

# UNIVERSAL MECHANISM OF REALIZATION OF NUCLEAR REACTIONS AT LOW ENERGY

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*Abstract.* A universal method of short-term and very significant attenuation and suppression of the tunneling effect in the nuclear interaction of low-energy particles by using coherent correlated states (CCS) formed with a weak controlled pulsed or periodic action on these particles or the environment is considered. This mechanism explains all the detected features of successful nuclear physics experiments conducted at low energy: an anomalously high probability of these reactions, a very significant suppression (as compared with similar reactions at high energy) of the accompanying gamma radiation, and the complete absence of radioactive daughter isotopes, formed in these reactions.

*Keywords:* nuclear reactions at low energy, coherent correlated states, the tunnel effect

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## 1. INTRODUCTION

Numerous successful experiments on realization of nuclear reactions at low energy (LENR), some of which have confidently emerged from the "child" age of laboratory experiments and have manifested themselves at the industrial level (this applies in particular to the experiments of A. Rossi), up to now are not based on a reliable theoretical model that adequately explains non-trivial results that are not consistent with the traditional models of nuclear physics.

Among well-known LENR problems, the most frequently considered is the reason of the anomalously high probability of overcoming the Coulomb potential barrier at interaction of charged particles with low energy. The "standard" approach of nuclear physics, which is very successful in the energy range  $E > 1$  keV, is unable to answer this question taking into account that the typical equilibrium particle energy in LENR problems does not exceed  $E \approx kT \approx 0.1$  eV, which leads to very small probability of a tunnel effect that is not comparable with the experimental results.

There are several dozen fairly well-developed theoretical models that, on completely different principles (including such exotic ones as taking into account the anisotropy of quark interaction, the existence of unknown elementary particles, or the presence of hypothetical super deep "Dirac" levels of electrons in a hydrogen atom) try to solve this paradox.

Other, even more exotic paradoxes (first of all, the complete absence of radioactive daughter isotopes in observed LENR reactions, which is completely unusual for "standard" nuclear physics; a very strong (by many orders of magnitude) suppression of gamma-radiation, concomitant and typical for specific nuclear reactions; very strong suppression of the neutron channel of the most studied reaction of dd fusion) in these models are practically not considered, confined to a very important but not the only problem of abnormally high transparency of the Coulomb barrier

Obviously, such one-sided approach is insufficient. The nontriviality of these unexplained paradoxes can not be ignored, since the lack of an adequate explanation of them is equivalent to the lack of understanding of these processes, and hence the impossibility of their optimization and safe large-scale use!

In [1-13], a general and rather universal mechanism for LENR optimization based on coherent correlated states (CCS) of interacting particles was considered. This mechanism provides a high probability of LENR and can be applied with the same efficiency to different experiments. It should be noted that the CCS method makes it possible to explain all mentioned above paradoxes on the basis of standard quantum mechanics and modern nuclear physics without involving fantastic heuristic models.

## 2. FORMALISM AND GENERAL REGULARITIES OF CCS USING IN QUANTUM-MECHANICAL SYSTEMS

In atomic and nuclear physics the well-known Heisenberg uncertainty relations are often used for the coordinate and momentum, as well as energy and time (1927)

$$\delta q \delta p \geq \hbar/2, \quad \delta E \delta t \geq \hbar/2 \quad (1a)$$

and its generalization, made in 1929 by Robertson for arbitrary dynamical variables  $A$  and  $B$

$$\delta A \delta B \geq |\langle [\hat{A}\hat{B}] \rangle|/2, \quad \delta K = \sqrt{\sigma_K}, \quad \sigma_K = \langle (\hat{K} - \langle K \rangle)^2 \rangle. \quad (1b)$$

In 1930, Schrödinger and Robertson independently generalized the relation (1b) and obtained a more universal inequality, called the Schrödinger-Robertson uncertainty relation [14, 15]

$$\sigma_A \sigma_B \geq |\langle [\hat{A}\hat{B}] \rangle|^2 / 4(1-r^2), \quad r = \sigma_{AB} / \sqrt{\sigma_A \sigma_B}, \quad \sigma_{AB} = (\langle \hat{A}\hat{B} + \hat{B}\hat{A} \rangle) / 2 - \langle A \rangle \langle B \rangle, \quad 0 \leq |r| \leq 1, \quad (2)$$

in which the value  $r$  is the correlation coefficient between  $A$  and  $B$ . This parameter determines the degree of mutual statistical connection between the dynamic variables  $A$  and  $B$  and determines the restriction on the product of the variances of these quantities. In the special case,  $A = q$ ,  $B = p$ ,  $\langle q \rangle = 0$ ,  $\langle p \rangle = 0$ ,  $\delta q = \sqrt{\sigma_q}$ ,  $\delta p = \sqrt{\sigma_p}$  relations (1) and (2) reduce to the modified Heisenberg uncertainty relation with the correlation coefficient  $r_{pq}$

$$\delta q \delta p \geq \hbar / 2 \sqrt{1-r_{pq}^2} \equiv \hbar^* / 2, \quad \hbar^* = G_{pq} \hbar, \quad G_{pq} = 1 / \sqrt{1-r_{pq}^2}. \quad (3a)$$

Accordingly, at  $A = E$ ,  $B = t$ , we have relation

$$\delta E \delta t \geq \hbar / 2 \sqrt{1-r_{Et}^2} \equiv \hbar^* / 2, \quad \hbar^* = G_{Et} \hbar, \quad G_{Et} = 1 / \sqrt{1-r_{Et}^2}, \quad (3b)$$

which depends on the correlation coefficient  $r_{Et}$ .

According to the generally accepted terminology, the term CCS refers to states for which relations (2) and (3) become equalities, but usually the term CCS is applied to any state with  $|r| \rightarrow 1$ . According to the same interpretation, the quantum state, which

converts relations (2) and (3) into equalities, is traditionally called the coherent state (CS) – a synonym for the "compressed" state, which is characterized by a minimum product of variances or rms fluctuations and maximum closeness to the classical state of the particle in the potential well. The principal difference between the uncertainty relations of Heisenberg-Robertson and Schrödinger-Robertson is clearly characterized by the coefficient of correlation efficiency  $G = 1/\sqrt{1-r^2}$  [9-12].

It increases from the value  $G = 1$  at  $r = 0$ , which corresponds to the absence of correlation, up to  $G \rightarrow \infty$  at full correlation (at  $|r| \rightarrow 1$ ).

The value of  $G$  characterizes the increase of the amplitudes of the fluctuations of dynamic variables  $A$  and  $B$ , and its importance follows from the following simple example demonstrating the efficiency of CCS for optimizing nuclear reactions at low energy.

In the case  $A = q, B = p, \langle q \rangle = 0, \langle p \rangle = 0$ , the following simple estimate for the lower limit (minimum value) of the kinetic energy fluctuation of a particle of mass  $M$  localized within the spatial interval  $\delta q$  follows from (3a)

$$\delta T^{(\min)} = (\delta p)^2 / 2M = G^2 \hbar^2 / 8M(\delta q)^2. \quad (4)$$

In particular, when a proton with a mass  $M_p$  is localized in an interatomic, typical for condensed matter, region  $a \approx 1.5\text{\AA}$  (in this case,  $\delta q \leq 0.75\text{\AA}$ ), the fluctuation of the kinetic energy of a particle that are in CCS with  $1 - |r| \approx 10^{-7}$  (which corresponds to a very large (but real) correlation efficiency  $G = 2240$ ) corresponds to the value  $\delta T_{|r|=10^{-7}}^{(\min)} \approx 5\text{keV}$ . It is necessary to note that this value can be obtained for an arbitrarily small (including zero) temperature of the medium in which the given potential well is located.

This value, even at this lower limit, is comparable with the optimum temperature of a thermonuclear plasma in a tokamak achieved by using of a very large real energy. If the correlation coefficient is greater, the value

$\delta T_r^{(\min)}$  will be even greater. For comparison, we point out that in the absence of CCS (ie, at  $r = 0$ ), the corresponding fluctuation of the kinetic energy  $\delta T_{r=0}^{(\min)} \approx 0.001\text{eV}$  will be incomparably smaller. It should be emphasized that usually [3, 7, 8] the real value of  $\delta T_{|r|\neq 0}$  is much higher  $\delta T_{|r|\neq 0}^{\min}$ .

