PHOTON LOCALIZATION IN OPTICAL METAMATERIALS WITH A RANDOM CLOSE TO ZERO REFRACTIVE INDEX

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Abstract. **The results of a theoretical and experimental study of the effect of photon localization in the composite layer of a nanomaterial (PMMA & Ag) with spherical silver nanoparticles are presented. Composite nanomaterial was a metamaterial with a random refractive index close to zero. The photon localization effect considered in this article is fundamentally different from the transverse localization of electromagnetic waves in 2D periodic structures, since it is considered in homogeneous transparent media far from the resonance of their structural elements. It is theoretically shown that at a vacuum-optical medium interface with a random refractive index close to zero the refraction of the wave occurs not according to Snell's law, but with the localization of photons, when the external plane wave is extinguished at a point. In accordance with the uncertainty relation photons propagate in the medium in all directions determined by a random angle of refraction. A condition under which the values of the angles of refraction of waves in a layer become complex quantities is derived. As a result, a parallel beam of light inside the layer is localized in a small region. A 1 mW helium-neon laser with longitudinal polarization of light and a wavelength of 632 nm has been used in experiment. The diameter of the laser beam was 1 mm. The radiation passing through the sample with a thickness of 10 μm in the longitudinal and transverse directions was recorded using a CD camera. When the sample is irradiated a photon localization region is formed in the composite layer, whose linear dimensions (8 μm) are comparable with the layer thickness.**

Keywords: **optical metamaterial with randomly close to zero refractive index, polymer composite nanomaterial, silver nanoparticles, polymethyl methacrylate, photon localization in the composite layer**

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Contents

1. INTRODUCTION

Among unusual transport properties for disordered materials, the phenomenon of Anderson localization predicted by Phillip Anderson in 1957 should be noted [1]. Anderson localization is a disordered phase transition in the behavior of electron transport, in which, unlike the classical diffusion regime with the well-known Ohm's law, in a localized state, the material behaves as an insulator. The effect is based on the interference of electrons that have passed multiple scattering by defects in a solid. However, the localization of electrons in a disordered solid has not been confirmed experimentally. The main condition for the localization of Anderson is the constancy in time of the potential of disorder in the medium, which cannot be fulfilled for electrons due to the inconstancy in time of phonons, loss of electron coherence during their inelastic interaction with the crystal lattice, the resulting nonlinearity due to multiparticle interactions leading to disorder potential. Unlike electrons, photons do not interact with each other. In this case, the phenomenon of photon localization takes place for multiple scattering of electromagnetic waves in a disordered medium. This makes the transfer of photons in disordered materials an ideal model system for studying the localization of Anderson. The first theoretical works on the localization of photons appeared in the mid-eighties of the twentieth century [2, 3]. The first successful experimental works confirming the phenomenon of transverse localization of photons in disordered media were made only 20 years later [4–6]. After that, the model of transverse localization of light in disordered materials became the main one. However, it is important to point out that the transverse photon localization model is associated with transport effects in one or two dimensions. At the same time, the 3*D* localization effect of Anderson with the help of short laser pulses propagating in a volume where the disorder of which is constant in time was studied in [7]. In [8] a review of works describing the interaction between disorder and nonlinearity, localization and enhanced transport of photons in photonic crystals, hypertransport stochastic acceleration of photons due to developing disorder and localization with quantum-correlated photons are given.

The condition of the transverse localization of photons observed in 2*D* disordered optical media can be represented as follows. Let a plane wave propagate along the z axis. The medium in which the wave propagates is a 2*D* periodic structure or photon lattice with modulation of the refractive index Δn. Even with a weak modulation of the refractive index of the order of 10^{-4}, it is possible to observe the localization of photons in such a medium. The transverse wave vector (along the y axis, x) is inversely proportional to the aperture of the wave packet $k_\perp \propto \frac{1}{A}$ and much less wave vector in homogeneous space $k_\perp << k = \omega n_0/c$ refractive index $n_0 > 1$. The length of the photon localization is $\xi_{2d} = l \times \exp\left(\frac{\pi}{2} k_\perp l\right)$, where l - photon free path. This condition can be fulfilled not only in 2*D* periodic structures (photonic crystals), but also in metamaterials with a refractive index $n_\perp << 1$. In this case, the condition is $k_\perp = \omega n_\perp / c << k = \omega n_0 / c$.

Nanotechnologies are known in which silver or gold nanoparticles are embedded in a dielectric matrix, for example, in polymethyl methacrylate [9, 10]. However, in these works the optical properties of the materials were not investigated. Based on the nanotechnologies developed by us [11, 12] samples were obtained, and in which layers of synthesized metamaterials (PMMA and Ag) with silver nanoparticles were deposited on different surfaces and the reflection and transmission spectra of these layers were studied. The study of these samples allowed us to detect unique optical phenomena such as interference of light in thick layers [13], violation of the principle of reversibility of light fluxes [14], amplification and focusing of light in nanostructures with a near-zero refractive index [15]. The basis of these optical properties is the effect of ideal optical transmission, when the reflection of the layer disappears, and the transmittance of the layer is equal to unity regardless of the wavelength, layer thickness, angle of incidence of external radiation and the refractive index of the framing media [16]. Based on a theoretical analysis of the experimental reflection and transmission spectra of composite layers (PMMA & Ag) of various thickness, it was concluded that the metamaterials synthesized by us have a random refractive index close to zero. Thus, in the reflection and transmission spectra of thick (considerably longer than the wavelength) composite layers (PMMA and Ag), interference minima and maxima were detected on the glass substrate. At the same time, in the layers of the polymethylmethacrylate matrix for the same thickness, there were no interference peaks. The arrangement of the minima and maxima in these spectra can be used to calculate the region of permissible values of the refractive index of the layer. In this case, the possible values of the refractive index of the layer are in the range (0, Δn_2), where $\Delta n_2 < 1$.

