces, it, according to the ideology of ESM, corresponds to the isotropic 5-energy-momentum-mass vector. This vector can be obtained by adding vectors (48) and (49) and adding a mass such that it becomes isotropic.

$$\begin{aligned}&\left(2\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}(\vec{k}_1 + \vec{k}_2, m_{1+2}c\right) = \\ &= \left(2\frac{\hbar\omega}{c}, \frac{\hbar\omega}{c}(\vec{k}_{1+2}\cos\frac{\alpha}{2}, m_{1+2}c\right).\end{aligned} \tag{50}$$

The mass value m_{1+2} must be such that the vector (50) is isotropic. Because

$$\begin{aligned}&|\vec{k}_1 + \vec{k}_2|^2 = 2 + 2(\vec{k}_1, \vec{k}_2) = \\ &= 2 + 2\cos\alpha = 4\cos^2\frac{\alpha}{2},\end{aligned} \tag{51}$$

for the mass of a system of two photons we obtain the expression

$$m_{1+2} = 2\frac{\hbar\omega}{c^2}\sin\frac{\alpha}{2}. \tag{52}$$

Here α is the angle between vectors $\vec{k}_1$, $\vec{k}_2$.

The isotropic 5-vector (59) now takes the form

$$\left(2\frac{\hbar\omega}{c}, 2\frac{\hbar\omega}{c}\cos\frac{\alpha}{2}, 2\frac{\hbar\omega}{c}\sin\frac{\alpha}{2}\right). \tag{53}$$

The expression (52) for the mass of a system of two photons can be obtained using the formula for the mass of a system of n particles [25, 26]

$$m^2c^4 = \left(\sum_{i=1}^{n} E_i\right)^2 + c^2\left(\sum_{i=1}^{n}\vec{p}_i\right)^2. \tag{54}$$

It is with the help of the formula (54) that the expression (53) for the mass of a system of two photons was obtained in [27].

Note that the question of the moment of a system of two photons was studied in [28].

CONCLUSION

According to the ideology of ESM the union of electromagnetic and gravitational fields occurs due to the fact that the interaction of particles

and fields changes their mass. Including the photon, getting into the medium, or in an external field, acquires mass. At the same time, it is being localized. In the empty Minkowski space, an infinite plane wave is compared to the photon, which contains the components $\vec{E}$ and $\vec{H}$ of the electric and magnetic fields. After the photon is exposed to external action, it is localized, acquires mass and, in addition to the fields $\vec{E}$ and $\vec{H}$, it acqires additional field components: the vector field $\vec{G}$ and the scalar field Q.

These 10 fields form a single object, they satisfy the extended system of Maxwell equations and can transform into each other. Each of them interacts with the environment in its own way and, thanks to the presence of additional components, they can penetrate through such barriers that are inaccessible to the usual electromagnetic field. An important role is played by the fact that photons have mass, and in addition to electromagnetic interaction, there is a gravitational interaction between photons and the external environment.

The appearance of a non-zero mass in a photon and the simultaneous change mass of other particles leads to a change in the nature of their interaction. The developed formalism of ESM allows us to take into account these changes.

The ideology of ESM and the formalism developed with its help can be useful in discussing a number of other problems in physics. One of them may be the construction of gauge theories for massive fields.

Also within the framework of the ESM there is an opportunity to take a new look at the nature of wave-particle dualism. The particle located far from the detectors is distributed over a large area of space and exhibits wave properties. Flying up to the detector, it is localized, and behaves like a corpuscle. This mechanism allows us to understand the nature of nonlocality.

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