For particles with a smaller mass (in particular, for an electron with a mass  $M_e$ ), the minimum fluctuation under the same conditions and the presence of a CCS corresponds to large relativistic energy

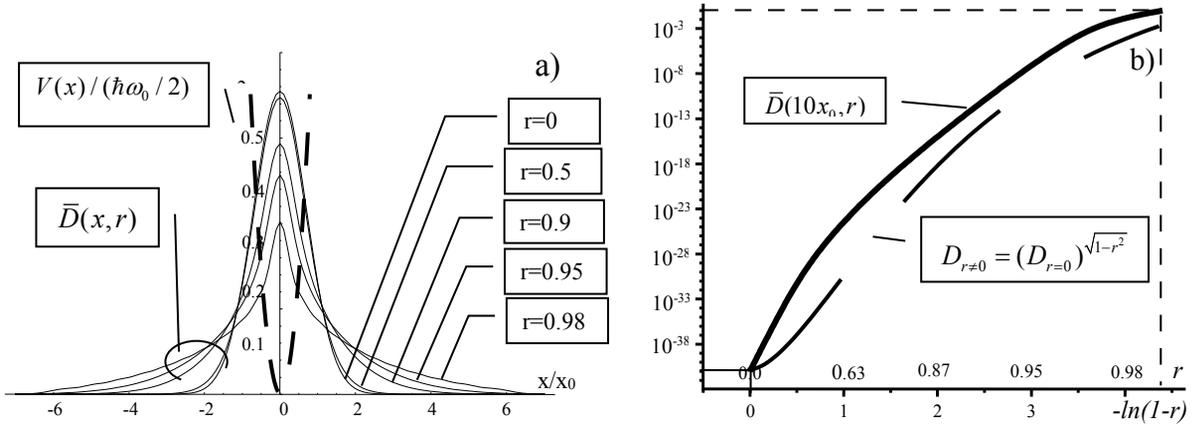
$$\begin{aligned} \delta T_{|r|=10^{-7}}^{(\min)} &= \sqrt{(\delta p)^2 c^2 + M_e^2 c^4} - M_e c^2 = \\ &= M_e c^2 [\sqrt{1 + G^2 \hbar^2 / 4M_e^2 c^2 (\delta q)^2} - 1] \approx \\ &\approx 2.23\text{MeV}, \end{aligned} \quad (5)$$

From the first hand it seems that this energy is sufficient for, for example, proton neutronization  $p + \beta^- + \delta T \rightarrow n + \nu$ . On the other hand, it will be shown below that such reaction due to fluctuations  $\delta T$  is impossible.

Another method of approximate estimation of the effectiveness of the CCS influence on the tunnel effect and subsequent nuclear transmutation is based on taking into account the formal substitution  $\hbar \rightarrow \hbar^* \equiv \hbar / \sqrt{1-r^2} \equiv G\hbar$  in the expression for the tunneling probability  $D$  through a high potential barrier. In [3, 7, 8], using an example of a particle localized in a parabolic well, it was shown that the direct use of such substitution in the formula for the probability of a tunnel effect in the sub-barrier region  $L(E)$  in a nuclear field of radius  $R$

$$\begin{aligned} D_{r \neq 0} &\approx \\ &\approx \exp \left\{ -\frac{2\sqrt{1-r^2}}{\hbar} \int_R^{R+L(E)} \sqrt{2M\{V(q) - E\}} dq \right\} = \\ &= (D_{r=0})^{\sqrt{1-r^2}} \equiv \sqrt[G]{D_{r=0}} \end{aligned} \quad (6)$$

is in good agreement with the results of an independent strict quantum-mechanical



**Fig. 1.** a) Average probability density  $\bar{D}(x,r) \equiv \langle |\Psi(x,t,r)|^2 \rangle_t$  in the potential well  $V(x) = (\hbar\omega_0/2)(x/x_0)^2$  and in the region under the barrier for correlated and uncorrelated states of a particle with different values; b) Dependence of the time-averaged probability density of particle localization (tunneling probability) on the correlation coefficient  $0 \leq r \leq 0.987$  in the subbarrier region with the coordinate  $x = 10x_0$ . The solid line is the result of the exact calculation, the discontinuous line is the result of the approximation (6).

calculation of  $D_{r \neq 0}$  under the condition  $D_{r=0} \ll 1$ .

**Fig. 1a** presents the results of the corresponding calculation showing the change in the time-averaged probability density of the subbarrier particle localization  $\bar{D}(x,r) \equiv \langle |\Psi(x,t,r)|^2 \rangle_t$  in the parabolic potential well  $V(x) = m\omega^2 x^2/2$ , and Fig. 1b – change of this value in the test point  $x = 10x_0$  deep under the barrier, depending on the correlation coefficient (here  $x_0 = \sqrt{\hbar/m\omega}$ ).

From the obtained data it follows that even with a relatively small increase in the correlation coefficient in the interval  $0 \leq r \leq 0.987$ , which corresponds to a small change in the correlation efficiency factor  $0 \leq G \leq 6.2$ , the barrier transparency increases from  $10^{-36}$  to 0.01. In a similar way, it is easy to determine that the probability of the tunnel effect  $D_{r=0,G=1} \approx 10^{-500}$ , typical for a low energy and large charges of interacting particles, increases to  $D_{G=100} \approx 0.3$  in the case of CCS formation, which allows to realize practically any nuclear reaction.

The physical mechanism providing such effect is associated with the formation of very large fluctuations of both momentum and energy of a particle in a potential well in a specially organized superposition coherent correlated state. This formation is related to the mutual enhancement (constructive

interference) of the partial fluctuations of the kinetic energy and momentum corresponding to different eigenstates of such superposition. The result of the interference is the formation of continuously repeated giant fluctuations of the kinetic energy, sufficient for passage through a high and wide potential barrier. It is interesting to note that the formation of a CCS is not connected with an increase of the average particle energy in a given system, but is due to the synchronization of a large number of random fluctuations.

A clear and slightly simplified illustration of such process can be obtained by analyzing the features of the formation of synchronized fluctuations of the kinetic energy of a particle in a one-dimensional parabolic potential well. Each of the  $N$  eigenfunctions of the particle  $\psi_n(q, t)$  in the potential well is characterized by the instantaneous value of the fluctuation of the momentum  $\Delta \vec{p}_n(t)$  with the dispersion  $\sigma_{p_n}(t) = \langle \{ \Delta \vec{p}_n(t) - \langle \Delta \vec{p}_n(t) \rangle \}^2 \rangle = \langle [\Delta \vec{p}_n(t)]^2 \rangle$ . In such system the mean value  $\langle \Delta \vec{p}_n(t) \rangle = 0$ . Formation of the coherent correlated state of the particle leads to phase coincidence and coherent addition (constructive interference) of the momentum fluctuations  $\Delta \vec{p}(t) = \sum_n^N \Delta \vec{p}_n(t)$  for a large number  $N \gg 1$  of different eigenfunctions  $\psi_n(q, t)$  forming a superpositional coherent

correlated state  $\Psi_{corr}(q, r, t)$ . A consequence of this interference is the condition  $\langle \Delta \vec{p}_n(t) \Delta \vec{p}_m(t) \rangle_{corr} > 0$ , that leads to the formation of very large fluctuations of the dispersion of the total momentum of the particle

$$\begin{aligned} \sigma_{p(corr)} &= \left\langle \left\{ \sum_n^N \Delta \vec{p}_n(t) \right\}_{corr}^2 \right\rangle = \\ &= \sum_n^N \sum_{m \neq n}^N \langle \Delta \vec{p}_n \Delta \vec{p}_m \rangle_{corr} + \sum_n^N \langle (\Delta \vec{p}_n)^2 \rangle \approx \\ &\approx N^2 \langle \Delta \vec{p}_n \Delta \vec{p}_m \rangle_{corr} + N \langle (\Delta \vec{p}_n)^2 \rangle \sim N^2, \quad N \gg 1. \end{aligned} \quad (7)$$

The presence of such fluctuations of the momentum at the moments of complete synchronization leads to a sharp increase of the probability of tunneling due to the short-term (fluctuation) formation of states with a very large kinetic energy fluctuation

$$\begin{aligned} \langle \Delta T(t) \rangle_{corr} &\equiv \langle (\Delta \vec{p}(t))^2 / 2M \rangle_{corr} = \\ &= N^2 \langle \Delta \vec{p}_n(t) \Delta \vec{p}_m(t) \rangle_{corr} / 2M + \\ &+ N \langle (\Delta \vec{p}_n)^2 \rangle / 2M \sim N^2 \end{aligned} \quad (8)$$

at constant (small) average kinetic energy of the particle. We note that, in the absence of coherence between different eigenstates, there is an obvious condition  $\langle \Delta \vec{p}_n \Delta \vec{p}_m \rangle_{noncoher} \equiv 0$ , that leads to a trivial relation