Known work on mathematical modeling of the types of structural elements for nanostructures [17, 18], in which structural elements in the form of spirals, rods, nanowires and other types, could be embedded in dielectric matrices in order to obtain metamaterials with zero dielectric permeability and magnetic permeability. However, such metamaterials with zero values and will strongly depend on the wavelength, have strong absorption and anisotropy, since the achievement of zero occurs in areas of anomalous dispersion near the natural frequencies of their structural elements. Optical peculiarity of metamaterials synthesized according to the technologies [11, 12] is that the structural elements in these metamaterials are spherical silver nanoparticles of small size (nanoparticle radius of about 2.5 nm). Isolated resonance of such nanoparticles is in the UV region, therefore, in the wavelength range of at least 450 to 1200 nm, these metamaterials are transparent metamaterials, in which the refractive index is significantly (hundreds of times) larger than the absorption index. Silver nanoparticles are uniformly distributed both over the depth of the layer and its surface, and only in a small neighborhood of nanoparticles, whose dimensions are much smaller than the wavelength, fluctuations in the displacements of nanoparticles relative to each other are taken into account. With a mass content of silver in the composite equal to 3%, the average distance between the centers of neighboring nanoparticles is equal to 30 ± 2 nm. At the same time, in the vicinity of nanoparticles in the region, the linear dimensions of which are significantly smaller than the wavelength, there are about 30 discretely distributed nanoparticles with fluctuating displacements from the equilibrium position. These fluctuations can be determined using the structure factor in the calculation of the lattice sums. A formula for the complex refractive index of the optical sphere with inclusions with allowance for the structural factor was derived in [19]. On the basis of this formula, in the particular case of M. Garnett transferring to the well-known formula [20], it was shown that the refractive index of the medium can reach zero and close to zero refractive indices in a wide range of wavelengths.

This article is devoted to a theoretical and experimental study of the effect of photon localization in the composite layer (PMMA and Ag) with spherical silver nanoparticles. The effect of localization of photons, considered in this article, is fundamentally different from the known transverse localization of electromagnetic waves in two-dimensional periodic structures, since it is considered in homogeneous transparent media far from the resonance of their structural elements.

2. THEORY OF PHOTON LOCALIZATION IN THE MEDIUM WITH RANDOMLY A NEAR-ZERO REFRACTIVE INDEX

2.1. Localization of photons at the interface of the vacuum-optical medium

The physical meaning of the effect of photon localization in media with a near-zero refractive index will be considered on the example of a single interface between optical media. It is known that for the boundary of two media with deterministic refractive indices n_1 and n_2 have a place rule of choice $n_1 sin\theta_1 = n_2 sin\theta_2$, where is the angle of incidence of the wave θ_1 and deterministic angle of refraction θ_2 can be determined with any accuracy [21]. The situation is different if one of the media, for example, medium 2 has a random refractive index n_2, accepting values in the range of acceptable values (0, Δn_2), where $\Delta n_2 < 1$. We assume that different values n_2, in the interval n_2, $n_2 + dn_2$, with the same probability is implemented

$dW(n_2) = f(n_2)dn_2$, where $f(n_2)$ – distribution function normalized by the normalization condition $\int_0^1 f(n_2)dn_2 = 1$ on the interval (0,1). In this case, the interval (0, Δn_2) is "cut out" from the general interval (0,1), depending on the layer thickness. With the help of experimental reflection spectra you can determine Δn_2 according to the formula $\Delta n_2 = \lambda_1\lambda_2/2d_2(\lambda_2 - \lambda_1)$, where d_2 – layer thickness, λ_1, λ_2 – wavelengths of neighboring minima in the reflection spectrum. In optical media with a random refractive index, in general, $n_1\sin\theta_1 \neq n_2\sin\theta_2$ [22]. We assume that n_2 and θ_2 are random values for fixed n_1 and θ_1. Moreover, we will assume that the quantities n_2 and θ_2 are statically independent quantities. The interface between two media becomes non-uniform. In this case, the inhomogeneity of the boundary is not associated with any geometric inhomogeneities, but is due to the random refractive index.

It is known [21] that when a plane electromagnetic wave interacts with a sharp interface between two media, the wave is extinguished at this boundary and, instead of it, an electromagnetic wave with a different speed propagates in the medium. This phenomenon is described using the Fresnel optics by the extinction theorem, in which the refractive index of the medium is a deterministic quantity, the interface is homogeneous, which allows us to accurately determine the direction of propagation of the refracted wave. In this case, the entire interface surface participates in the wave cancellation. The situation is different if the medium has a random refractive index in the region of quasi-zero values ($\Delta n_2 < 1$) when falling on the boundary of the light beam $z = 0$ with a certain cross-sectional area S_1. Then, in accordance with the extinction theorem, as shown below, the extinction of the external wave will occur at a point x located at the intersection of the plane of incidence and the surface $z = 0$, satisfying the condition $\Delta n_2 x k_0 \to 0$, where Δn_2 determines the interval (0, Δn_2) of permissible values of the random refractive index of the medium $k_0 = \omega/c$, where c - the speed of light in vacuum, ω - the frequency of the external wave. This means that the external beam of light is focused in close proximity x, and in accordance with the uncertainty relation for the coordinate and momentum of the photons, the light flux with some other cross-sectional area will propagate in the medium.