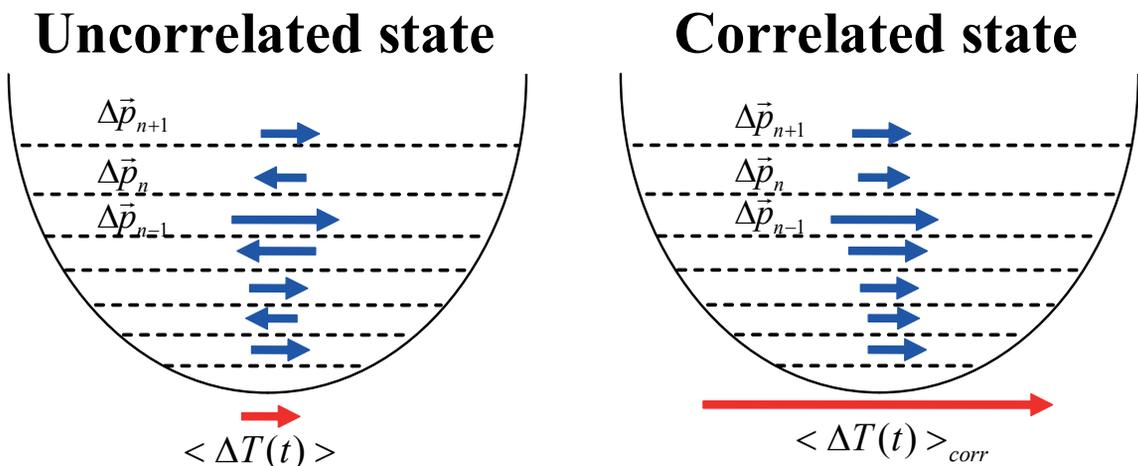
$$\begin{aligned} \langle \Delta T(t) \rangle_{noncoher} &= N \langle (\Delta \vec{p}_n)^2 \rangle / 2M = \\ &= N \langle \Delta T_n(t) \rangle. \end{aligned} \quad (9)$$

An exact calculation using the density matrix leads, naturally, to the similar results. These results are presented in a symbolic form in **Fig. 2**.

The above discussion of the CCS specifics concerns the problem of optimizing the tunnel effect, which is the main problem, but not the only one, characterizing the features of LENR.

It is easy to see that other features of LENR (first of all the absence of daughter radioactive isotopes) also follow from the specifics of CCS. In particular, from the basic relation (3b) it follows that the possibility of realizing the LENR due to the virtual kinetic energy  $\delta E \equiv \delta T_{|r| \neq 0}$  is limited by the conservation laws for the entire system. Very important is the fact that this virtual energy "exists" in the given system (that is, it can have a certain influence on different processes) a finite time  $\delta t$ . As a consequence, any process using  $\delta E$  can be realized only if during the reaction carried out by this virtual energy, the reaction energy  $\Delta E$  that is not less than  $\delta E$  is released and the time of "return" to the system under consideration of this virtual energy (in fact, the duration of the reaction with the release of energy) does not exceed the value of  $\delta t$ .

This result, with respect to the nuclear reaction, corresponds to the fact that the

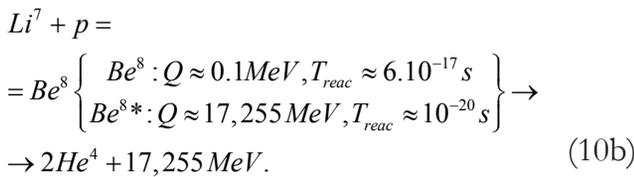
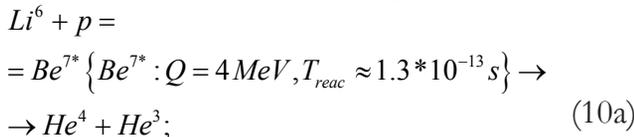


**Fig. 2.** The scheme for the formation of large fluctuations of the kinetic energy of a particle in a coherent correlated state (on the right), due to the synchronization of fluctuations at different energy levels of the superposition state.

total time of the reaction  $T_{\text{total}}$  (including the time of approach of the particle to the barrier  $t_1$ , the duration of the passage through the barrier  $t_2$  and the time of the reaction itself with the release of  $T_{\text{reac}}$  energy) should not exceed  $\delta t$ . This requirement, taking into account the very short duration  $\delta t$  of a large fluctuation amplitude  $\delta T_{|r|\neq 0}$ , imposes very stringent conditions on such processes and automatically excludes the possibility of non-optimal reactions.

The scheme of this scenario is shown in Fig. 3.

For example, let us consider the features of the course of LENR reactions involving protons and isotopes  ${}^6\text{Li}$  and  ${}^7\text{Li}$ . The general scheme of these reactions is the following



In the "ordinary" uncorrelated state, the probability of the tunneling effect for these reactions at a low temperature of 300...1000 K is extremely small and does not exceed  $D_{r=0} \approx 10^{-200} \dots 10^{-100}$ . Let us take as an estimate that for the rapid realization of these reactions it is necessary

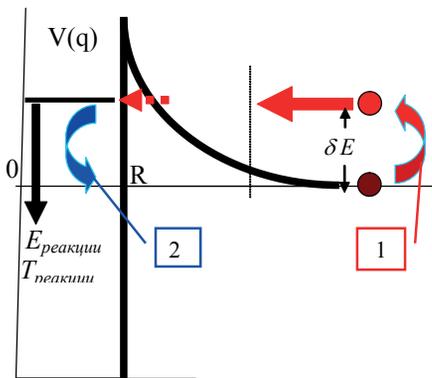


Fig. 3. The scheme of "birth" and "disappearance" of virtual energy during the motion of a particle through the Coulomb barrier with a subsequent short-term nuclear reaction. 1 - "Birth" of virtual energy  $\delta E$ ; 2 - "disappearance" of virtual energy  $\delta E$  within the interval  $\delta t$  after "birth".

for proton to have an energy  $\delta E \approx 10 \text{ KeV}$ . In the case of using the Heisenberg uncertainty relation  $\delta E \delta t_{r=0} \geq \hbar/2$ , such fluctuation can exist for a time  $\delta t_{r=0} \approx \hbar/2\delta E \approx 5 \cdot 10^{-21} \text{ s}$ . At this energy, the minimum total reaction time is equal to  $T_{\text{total}} = T_{\text{reac}} + t_1 + t_2 \approx T_{\text{reac}} + L(\delta E)/v(\delta E) \approx 10^{-18} \text{ s}$ .

It is seen from the reaction schemes (10a) and (10b) that for such value of  $T_{\text{total}}$  the necessary condition  $T_{\text{total}} < \delta t_{r=0}$  is not satisfied for both reactions and they are impossible.

In the correlated state, with an achievable value  $r \approx 0.99999$ , the same energy fluctuation  $\delta E \approx 10 \text{ KeV}$  can exist during  $\delta t_{r=0.99999} \approx \hbar/2\delta E\sqrt{1-r^2} \approx 2.5 \cdot 10^{-18} \text{ s}$ . Comparing this value with the total duration  $T_{\text{total}} \approx 10^{-18} \text{ s}$  of the reaction (10b)  ${}^7\text{Li} + p = 2{}^4\text{He}$ , we come to the conclusion that  $T_{\text{total}} \approx 10^{-18} \text{ s} < \delta t$  (ie the total reaction time is less than the fluctuation time that stimulates this reaction), and the flow of such a reaction is consistent with the law of conservation of energy and the corresponding uncertainty relation.

In contrast, for reaction (10a), the opposite condition  $T_{\text{total}} \approx 10^{-13} \text{ s} \gg \delta t$  takes place and such reaction is impossible in principle due to the formation of CCS.

These results fully coincide with the data of very detailed experiments [15] conducted for 32 days in Lugano for the examination of the A. Rossi installation, in which very efficient processing of the  ${}^7\text{Li}$  isotope was observed, which is characterized by a short reaction time, and complete absence of reactions involving the  ${}^6\text{Li}$  isotope.

Similarly, it is easy to show that this same selection rule prohibits the realization of LENR in reaction channels involving any other isotopes and elements that have a longer reaction time exceeding  $\delta t$ . It is obvious that, taking into account the smallness of  $\delta t$  even in systems with a large correlation coefficient, the reactions passing through the stage of formation of long-lived radioactive isotopes fully fall under this prohibition.

This mechanism also makes it possible to understand why gamma radiation is strongly suppressed in LENR reactions. The fact is that the majority of gamma transitions in nuclei are characterized by a lifetime  $\tau \geq 10^{-13} \dots 10^{-15}$  s, which significantly exceeds the duration of the existence of fluctuations for coherent correlated states, and because of this, such processes also have low probability.

It is also worth to note that the same regularities distinguish the flow of any reactions using virtual energy from reactions involving really accelerated particles. The main differences are related to the ban on both the implementation of any endoenergetic reactions and reactions with the formation of a long-lived intermediate state of the nucleus with a lifetime  $\tau \gg \delta t$ .

### 3. METHODS FOR FORMATION OF COHERENT CORRELATED STATES IN REAL SYSTEMS

The basic model for the CCS analysis is the nonstationary harmonic oscillator.