Taking into account the above averaging procedure, we represent the wave in a medium with a random refractive index as a wave packet for the electric fields of a refracted electromagnetic wave, determined using the refractive indices with a random n_2 and θ_2. Then on the border $z = 0$ we obtain the following relationship between the amplitude of an external plane wave E_{10}^+ and T_0 the amplitude of the refracted wave:

$$E_{10}^+ \exp(-ik_0 x \sin\theta_1) \propto - \\ -\frac{\sin(\varphi_2 + \theta_2)}{2\cos\varphi_2 \sin\theta_2} T_0 F(x, \Delta n_2), \tag{1}$$

where the angle φ_2 is defined by equality $n_2\sin\theta_2 = \sin\varphi_2$, and the vector $\vec{s}$ has components:

$$s_x = -\sin\varphi_2,\ s_y = 0,\ s_z = -\cos\varphi_2, \tag{2}$$

and the function is:

$$F(x, \Delta n_2) = \frac{\cos(-k_0 x \Delta n_2 \sin\theta_2) - 1}{-k_0 x \sin\theta_2} + \\ +i\frac{\sin(-k_0 x \Delta n_2 \sin\theta_2)}{-k_0 x \sin\theta_2}. \tag{3}$$

At point $k_0 x \Delta n_2 \sin\theta_2 = 0$ function (3) reaches a maximum equal to Δn_2, and at points $k_0 x \Delta n_2 \sin\theta_2 = \pi m$, where $m = \pm1, \pm2\ldots$ makes vanish. At the same time, it $\Delta x = 2\pi/k_0\Delta n_2\sin\theta_2$ represents the spatial extent of the wave packet. The smaller $k_0\Delta n_2\sin\theta_2$, then the greater the spatial extent of this wave packet. Given that $k_0\Delta n_2\sin\theta_2 = (\Delta p_x)/2\pi\hbar$ we obtain the uncertainty relation for the pulse and the photon coordinates

$$\Delta x \Delta p_x = 2\pi\hbar. \tag{4}$$

Thus, at the boundary of the vacuum-optical medium with a random refractive index close to zero, the wave refraction does not follow the Snell law, but with the localization of photons, when the external plane wave is extinguished at a point at $k_0 x \Delta n_2 \sin\theta_2 \to 0$, and then according to the uncertainty relation (4) wave is distributed in the medium in all directions determined by a random angle of refraction θ_2.

2.2. Localization of photons inside the composite

We describe the optical properties with a random refractive index close to zero using the integro-differential equation for the propagation of electromagnetic waves [21, 23], and using the procedure of converting the volume integral in this equation into surface integrals for the boundaries 1-2 and 2-3 of the plane-parallel layer 2. We assume that the medium 1, from which the external radiation falls, is a vacuum, and the medium 3 is a semi-infinite optical medium with a refractive index n_3. The field in the composite layer is represented as a superposition of the electric fields $\boldsymbol{E}_2^+$ and $\boldsymbol{E}_2^-$, where $\boldsymbol{E}_2^-$, is the electric field strength of wave, refracted at the boundary of 1-2 layers, and $\boldsymbol{E}_2^-$ is the electric field strength of wave reflected from the border 2-3 of this field. We write the electric field strength in the layer as follows:

$$\boldsymbol{E}_2^{s,p} = f_+^{s,p}\boldsymbol{E}_1^+ a^+(x,z) + f_-^{s,p}\boldsymbol{E}_1^+ a^-(x,z),$$

where the indices correspond to s and p to polarized waves, $\boldsymbol{E}_1^+$ is the electric field strength of the external wave,

$$a^{\pm}(x,z) = \frac{1}{ik_0(\boldsymbol{rs}_T^{\pm})}(\exp[k_0\Delta n_2(\boldsymbol{rs}_T^{\pm})]-1), \quad (5)$$

$k_0 = 2\pi/\lambda$, λ – external radiation wavelength, r – radius-vector of observation points inside the layer, x_2 - the plane of incidence. At the same time $(\boldsymbol{rs}_T^{\pm}) = -x\sin\theta_2 \mp z\cos\theta_2$, $\boldsymbol{s}_T^{\pm}$ – unit vectors along the directions of propagation of the refracted and reflected waves inside the layer, θ_2 is the angle of refraction.

The refractive indexes $f_+^{s,p}$ and reflections $f_-^{s,p}$ inside the layer after the necessary calculations are defined as

$$\begin{aligned} f_+^{s,p} &= t_{23}^{s,p}t_{12}^{s,p}\Delta n_2 + \\ &+\frac{it_{12}^{s,p}t_{23}^{s,p}}{2k_0d_2\cos\theta_2}\ln\left(\frac{1+r_{12}^{s,p}r_{23}^{s,p}\exp[2ik_0d_2\Delta n_2\cos\theta_2]}{1+r_{12}^{s,p}r_{23}^{s,p}}\right), \\ f_-^{s,p} &= \frac{t_{23}^{s,p}t_{12}^{s,p}}{r_{12}^{s,p}}\Delta n_2 - \\ &-\frac{it_{12}^{s,p}t_{23}^{s,p}}{2k_0d_2r_{12}^{s,p}\cos\theta_2}\ln\left(\frac{r_{12}^{s,p}r_{23}^{s,p}+\exp[2ik_0d_2\Delta n_2\cos\theta_2]}{1+r_{12}^{s,p}r_{23}^{s,p}}\right), \end{aligned} \quad (6)$$

where the non-Fresnel reflection and refraction coefficients at the boundaries of the layer in the case of s-polarization of the waves has the form [24]:

$$\begin{aligned} r_{12}^s &= \frac{n_1\cos\varphi_2 - n_2\cos\theta_2}{n_1\cos\varphi_2 + n_2\cos\theta_2}, \quad r_{23}^s = \frac{n_2\cos\theta_2 - n_3\cos\theta_3}{n_2\cos\theta_2 + n_3\cos\theta_3}, \\ r_{13}^s &= \frac{n_1\cos\theta_1 - n_3\cos\theta_3}{n_1\cos\theta_1 + n_3\cos\theta_3}, \quad t_{12}^s = \frac{2n_1\cos\varphi_2}{n_1\cos\varphi_2 + n_2\cos\theta_2}, \\ t_{23}^s &= \frac{2n_2\cos\theta_2}{n_2\cos\theta_2 + n_3\cos\theta_3}, \quad t_{13}^s = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_3\cos\theta_3}, \end{aligned} \quad (7)$$

where n_1, n_3 are the refractive indexes of optical media 1 and 3 above and below the layer, respectively, θ_1 is the angle of incidence of external radiation, θ_3 is the angle of refraction in the underlying medium 3.

Similarly, we have the following formulas for the reflection and refraction coefficients at the boundaries of 1-2 and 2-3 layers for p-polarization of waves:

$$\begin{aligned} r_{12}^p &= \frac{n_1\cos\theta_2 - n_2\cos\varphi_2}{n_1\cos\theta_2 + n_2\cos\varphi_2}, \quad r_{23}^p = \frac{n_2\cos\theta_3 - n_3\cos\theta_2}{n_1\cos\theta_3 + n_3\cos\theta_2}, \\ r_{13}^p &= \frac{n_1\cos\theta_3 - n_3\cos\theta_1}{n_1\cos\theta_1 + n_3\cos\theta_3}, \quad t_{12}^p = \frac{2n_1\cos\varphi_2}{n_1\cos\theta_2 + n_2\cos\varphi_2}, \\ t_{23}^p &= \frac{2n_2\cos\theta_2}{n_2\cos\theta_3 + n_3\cos\theta_2}, \quad t_{13}^p = \frac{2n_1\cos\theta_1}{n_1\cos\theta_3 + n_3\cos\theta_1}. \end{aligned} \quad (8)$$

Formulas (7), (8) are obtained in [24] using the extinction theorem separately for boundaries 1-2 and 2-3 for heterogeneous interfaces on which the Snell law is violated due to the random refractive index of medium 2. In this case the angle of incidence is determined from the ratio $n_1\sin\theta_1 = n_3\sin\theta_3$, and the angle φ_2 is determined from the relation $n_2\sin\theta_2 = \sin\varphi_2$. The last relation to the calculation of surface integrals in the cancellation theorem by the stationary phase method is related [21]. In the transition to deterministic values n_2 and θ_2 formulas (7), (8) coincide with the corresponding Fresnel formulas, in which $\varphi_2 = \theta_1$ and $n_1\sin\theta_1 = n_2\sin\theta_2$. In the non-Frenelle formulas, considering the reflected radiation with the help of the corresponding surface integral and placing the observation points above the layer, the angle φ_2 is related to the random reflection angle θ_R as $\varphi_2 = \pi - \theta_R$.

The reflectivity of a layer with a random refractive index close to zero in the interval (0, Δn_2) is $\Delta n_2 < 1$ calculated as

$$R_{123} = R_{123}^p \cos^2\alpha_i + R_{123}^s \sin^2\alpha_i, \tag{9}$$

where α_i is the angle between the electric vector of the electromagnetic wave and the plane of incidence,

$R_{123}^{s,p} = \left| r_{123}^{s,p} \right|^2$ and

$$r_{123}^{s,p} = r_{12}^{s,p}\Delta n_2 - -i\frac{1-(r_{12}^{s,p})^2}{r_{12}^{s,p}\frac{4\pi}{\lambda}d_2\cos\theta_2}\ln\left(\frac{1+r_{12}^{s,p}r_{23}^{s,p}\exp\left[\frac{4\pi}{\lambda}id_2\Delta n_2\cos\theta_2\right]}{1+r_{12}^{s,p}r_{23}^{s,p}}\right). \tag{10}$$

For natural light $\sin^2\alpha_i = \cos^2\alpha_i = 1/2$ [21].

The transmittance of the composite layer is determined using the following formulas:

$$T_{123} = T_{123}^s \sin^2\alpha_i + T_{123}^p \cos^2\alpha_i,$$

where

$$T_{123}^{s,p} = \frac{n_3\cos\theta_3}{n_1\cos\theta_1}\left| t_{123}^{s,p} \right|^2, \tag{11}$$

with optical transmittance layer

$$t_{123}^{s,p} = -i\sqrt{\frac{n_1\cos\theta_1}{n_3\cos\theta_3}} + \frac{t_{12}^{s,p}t_{23}^{s,p}}{\frac{4\pi}{\lambda}id_2\cos\theta_2\sqrt{-r_{12}^{s,p}r_{23}^{s,p}}}\times$$
$$\times\left\{\ln\frac{1+\exp\left[\frac{2\pi}{\lambda}id_2\cos\theta_2\Delta n_2\right]\sqrt{-r_{12}^{s,p}r_{23}^{s,p}}}{1+\sqrt{-r_{12}^{s,p}r_{23}^{s,p}}} - \right. \tag{12}$$
$$\left. -\ln\frac{1-\exp\left[\frac{2\pi}{\lambda}id_2\cos\theta_2\Delta n_2\right]\sqrt{-r_{12}^{s,p}r_{23}^{s,p}}}{1-\sqrt{-r_{12}^{s,p}r_{23}^{s,p}}}\right\},$$

taking into account the fact that $\Delta n_2 < 1$ we have $r_{12}^{s,p}r_{23}^{s,p} < 0$.

When calculating the reflectivity and transmittance of the layer in the non-Frennel coefficients $r_{1k}^{s,p}$ and $t_{1k}^{s,p}$ (i, k = 1, 2, 3), the values n_2 will be considered equal Δn_2, since the main dependence on the random refractive index of the layer is determined by exponential factors in the reflection and transmission coefficients of the layer. The presence of the first term on the right-hand side of formula (12) is due to the effect of ideal optical transmission when the refractive index tends to zero from the range of permissible values (0, Δn_2).

Formulas (9), (11) satisfactorily describe the experimental reflection and transmission spectra of Fig. 5, 6. Indeed, as can be seen from Figs 5, 6, for some values of the layer thickness, the interference of light in the reflection and transmission spectrum disappears. This is because the angle of refraction θ_2 becomes complex. For complex angles of refraction, we have the following relations:

$$\theta_2 = \theta_2^{'} - i\theta_2^{''},\ \cos\theta_2 = ish\theta_2^{''} = i\sqrt{x_2^2-1},$$
$$x_2 = ch\theta_2^{''},\ \theta_2^{'} = \pi/2,\ \sin\theta_2 = x_2, \tag{13}$$
$$\cos\varphi_2 = \sqrt{1-(n_2)^2x_2^2},\ \sin\varphi_2 = n_2x_2,$$

where $\theta_2^{'}$ is the actual angle of refraction, which determines the direction of wave propagation along the layer, $\theta_2^{''}$ is the angle in the direction of which the wave attenuation occurs. The sign in front of the root in these relations is chosen so that the wave is damped, regardless of its direction of propagation.