It was shown in [3-13] that the simplest method of excitation of CCS for a particle is associated with a nonstationary deformation of the harmonic potential  $V(q,t) = M\omega^2(t)q^2/2$ , in the field of which this particle is located. In a symmetric nonstationary parabolic potential well for which  $\langle q \rangle = 0$ ,  $\langle p \rangle = 0$ , the wave function of the particle, which was in the ground state prior to the deformation, depends on the correlation coefficient and has the form [1-3, 4, 7]

$$\Psi_0(q,t) = \frac{1}{\sqrt{2\pi\sigma_q}} \exp \left[ -\frac{q^2}{4\sigma_q} \left( 1 - \frac{ir(t)}{\sqrt{1-r(t)^2}} \right) \right]. \quad (11)$$

The explicit form of the correlation coefficient

$$r = \text{Re} \left\{ \left. \varepsilon^* \frac{d\varepsilon}{dt} \right| \varepsilon^* \frac{d\varepsilon}{dt} \right\}, \quad (12)$$

and also the compression coefficient  $k$ , which determines the ratio of the dispersions of the coordinate and momentum of the particle

$$k = \sigma_q / \sigma_p = |\varepsilon / (d\varepsilon/dt)|^2, \quad (13)$$

and the values of these dispersions

$$\begin{aligned} \sigma_q &\geq (\hbar/2)\sqrt{k/(1-r^2)}, \\ \sigma_p &\geq (\hbar/2)\sqrt{1/k(1-r^2)}, \end{aligned} \quad (14)$$

can be found on the basis of the solution of the equation of a classical oscillator with a variable frequency in the presence of an external force

$$\frac{d^2\varepsilon}{dt^2} + \omega^2(t)\varepsilon = f(t) \quad (15)$$

at initial conditions

$$\varepsilon(0) = 1, \quad \left. \frac{d\varepsilon}{dt} \right|_0 = i. \quad (16)$$

In equations (11)-(16) and the following relations,  $\omega(t)$  is the dimensionless frequency normalized to the characteristic oscillator frequency  $\omega_0$ ;  $t$  is a dimensionless (normalized to  $\omega_0^{-1}$ ) time;  $\varepsilon(t)$  is the dimensionless (normalized to  $q_0 = \sqrt{\hbar/M\omega_0}$ ) complex coordinate of the particle;  $M$  is the reduced mass of the particle.

In the general case, the solution of equation (14) has the form  $\varepsilon(t) = e^{\varphi(t)}$ ,  $\varphi(t) = \alpha(t) + i\beta(t)$ .

Substituting this solution into (12) and (15), using the initial conditions following from (16)  $\varphi(0) = \alpha(0) = \beta(0) = 0$ ,  $\left. \frac{d\varphi}{dt} \right|_0 = i$ ,  $\left. \frac{d\alpha}{dt} \right|_0 = 0$ ,  $\left. \frac{d\beta}{dt} \right|_0 = 1$  (17) and separating the real and imaginary parts of the resulting equation, we find

$$\begin{aligned} \frac{d^2\alpha}{dt^2} + \left( \frac{d\alpha}{dt} \right)^2 - \exp(-4\alpha) &= -\omega^2(t), \\ \beta(t) &= \int_0^t \exp\{-2\alpha(t')\} dt', \end{aligned} \quad (18)$$

$$|r| = \sqrt{\left( \frac{d\alpha}{dt} \right)^2 \exp(4\alpha) / \left\{ 1 + \left( \frac{d\alpha}{dt} \right)^2 \exp(4\alpha) \right\}}. \quad (19)$$

It is clear from (19) that obtaining the limiting value  $|r| \rightarrow 1$  is possible only if the condition  $(d\alpha/dt)^2 \exp(4\alpha) \gg 1$  is satisfied.

The system of equations (18), (19) is equivalent to equation (15), but it is more convenient for analysis and allows us to find the exponent of the amplitude of oscillation  $\alpha(t)$

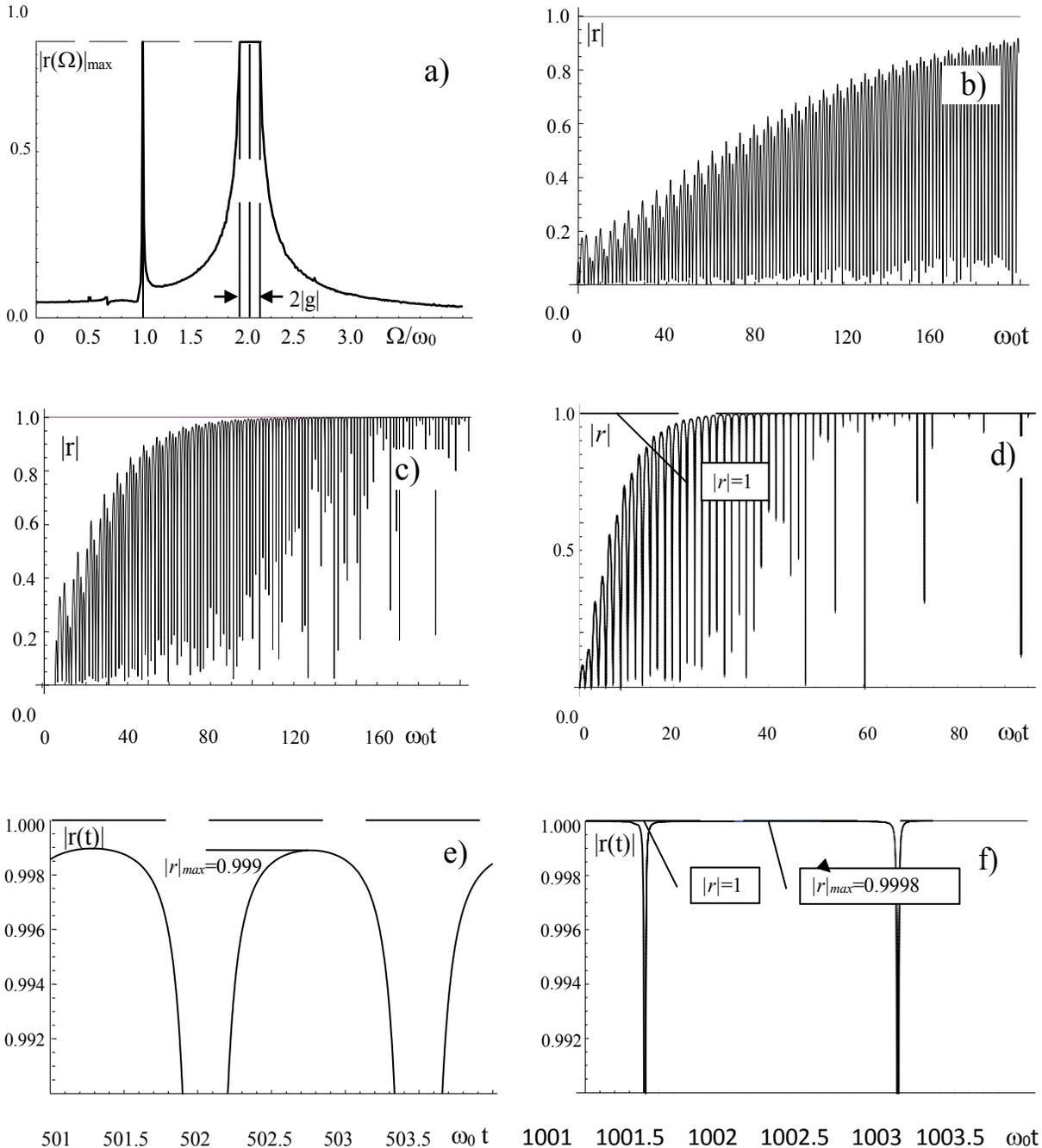
from (18), and then based on the given law of variation  $\omega(t)$  to find  $r(t)$  from equation (19).

A study of specific mechanisms for the formation of CCS under various modes of deformation of a potential well, as well as an analysis of the specifics of the manifestation of

this state in model and real systems, was carried out in [4-13].

### 3.1. FORMATION OF CCS FOR PERIODICAL MODULATION OF PARAMETERS OF THE HARMONIC OSCILLATOR

In [4-8, 10], the features of the formation of CCS for the particle in the case of a weak periodic effect



**Fig. 4.** a) - resonant structure of the dependence of the maximum of the correlation coefficient on the frequency  $\Omega$  for periodic modulation  $\omega(t) = \omega_0(1 + g\cos\Omega t)$  of the potential well parameters; b) -f) are the dependence of the correlation coefficient on time for modulation at the fundamental frequency  $\Omega = \omega_0$  at  $g = 0.1$  and at  $g = 0.2$ (c) and the parametric resonance frequency  $\Omega = 2\omega_0$  at  $g = 0.1$  for time intervals  $\omega_0 t \leq 100$ (d),  $\omega_0 t \approx 500$ (e) u  $\omega_0 t \approx 1000$ (f).

on the parameters (in particular, the frequency) of the harmonic oscillator  $\omega(t) = \omega_0(1 + g\cos\Omega t)$  in the case  $|g| \ll 1$  were examined in details. It follows from the solution of Eqs. (12)-(16) that the process of CCS formation for such modulation of the parameters of the potential well is characterized by the presence of the main (for  $\Omega \approx \omega_0$ ) and parametric ( $\Omega \approx 2\omega_0$ ) resonances, beyond which the efficiency of this process very sharply decreases, although it remains nonzero [10]. Within these resonances  $|r| \rightarrow 1$  as the modulation duration increases (see Fig. 4a). It is interesting to note that the frequency half-width of the main resonance is very small ( $|\delta\Omega| \ll g\omega_0$ ), and the parametric resonance is characterized by a resonance curve having a flat vertex of width  $|\delta\Omega| = 2g\omega_0$ .