Let us calculate the reflective and transmission capacity of the layer using formulas (9), (11), taking into account the above selective properties of these formulas, when the maxima $R_{123}^{s,p}$ and are $T_{123}^{s,p}$ reached at $x_2 \to 1$. In this case, after eliminating the type uncertainty (0/0), we obtain from (10), (12) the following formulas:

$$r_{123}^s = \Delta n_2 - i\frac{\Delta n_2}{\cos\varphi_2}\frac{1}{k_0d_2}\beta_{123}^s,$$
$$r_{123}^p = \Delta n_2 + i\frac{1}{\Delta n_2\cos\varphi_2}\frac{1}{k_0d_2}\beta_{123}^p, \tag{14}$$

where

$$\beta_{123}^s = \ln\left(1 - i\frac{(2\pi/\lambda)d_2}{(1/\cos\varphi_2)+(1/n_3\cos_2)}\right),$$
$$\beta_{123}^p = \ln\left(1 - i\frac{(2\pi/\lambda)d_2(\Delta n_2)^2}{(n_3/\cos\theta_3)+(1/\cos\varphi_2)}\right).$$

The transmittance of the layer when $x_2 \to 1$ after eliminating type uncertainty (0/0) are:

$$t_{123}^s = i\sqrt{\frac{n_1\cos\theta_1}{n_3\cos\theta_3}} - i\frac{\Delta n_2}{n_3\cos\theta_3(\pi/\lambda)d_2}(\alpha_{123}^s - \gamma_{123}^s),$$
$$t_{123}^p = i\sqrt{\frac{n_1\cos\theta_1}{n_3\cos\theta_3}} - i\frac{1}{\Delta n_2\cos\theta_3(\pi/\lambda)d_2}(\alpha_{123}^p - \gamma_{123}^p), \tag{15}$$

where $\alpha_{123}^{s,p}$ and $\gamma_{123}^{s,p}$ correspond to the first and second terms in braces in the formula (12). When $x_2 \to 1$ we get that $\alpha_{123}^{s,p}$ vanishes, and

$$\gamma_{123}^{s} = \ln\left(1 + i\frac{(2\pi/\lambda)d_2}{(1/\cos\varphi_2) + (1/n_3\cos\theta_3)}\right),$$
$$\gamma_{123}^{p} = \ln\left(1 + i\frac{(2\pi/\lambda)d_2(\Delta n_2)^2}{(1/\cos\varphi_2) + (n_3/\cos\theta_3)}\right). \quad (16)$$

The presence of the first term on the right-hand side of formulas (15) is due to the fact that with accurate refractive index vanishing from the integration region $(0, \Delta n_2)$, it is necessary to eliminate the type uncertainty (0/0) in the corresponding layer transmittance. As shown in [16], when the refractive index of the layer is exactly zero, the reflection coefficient of the layer is zero, and the transmittance of the layer is 1, which corresponds to the effect of an ideal optical enlightenment.

Consider the optical properties of the composite layer at complex angles of refraction determined by relations (13). This case corresponds to the minimum transmission of the layer, since in the case of complex angles of refraction a wave propagates along the layer at $\theta_2^{'} = \pi/2$. In this case, part of the external light wave is converted into side waves. In the case of complex angles of refraction, functions (5), which determine the spatial distribution of the field inside the layer, are equal to each other, i.e. $a^+ = a^- = a(x,z)$.

Then

$$|\alpha|^2 = \frac{1}{x^2x_2^2 + |z|^2(x_2^2-1)}\left(\exp\left[-2k_0\Delta n_2|z|\sqrt{x_2^2-1}\right] + 1 - 2\cos(k_0\Delta n_2 x^2 x_2^2)\exp\left[-k_0\Delta n_2|z|\sqrt{x_2^2+1}\right]\right).$$

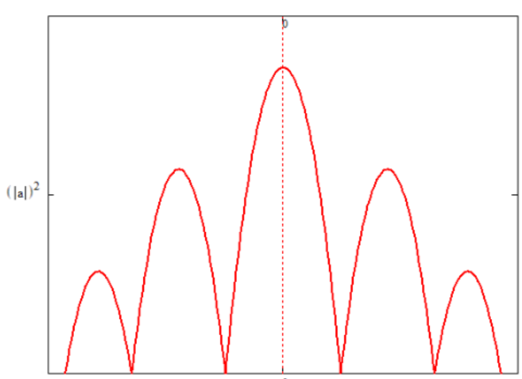

Fig. 1. *Spatial distribution of photons inside the composite layer (on the x-axis - wave phase).*

When $x_2 \to 1$, this function takes the following form:

$$|\alpha|^2 = \frac{2 - 2\cos(k_0\Delta n_2 x)}{x^2}. \quad (17)$$

Fig. 1 shows the dependence of this function on a dimensionless variable or phase of the wave $k_0\Delta n_2 x$.. When $k_0\Delta n_2 x = 2\pi m$, where $m = \pm1, \pm2\ldots$, is the function $|a|^2 = 0$, when $k_0\Delta n_2 x = \pm3\pi$, $\pm5\pi\ldots$ this function is equal $|a|^2 = 4(k_0\Delta n_2)^2/(3\pi)^2$, $4(k_0\Delta n_2)^2/(5\pi)^2$ etc. The linear dimensions of the localization volume are determined with the help of the main maximum of the function (17) as $\Delta x = 2\lambda/(\Delta n_2)$.

3. EXPERIMENTAL DETECTION OF EFFECT OF PHOTON LOCALIZATION IN A COMPOSITE LAYER PMMA&AG

The optical scheme for the localization of photons with a normal incidence of a laser beam on the surface of a composite layer 10 microns thick on a glass substrate is shown in **Fig. 2**. A helium-neon laser with a power of 1 mW with longitudinal polarization of light and a wavelength of 632 nm was used as a coherent radiation. The diameter of the laser beam was 1 mm. Radiation transmitted through the sample in the longitudinal and transverse directions was recorded using a CD camera.