From the same calculations it follows that as the modulation duration increases, the maximum values of the correlation coefficient  $r(t)_{\max}$  increase rapidly. The largest rate of  $r(t)_{\max}$  increase corresponds to the frequency  $\Omega = 2\omega_0$  and it rises sharply with the frequency modulation index  $g$ , reaching the value  $|r|_{\max} = 0.999$  (which corresponds to  $G_{\max} \approx 22$ ) at  $\omega_0(t) \approx 500$  and  $|r|_{\max} = 0.9997$  ( $G_{\max} \approx 41$ ) at  $\omega_0(t) \approx 1000$ .

Obviously, an analogous structure with two maxima of different amplitude and area (Fig. 4a) corresponds to the dependence of the tunneling effect on the modulation frequency.

This result can explain completely the results of the experiments [16] on stimulation of LENR under the synchronized action of two laser beams generated by low-power laser diodes ( $P \approx 20$  mW) with close frequencies to the Pd surface of a cathode located in heavy water in an electrolytic cell (Fig. 5)

When the polarization of these beams coincides, the generation of a difference frequency that acts on electrons in conduction band of the cathode takes place. Action of this low-frequency field leads to the periodic modulation of the potential well parameters for localized deuterium ions in the palladium lattice. Selecting the appropriate pairs of such

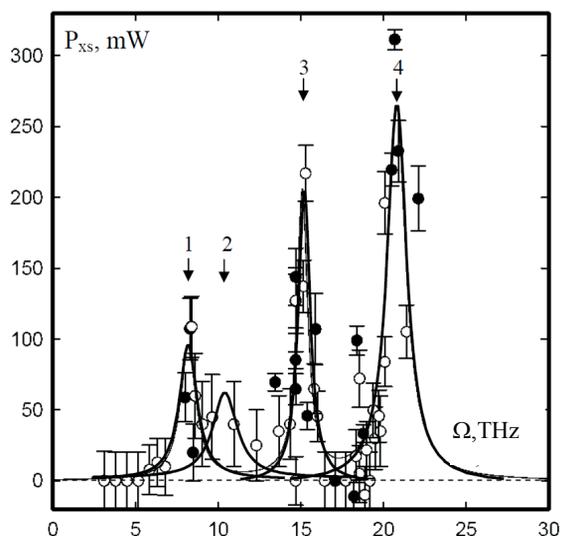
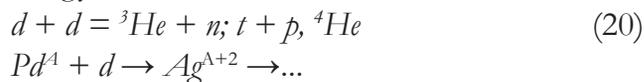


Fig. 5. Dependence of the power released upon exposure to the surface of the palladium cathode saturated in the process of electrolysis by deuterium in the volume of heavy water, versus difference frequency  $\Omega$  of radiation from two low-power laser diodes [16].

diodes, the authors investigated the dependence of the energy release in such system on this difference frequency in the interval 5...25 THz and found 4 resonance peaks of energy release with frequencies  $\Omega_1 \approx 7.8...8.2$ ,  $\Omega_2 \approx 10.2...10.8$ ,  $\Omega_3 \approx 15.2...15.6$  and  $\Omega_4 \approx 20.2...20.8$  THz, having a different amplitude.

The authors [16] could not give any adequate interpretation of these experiments. It is easy to see that these results agree very well with the data of the calculations presented above if we assume that the energy release is associated with the stimulation of nuclear reactions at low energy



in the volume of palladium saturated with deuterium.

Analysis of the deuterium vibrational structure in the Pd matrix shows that the frequencies  $\omega_1 \approx 7.8...8.2$  THz and  $\omega_2 \approx 10.2...10.8$  THz correspond to the intrinsic vibrations of deuterium ions in the Pd lattice. Each of these ions is, in fact, a harmonic oscillator.

Comparing the dependences of the power output, shown in Fig. 5, and, accordingly, the structure of the frequency dependence of the correlation coefficient (Fig. 4a), it is

easy to verify that first and third peaks in Fig. 5 correspond to the pair determined by the basic  $\Omega = \omega_1 \approx 7.8...8.2$  THz and parametric  $\Omega = 2\omega_1 \approx 15.2...15.6$  THz frequency resonances of the correlation coefficient formation, and second and fourth peaks correspond to another pair (basic  $\Omega = \omega_1 \approx 10.2...10.8$  THz and parametric  $\Omega = 2\omega_1 \approx 20.2...20.8$  THz frequency resonances).

The ratio of the amplitudes of the maxima of energy release in Fig. 5 fully corresponds to the results presented in Fig. 4 – the first, lower peak of each pair corresponds to the lower efficiency of the formation of the CCS at the frequency of the main resonance, and the second, higher – to greater efficiency at the parametric resonance frequency.

**3.2. FEATURES OF CCS FORMATION AT LIMITED INCREASE OR DECREASE OF THE WIDTH OF A PARABOLIC POTENTIAL WELL**

Let's find solution of equations (18)-(19) with a limited (in the range from  $L_0$  to  $L_{max} \equiv L_0(1 + g^{(+)})$ ) increase of the width of parabolic well

$$L(t) = L_0(1 + g^{(+)}) / (1 + g^{(+)}e^{-t/T}), \tag{21a}$$

which corresponds to a decrease of the frequency of the oscillator

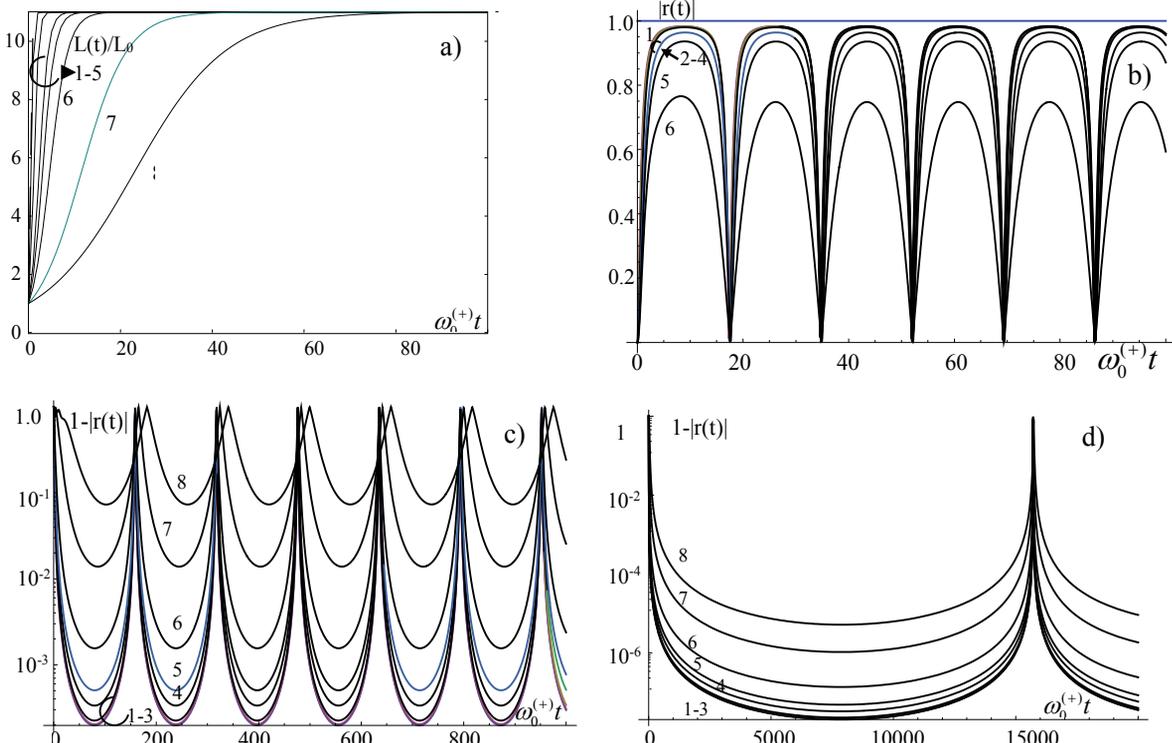
$$\omega(t) = \omega_0^{(+)}(1 + g^{(+)}e^{-t/T}) / (1 + g^{(+)}) \tag{21b}$$

from  $\omega(0) = \omega_0^{(+)}$  to  $\omega(\rightarrow \infty) \equiv \omega_{min} = \omega_0^{(+)} / (1 + g^{(+)})$ . Here  $g^{(+)} = L_{max} / L_0 - 1$  and  $g^{(+)} \approx L_{max} / L_0$  if  $L_{max} \gg L_0$ .

The value of  $T$  determines the characteristic deformation time (size increase) of the well.