Experimental reflection and transmission spectra of layers (PMMA & Ag) of various thickness on a glass substrate were measured on an Ocean Optics QE65000 spectrophotometer in

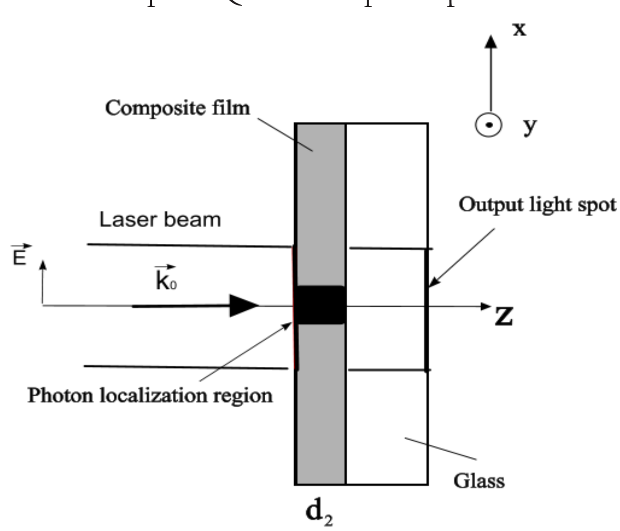

Fig. 2. *Optical scheme of laser irradiation of the PMMA&Ag layer.*

a wide wavelength range from 300 to 1000 nm. For comparison, the reflection and transmission spectra of the layers of the material of the PMMA matrix of the same thickness were measured separately on glass with a normal incidence of light.

When the sample is irradiated in the composite layer, a photon localization region is formed, the linear dimensions of which are comparable with the layer thickness d_2 = 10 μm (see Fig. 7).

4. RESULTS AND DISCUSSION

Fig. 3 presents the theoretical spectra composite layer reflections calculated according to the formulas (9) and (10), depending on the length wave and parameter selection x_2. As can be seen from the figure as x_2 is closer to one the reflectivity of the layer decreases. When x_2 = 1 the reflectivity of the layer with numerical values of physical quantities indicated in **Fig. 4** at a wavelength of λ = 632 nm equal to R_{123} = 1.926%.

Fig. 4 presents theoretical transmittance spectra of the composite layer, calculated using formulas (15) and (16). Using formulas (15), (16), we define the transmittance value of this layer at x_2 = 1.01, equal to T_{123} = 95.37% (Fig. 4*a*). As can be seen from Fig. 4*b* with x_2 = 1.0001 throughput layer increases significantly, reaches units and practically does not depend on the wavelength in wide wavelength range from 400 to 1200 nm.

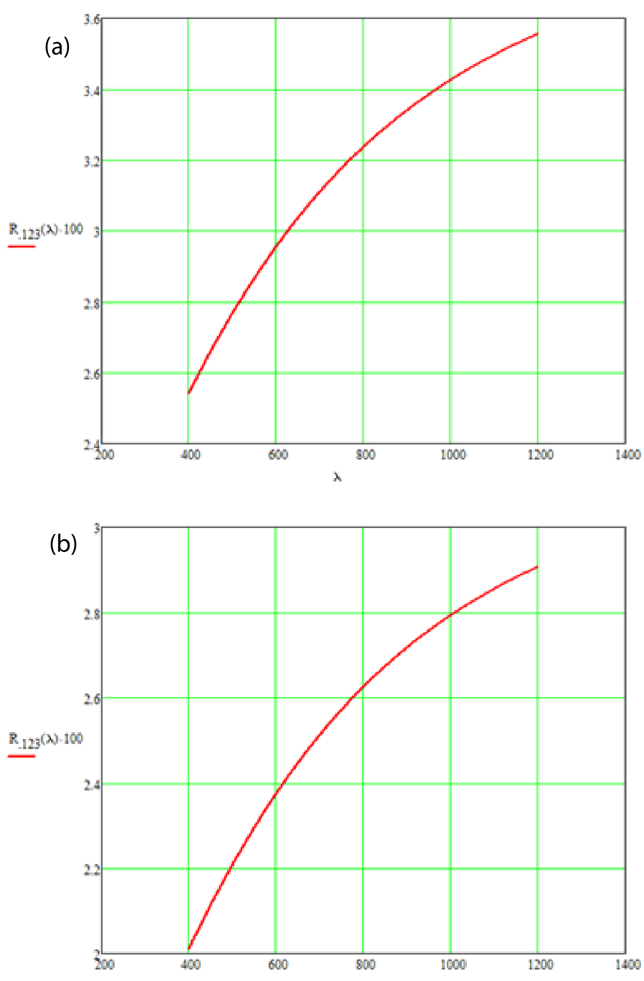

Fig. 3. *The theoretical reflectivity of a composite layer with a random refractive index close to zero, depending on the wavelength for a) x_2 = 1.01 and b) x_2 = 1.0001. The layer thickness is d_2 = 10 μm, the refractive index of the glass substrate is n_2 = 1.5, Δn_2 = 0.1625, the wavelengths are indicated in nanometers.*