**Fig. 6** shows the dependence of the correlation coefficient on the time of a monotonic increase of the width of the potential well in the interval  $L_{max} / L_0 = 11...10^4$  for different characteristic durations  $T = (10^{-1} / \omega_0^{(+)})... (10 / \omega_0^{(+)})$  changes of this width. Such change of the size of the well  $L_{max} / L_0$  corresponds to the change of interval  $\omega_0^{(+)} / \omega_{min} = 11...10^4$  of oscillation frequency of the particle in the well.

From these results follows that if the interval  $L_{max} / L_0$  increases, the amplitude of the correlation coefficient oscillations also increases greatly to the maximum possible value  $|r|_{max} \rightarrow 1$ . Narrow dips in the graph of  $|r(t)|$  are a consequence of the rapid interference transitions between the values of  $r(t)$  and  $-r(t)$  with time increase. With



**Fig. 6.** The time dependence of the width of the potential well (a) and the correlation coefficient (b) at the expansion of the well in the range:  $g^{(+)} = 10, L_{max} / L_0 = 11$  (b);  $g^{(+)} = 100, L_{max} / L_0 \approx 100$  (c);  $g^{(+)} = 10^4, L_{max} / L_0 \approx 10^4$  (d). Graphs 1-8 correspond to the values  $T\omega_0^{(+)} = 0.1, 0.25, 0.5, 1.0, 1.33, 2, 5, 10$ .

the increase  $|r(t)|_{\max} \rightarrow 1$ , the width of these gaps tends to zero.

Another important factor of  $|r(t)|_{\max}$  increase is the use of the minimum deformation time  $T$  of the well.

In particular, for a relatively small change of the size of the well (at  $L_{\max}/L_0 = 11$  and  $T = (0.1...1)/\omega_0^{(+)}$ , the maximum values of the correlation coefficient  $|r|_{\max}$  and the correlation efficiency  $G_{\max} = 1/\sqrt{1-r_{\max}^2}$  do not exceed, respectively,  $|R|_{\max} \approx 0.98$  and  $G_{\max} \approx 5$ . If this interval is increased to  $L_{\max}/L_0 = 10^4$  (this corresponds, for example, to an increase of the width of the microcrack from the "seed" value  $L_0 \approx 5...10\text{\AA}$  to  $L_{\max} \approx 5...10\ \mu\text{m}$ ), we have  $1 - |r|_{\max} \approx 2 \cdot 10^{-7}$  and  $G_{\max} \approx 1600$ .

The hypothetical case of even greater change  $L_{\max}/L_0 = 10^5$  corresponds to a CCS with close to ideal characteristics  $1 - |r|_{\max} \approx 10^{-9}$  and  $G_{\max} \approx 20000$ !

We can emphasize another feature of the process of formation of the CCS. The maximum current correlation coefficient  $|r(t)|_{\max}$  is provided through a time interval much greater than the value of  $T$ , which determines the duration of a significant change in the width of the well. It follows directly from the analysis of the data presented in Fig. 5. In particular, the first maxima of  $|r(t)|$  and  $G(t)$  correspond to time of CCS formation and are equal to  $t_c \approx 750/\omega_0^{(+)}$ ,  $7500/\omega_0^{(+)}$ ,  $75000/\omega_0^{(+)}$  for  $L_{\max}/L_0 = 10^3$ ,  $10^4$ ,  $10^5$ , respectively. For the appearance of the following maxima of  $|r(t)|_{\max}$  and  $G_{\max}(t)$  more time is required.

These values of  $t_c$  essentially exceed the time of a significant change of the width of the well  $T = (0.1/\omega_0^{(+)})...(10/\omega_0^{(+)})$ . The situation corresponds to the condition  $T \ll t_c$ , at which rapid deformation of the well occurs first, and then a slow process of formation of the CCS takes place. Obviously, the process of deformation of a well with  $T \rightarrow 0$ , which is close to its instantaneous expansion in the interval from  $L_0$  to  $L_{\max}$  is close to the ideal case.

Examined above scenario of LENR optimization at expanding of the potential well agrees with experiments in metalhydrides (in particular, with A.Rossi's experiments), when the formation of unsteady (rapidly growing) microcracks in the volume of metal, in which hydrogen ions are localized, occurs in the process of hydrogenation. In addition, such scenario can "work" in natural dynamic systems such as cell division, when, for example, atoms or ions of hydrogen are in the space between separating cells.

b) An alternative mode of CCS formation due to the influence on the parameters of the potential well, in which the particle is located, is the reduction of its width. Let us find the solution of the system of equations (18)-(19) with a limited decrease in the width of the potential well

$$L(t) = L_0(1 + g^{(-)}e^{-t/T})/(1 + g^{(-)}) \quad (22a)$$

from  $L_0$  to  $L_{\min} \equiv L_0/(1 + g^{(-)})$ , which corresponds to an increase in the frequency of the oscillator

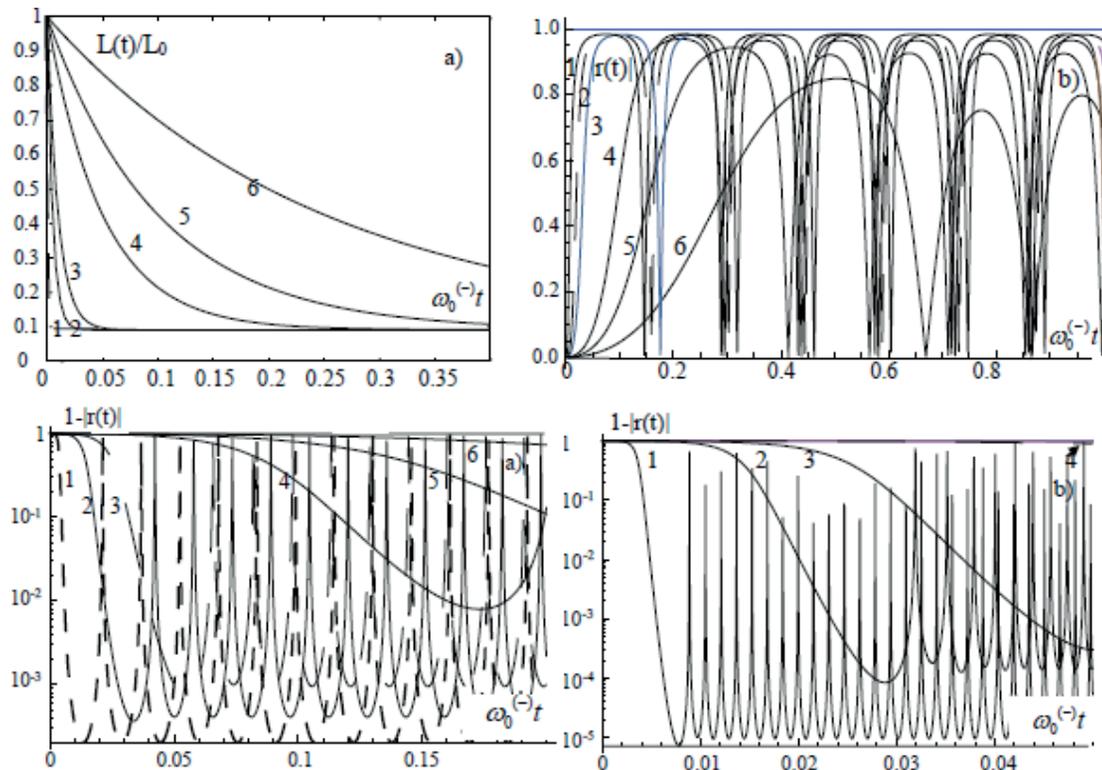
$$\omega(t) = \omega_0^{(-)}(1 + g^{(-)})/(1 + g^{(-)}e^{-t/T}) \quad (22b)$$

from  $\omega(0) = \omega_0^{(-)}$  to  $\omega_{\max} \equiv \omega_0^{(-)}(1 + g^{(-)})$ .

Here  $g^{(-)} = (L_0/L_{\min} - 1)$  and  $g^{(-)} \approx L_0/L_{\min}$  if  $L_0 \gg L_{\min}$ .

The results of calculation of the coefficient  $g^{(-)} = 10, 10^2, 10^3$  for three values, which correspond to similar decreases of the size of the parabolic well, and, accordingly, an increase of the oscillation frequency within this well, as well as different values of the characteristic duration  $T$  of the well compression, are shown in **Fig. 7**. It follows from these results that the maximum value of the correlation coefficient, as in the case of expanding well, increases with the increase of the compression interval  $L_{\max}/L_0$  and decrease of the compression time  $T$ .

E.g. at a relatively small compression of the well in the interval  $L_0/L_{\min} = 11$  and at  $T = (0.001/\omega_0^{(-)})...(0.01/\omega_0^{(-)})$  the maximal values of the correlation coefficient and the correlation efficiency coefficient don't exceed, respectively,  $|r|_{\max} \approx 0.98$  and  $G_{\max} = 1/\sqrt{1-r_{\max}^2} \approx 5$ . With an increase of the compression interval to  $L_0/L_{\min} = 10^3$ , for example, by the reducing of the width of the



**Fig. 7.** The change in the width of the compressing potential well (a) and the correlation coefficient for:  $g^{(-)} = 10, L_0/L_{min} = 11$  (b);  $g^{(-)} \approx L_0/L_{min} = 10^2$  (c);  $g^{(-)} \approx L_0/L_{min} \approx 10^3$  (d). Graphs 1-6 correspond to the values  $T\omega_0^{(-)} = 0.001, 0.005, 0.01, 0.05, 0.1, 0.25$ .

microcrack in the matrix from 1  $\mu\text{m}$  to 10 $\text{\AA}$ , we have  $1 - |r|_{\text{max}} \approx 10^{-5}$  and  $G_{\text{max}} \approx 220$  at  $T = 0.001/\omega_0^{(-)}$  and  $1 - |r|_{\text{max}} \approx 10^{-4}$ ,  $G_{\text{max}} \approx 70$  at  $T = 0.005/\omega_0^{(-)}$ .

It should be noted that if the potential well is compressed in this interval, the initial frequency  $\omega_0 \equiv \omega_0^{(-)}$  (before the compression of the well) is in  $L_0/L_{min} = 10^3$  times smaller than the initial frequency  $\omega_0 \equiv \omega_0^{(+)}$  (before the expansion) at a similar expansion of the well in the same interval from 10 $\text{\AA}$  to 1  $\mu\text{m}$ . This circumstance must be taken into account when comparing graphs  $r(t)$ , which determine the time dependence of the process of formation of CCS with the increase and decrease of potential well sizes.

At an even larger value of the compression interval  $L_{\text{max}}/L_0$  and with the corresponding shortening of the compression process duration  $T$ , the values of  $|r|_{\text{max}}$  and  $G_{\text{max}}$  increase as effectively as in the case of an expansion of the well.

It should be noted that considered mechanism of CCS formation is realized when microcracks are "healed" in a number of materials and during the growth of biological cultures (in particular,

during the division of DNA, on membrane surfaces, etc.).

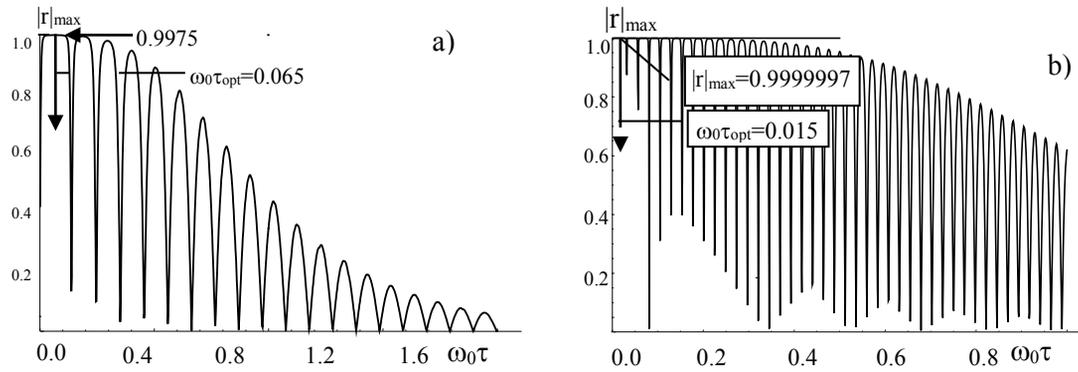
### 3.3. FORMATION OF CCS AT PULSE MODULATION OF POTENTIAL WELL PARAMETERS

Another alternative method of CCS excitation is connected with a pulsed change of the frequency of an equivalent harmonic oscillator – rapid deviation (usually an increase) of the frequency from a stationary value, followed by rapid return to this value. In particular, papers [11, 13] consider the features of CCS formation for different structures, durations and amplitudes of such change. **Fig. 8** shows the dependence of the maximum  $|r(t)|_{\text{max}}$  and the time-averaged  $\langle |r(t)| \rangle$  correlation coefficient calculated on the basis of relations of the type (18)-(19) on the pulse time width  $\tau$  of the frequency modulation pulse

$$\omega = \omega_0(1 + f(t)), f(t) = ge^{-(t-t_0)^2/2\tau^2}, t_0 \gg \tau \quad (23)$$

at different amplitudes of this pulse.

Direct numerical calculation on the basis of equations (18)-(19) has shown that the action of such Gaussian pulse  $f(t)$  leads to the rapid formation of a CCS with the maximum values of the correlation coefficient  $|r(t)|_{\text{max}}$ ,



**Fig. 8.** Dependence of the maximum value  $|r(t)|_{max}$  of the correlation coefficient versus duration of the frequency modulation pulse (18) for different amplitude of this pulse:  $g = 10$  (a);  $50$  (b).

which correspond to a very large coefficient of correlation efficiency  $G$ , which provides great transparency of the potential barrier. In particular, at increases of the dimensionless pulse amplitude (increase of the frequency from the initial value) from  $g = 10$  to  $g = 50$ , the value of  $G$  increases from  $G \approx 14$  to  $G \approx 1300$ . According to the estimates made above, it provides an increase of the probability of the tunnel effect in the interaction of particles at low energy from  $D_{r=0} \approx 10^{-500}$  in the absence of such action to  $D_{r=0.9975} \approx 10^{-35}$  at  $g = 10$  and to  $D_{r=0.9999997} = 0.3$  at  $g = 50$ .

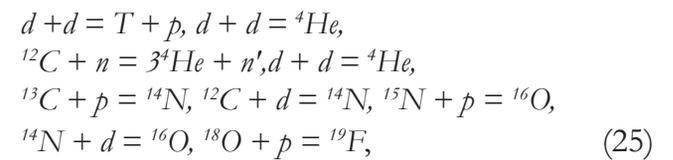
The mechanism of the formation of a CCS under a pulsed action on a particle [11, 13] can be realized, for example, as by the shock deformation of the lattice under the action of shock waves, as under the action of a pulsed magnetic field on free charged particles

A typical example of such external action is an electric discharge in a gas or liquid. The current  $J(t)$  of the discharge is accompanied by the formation of a pulsed azimuthal magnetic field  $H(\vec{r}, t)$  in which the motion of the ions corresponds to tunable cyclotron resonance, and the system itself is a complete (formal) analog of the nonstationary harmonic oscillator with the same Hamilton operator with the same wave functions and energy spectrum. Used above formalism for the formation of a CCS in a nonstationary harmonic oscillator can be fully applied to such system, taking into account the obvious change of the initial frequency  $\omega(t) = |q|H(t)/Mc$ .

The results obtained above can be directly used for this case, if we assume that

$$\omega(t) = \omega_0(1 + f(t)), \omega_0 = |q|H_0/Mc, H(t) = H_0(1 + f(t)), H_{max} = H_0(1 + g). \quad (24)$$

Under the action of the pulsed magnetic field, a peculiar "deformation" of this equivalent oscillator and a very effective formation of the CCS take place. Such scenario explains well [13], for example, the generation of neutrons and other isotopes in air during lightning on the base of reactions



and also the well-known experiments of R. Mills (J. Mols, J. Lotoski, Y. Lu. Brilliant Light Power [18]) on stimulation of large energy release in an electric discharge in a gaseous medium.

### 3.4. THE INFLUENCE OF DAMPING AND RANDOM FORCE ON FORMATION OF COHERENT CORRELATED STATE OF PARTICLES IN PARABOLIC WELL

The presence of fluctuations and damping can have a significant effect on the process of CCS formation. The most reasonable method for taking into account the attenuation of a quantum oscillator is the introduction of a thermostat and the use of a density matrix apparatus, which necessitates the use of a large number of longitudinal  $T_{ij}$  and transverse  $\tau_{ij}$  relaxation times. These values are most often found semiempirically. This method greatly complicates the solution and makes it much less clear if you

stay within the framework of a model close to the classical harmonic oscillator. On the other hand, it is well known that in a classical harmonic oscillator damping can be taken into account by introducing a phenomenological braking force  $\vec{F}_d = -2\gamma d\vec{q}/dt$  with a single phenomenological coefficient  $\gamma$ .

An acceptable alternative to the density matrix method is the simulation of a phenomenological nonstationary quantum-mechanical Hamiltonian, from which an equation of motion can be obtained, in the form corresponding to a classical oscillator with damping. This condition corresponds to the Caldirola-Kanai Hamiltonian, which takes into account the effect of the external force  $\vec{F}_d$  and the phenomenological braking force on a particle in the parabolic potential, and has the form

$$\hat{H}(\hat{x}, t) = \frac{\hat{p}_x^2}{2M} e^{-2\gamma t} + \frac{M\omega^2(t)\hat{x}^2}{2} e^{2\gamma t} - F(t)\hat{x}e^{2\gamma t}. \quad (26)$$

In this relation, the canonical (generalized) momentum  $\hat{p}_x$  is connected with the "physical" momentum  $p_{(k)x} = Mdx/dt$  by the relation  $\hat{p}_x = e^{2\gamma t} \hat{p}_{(k)x}$ .