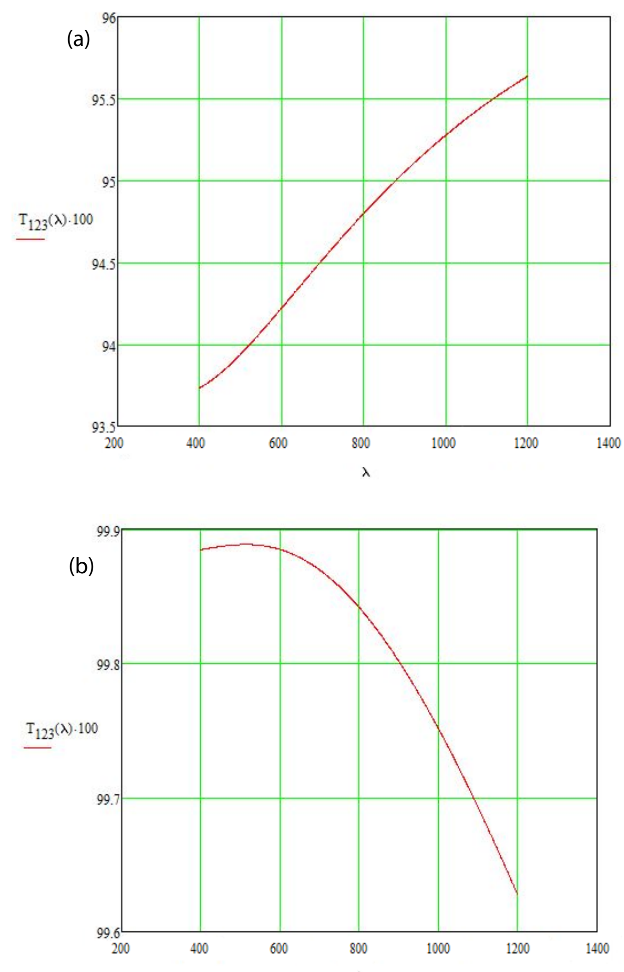

Fig. 4. *The theoretical transmittance of the composite layer with a random refractive index close to zero, depending on the wavelength for a) x_2 = 1.01 и b) x_2 = 1.0001. The layer thickness is d_2 = 10 μm, the refractive index of the glass substrate is n_2 = 1.5, Δn_2 = 0.1625, the wavelengths are indicated in nanometers*

The ratio between the reflectivity and the transmittance of a composite layer with a random close to zero refractive index is written as follows:

$$R_{123} + T_{123} + W_{123} = 1, \qquad (18)$$

where W_{123} is the part of external radiation propagating along the layer. Using the limit values R_{123} = 0.019 and T_{123} = 0.95 for a sample with a layer thickness of d_2 = 10 μm for a normal incidence of a laser beam with a wave length λ = 632 nm, we get W_{123} = 0.031.

Wave flow effect for the flat surface of the composite layer (PMMA & Ag) was discovered by us experimentally [16].

Fig. 5 shows the experimental reflection and transmission spectra of samples with composite Ag&PMMA layers on glass substrates with respect to the spectral values of reflection and transmission of a glass substrate. When the layer thickness of the d_2 = 5.20 mkm, as can be seen from Fig. 5*a*, the interference of light is detected and the refractive index of the composite can be calculated from the location of the minima. The formation of these interference minima is due to the fact that the range of permissible values (0, Δn_2) of the random refractive index of the medium is such that $\Delta n_2 << 1$.

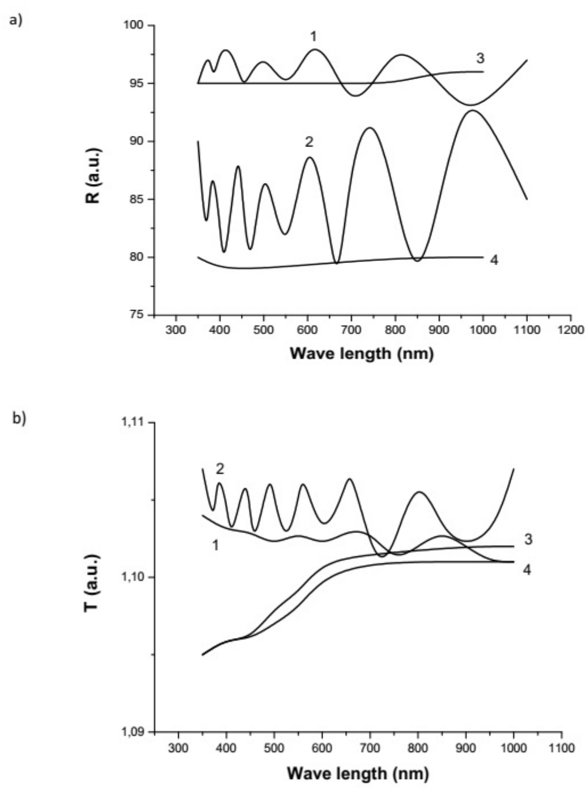

Fig. 5. *The experimental reflection spectra (a) and transmittance (b) of the composite layers (PMMA & Ag) with different thickness of silver nanoparticles on a glass substrate. The spectra are presented in relative units with respect to the optical transmission and reflection of a glass substrate 2 mm thick. The relative reflectivity of the glass substrate in Fig. 5a is 100, and its own value is 0.07. Curve 1 - layer thickness d = 5 μm; curve 2- d = 20 μm; curve 3- d = 10 μm; curve 4- d = 50 μm*

In the experimental reflection and transmission spectra in Fig. 5, where the interference of light is observed in formulas (9), (11), the angles of refraction should be considered valid. We define these angles using the following relations:

$$\cos\theta_2 = \sqrt{1-y_2^2}, \;\; y_2 = \sin\theta_2, \;\; y_2 \le 1,$$

$$\sin\varphi_2 = \Delta n_2 y_2, \;\; \cos\varphi_2 = \sqrt{1-(\Delta n_2)^2 y_2^2}.$$

The value $x_2 = 1$ separates the areas of complex and real angles of refraction in the layer. At $x_2 > 1$ angles of refraction θ_2 are complex, and at $x_2 < 1$ angles of refraction are real. When $x_2 = 1$ the angle θ_2'' that determines the direction of attenuation of the wave is equal to zero, that is the attenuation of the wave propagating along the layer occurs in the direction perpendicular to the surface of the layer.

The angle θ_2'' is a random variable, varying from $-\pi/2$ to $+\pi/2$, including θ_2'' = 0. In this case, the value $x_2 = \mathrm{ch}\,\theta_2''$ varies from 0 to 2.53. However, as can be seen from formulas (10), (12), the reflection and transmission coefficients of a layer with a random close to zero refractive index have a sharp maximum at x_2 = 1, that is, at θ_2'' = 0. This means that when a random variable x_2 changes, a value x_2 = 1 is most likely to be realized, which allows us to simplify the calculation of the reflectance and transmittance of the layer, as well as the field inside the layer with reflection and refraction coefficients $f_{\pm}^{s,p}$ (see relation (6)).

As follows from the transmission spectra of Fig. 5*b* of composite layers (PMMA&Ag) with silver nanoparticles at a thickness of d_2 = 10, 30, 50 μm, the relative transmittance of the layers with respect to the optical transmittance of the glass substrate is greater than 1 and practically does not depend on the wavelength. This means that in these layers an amplified optical transmission of light is detected in a wide range of wavelengths. In the polymer layers of the same thickness, the relative transmittance $T_{PMMA/glass}/T_{glass} < 1$, that is, the deposition of polymer layers on the surface of the glass substrate leads to additional absorption of *PMMA/glass* samples. Thus, the addition of silver nanoparticles to the polymer matrix leads to an increase in the transparency of thick composite layers synthesized according to the nanotechnology developed by us.

Fig. 6 shows the experimental reflection and transmission spectra of samples with layers of PMMA polymer on glass substrates with respect to the spectral values of reflection and transmission of the glass substrate itself. In the polymer layers of the same thickness, as can be seen from Fig. 6, the interference of light is not detected, since the refractive index of this polymer is 1.49.

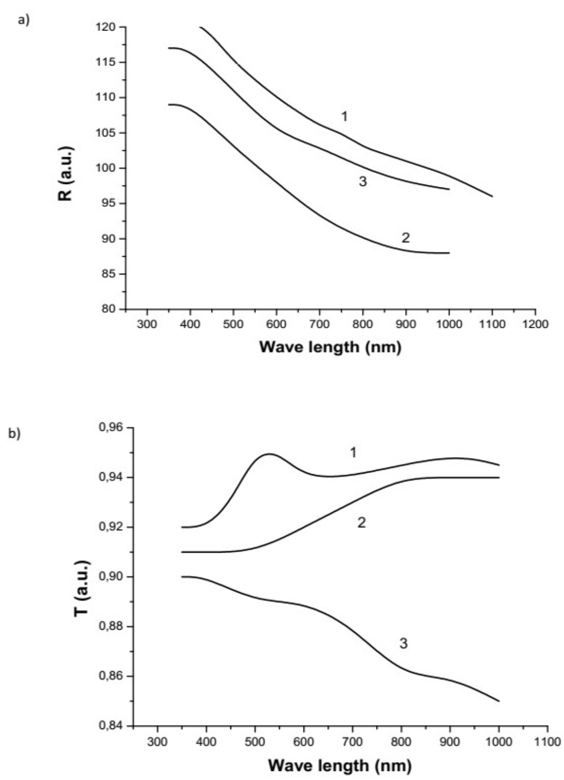

Fig. 6. *Experimental reflection (a) and transmittance (b) spectra of polymer layers of different thickness on a glass substrate. The experimental spectra are presented in relative units with respect to the reflectivity and transmittance of the glass substrate. The relative reflectivity of the glass substrate in Fig. 6a is 100, and its own value is 0.07. Curve 1 - layer thickness d = 5 μm; curve 2- d = 10 μm; curve 3- d = 50 μm.*

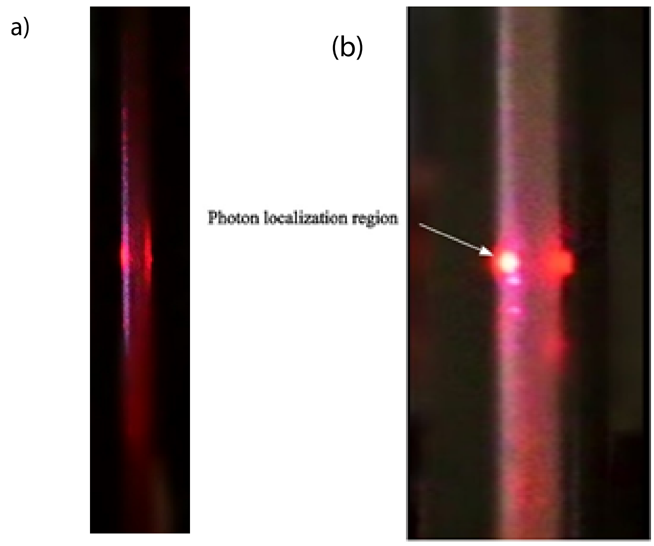

Fig. 7. *Localization of photons in the PMMA&Ag layer at a normal incidence of a laser beam on the surface of the layer (photo): a) Photo of a PMMA/glass sample with a polymer layer; b) Photo sample (PMMA & Ag) / glass with a composite layer. The arrow indicates the region of photon localization.*

Fig. 7*a* shows a photograph of a PMMA polymer layer on glass, where the region of photon localization is absent. Fig. 7*b* shows a photograph of the photon localization region in the composite layer. We will consider the effect of photon localization in the composite layer as a spatial transformation of the energy density of external radiation, when the beam of external radiation is localized in a small region inside the layer. At $\lambda = 632$ nm and for $\Delta n_2 = 0.1625$ a composite layer with a thickness of $d_2 = 10$ μm, we obtain a $\Delta x = 7.778$ μm when linear dimensions of the photon localization region smaller than the layer thickness. Thus, the photon localization region is much smaller than the diameter of the laser beam irradiating the composite layer.

5. CONCLUSION

This article presents theoretical and experimental evidence for the existence of a photon localization effect in a metamaterial layer with a random refractive index close to zero. The detection of this effect indicates that in such optical media the usual beam path characteristic of Fresnel optics is violated. As a result, a parallel beam of light inside the layer is localized in a small region.

A violation of the principle of reversibility of light fluxes in the composite layer was experimentally discovered in [14], and the explanation of this phenomenon was associated with the effect of photon localization. This article provides direct evidence of the existence of a photon localization effect in optical media with a random refractive index close to zero.

As follows from the conservation law (18), the localization of photons inside the composite layer is accompanied by light bending around the flat surface of the layer.

Acknowledgments

The work was performed as part of the state task.

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