This Hamiltonian is Hermitian, its eigenvalues are real, and the eigenfunctions are bounded and normalized. The use  $\hat{H}(t)$  of the form (26) does not violate the canons of quantum mechanics. The validity of the use of such Hamiltonian for the analysis of systems with dissipation at a variable frequency was discussed in many papers (in particular, in [1, 2, 9-11]).

Based on the Hamiltonian (26) and taking into account the general rule for constructing the equation of motion for an arbitrary operator

$$\frac{d\hat{L}}{dt} = \frac{\partial \hat{L}}{\partial t} + \frac{1}{i\hbar} [\hat{L}\hat{H}(t)] \quad (27)$$

we can obtain the equation of motion for the coordinate operator  $\hat{x}$

$$\frac{d^2\hat{x}}{dt^2} - \frac{1}{i\hbar} \frac{\partial}{\partial t} [\hat{x}\hat{H}(t)] + \frac{1}{\hbar^2} [[\hat{x}\hat{H}(t)]\hat{H}(t)] = 0, \quad (28)$$

which leads to the dimensionless equation of a classical harmonic oscillator with damping, an arbitrary external force, and the necessary initial conditions

$$\begin{aligned} \frac{d^2\varepsilon}{dt^2} + 2\gamma \frac{d\varepsilon}{dt} + \omega^2(t)\varepsilon &= f(t), \\ \varepsilon(0) = 1, \quad \left. \frac{d\varepsilon}{dt} \right|_0 &= i, \quad \omega(0) = 1. \end{aligned} \quad (29)$$

This equation is an obvious generalization of equation (15).

In equation (29) and in the following relations,  $f(t) = F(t)/\sqrt{\hbar M \omega_0^3}$  is a dimensionless external (including stochastic) force;  $\gamma$  is the dimensionless attenuation coefficient normalized to  $\omega_0$ .

To solve a specific problem of the CCS formation process at the presence of damping, variable frequency, and stochastic effects, we used more simple method of equation (29) analysis, which is connected with its transformation into equations for the corresponding reciprocal and mixed moments of quantities  $\hat{q}$  and  $\hat{p}_q$  (in dimensionless form  $\varepsilon$  and  $d\varepsilon/dt$ ), appearing in (29), and taking into account the correlation characteristics of the function  $f(t)$ . A similar method can also be used at the presence of a random perturbation of the variable oscillator frequency  $\omega(t)$ .

Let us consider the evolution of a nonstationary oscillator with damping under the action of a random stationary delta-correlated force  $f(t)$  with characteristics

$$\langle f(t) \rangle_f = 0, \quad \langle f(t_1)f(t_2) \rangle_f = 2S\delta(t_1 - t_2) \quad (30)$$

corresponding to averaging over the realization of a random force with the intensity  $S$ .

An explicit form of the dependence  $S$  on the parameters of a low-pressure plasma or gas was obtained in [9-11]

$$\begin{aligned} S &= \frac{1}{2} \int_{-\infty}^{\infty} d\tau \left\langle \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} f(t)f(t+\tau)dt \right\rangle_f \approx \\ &\approx \frac{M^* \sigma n}{2\hbar\omega_0^2} \langle (\Delta v)^2 \rangle_f \langle |v| \rangle_f. \end{aligned} \quad (31)$$

Here  $1/\Delta t = \sigma n |v| / \omega_0$  is the dimensionless collision frequency of atoms in the medium (in this case – in a gas with a particle concentration  $n$ ),  $\sigma \approx 3 \cdot 10^{-16}$  cm<sup>2</sup> is the total cross section for elastic scattering of atoms at low energy,  $M^* = M/(1 + M/M_a)$  is the reduced mass at

the collision of the particle with another particle of the medium, and  $\Delta v$  is the change of particle velocity in an elastic collision.

After introduction of the functions

$$\begin{aligned} \mu_{00} &= \varepsilon^* \varepsilon, \mu_{01} = \varepsilon^* \frac{d\varepsilon}{dt}, \\ \mu_{10} &= \frac{d\varepsilon^*}{dt} \varepsilon = \mu_{01}^*, \mu_{11} = \frac{d\varepsilon^*}{dt} \frac{d\varepsilon}{dt}, \end{aligned} \quad (32)$$

which includes a combination of dimensionless coordinates and particle pulses, it is possible to obtain a system of equations for the mixed  $m_{ii} = \langle \mu_{ii} \rangle$  and mutual  $m_{i \neq j} = \langle \mu_{i \neq j} \rangle$  moments of the quantities  $\varepsilon$  and  $d\varepsilon/dt$  corresponding to the dimensionless coordinate and momentum of the particle. After additional averaging of all components of these equations by the realization of the random force  $f(t)$ , we can obtain the resulting system of equations for the moments  $m_{ij} = \langle \mu_{ij} \rangle_f$

$$\frac{dm_{00}}{dt} = m_{01} + m_{01}^*, \quad (33a)$$

$$\frac{dm_{01}}{dt} = m_{11} - 2\gamma m_{01} - \omega^2(t)m_{00}, \quad (33b)$$

$$\frac{dm_{11}}{dt} = -4\gamma m_{11} - \omega^2(t)\{m_{01} + m_{01}^*\} + 2S. \quad (33c)$$

The solutions of this system satisfy the initial conditions for the moments

$$m_{00}(0) = 1, m_{01}(0) = i, m_{01}^*(0) = -i, m_{11}(0) = 1, \quad (34)$$

directly following from the initial conditions for  $\varepsilon$  and  $d\varepsilon/dt$ .

The solution of the system of equations (33) and the corresponding correlation coefficient

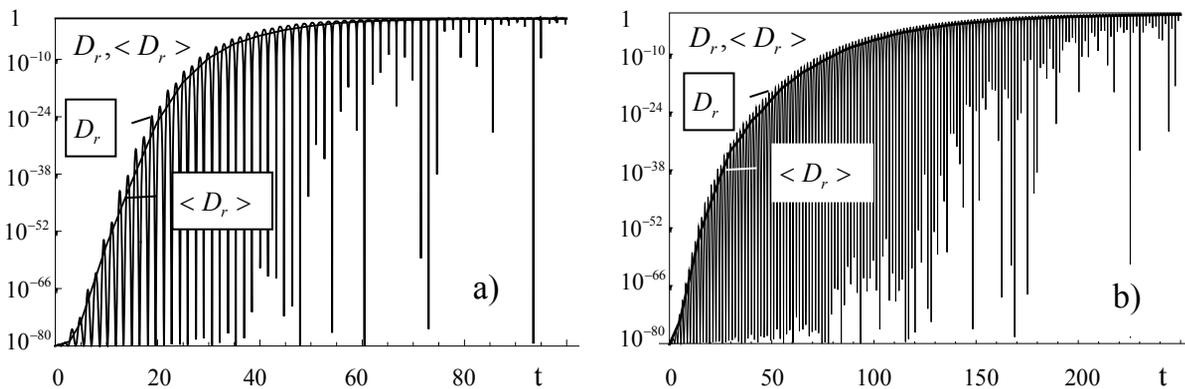
$$r(t) = \frac{m_{01} + m_{10}}{2\sqrt{m_{00}m_{11}}} \equiv \frac{m_{01} + m_{01}^*}{2\sqrt{m_{00}m_{11}}}, \quad (35)$$

can be found for a given law of variation of the oscillator frequency  $\omega(t)$ . Using this coefficient and the wave function of the CCS, it is possible to calculate the fluctuation parameters of a quantum oscillator, and, using an approximate relation  $D_{r \neq 0} \approx (D_{r=0})^{\sqrt{1-r^2}}$ , estimate the change in the transparency of the potential barrier.

Such analysis was carried out in [9-11], where it was shown that the presence of such fluctuations (for example, due to the collision of an ion located in the field of a variable harmonic oscillator with extraneous atoms) can significantly complicate the process of CCS formation and reduce the maximum value of the correlation coefficient.

**Fig. 9** presents one of the many results of the analysis of the action of random force and dephasing fluctuations on the process of CCS formation with periodic modulation of parabolic potential well (at the frequency  $\omega(t) = \omega_0(1 + g\cos\Omega t)$  change) on the parametric resonance frequency  $\Omega = 2\omega_0$  at the absence and presence of random force.

It can be seen that the presence of a random fluctuating force slows down the increase of the correlation coefficient, and in some cases restricts it to a fixed level. These questions are discussed in details in [5, 9, 11].



**Fig. 9.** The dependence of the current  $D_r$  (oscillating function) and the averaged  $\langle D_r \rangle$  (monotonic function) of the tunnel effect probabilities versus duration of periodic frequency modulation in the case of: a) the absence of random force and the presence of attenuation with parameters  $2\gamma = g = 0.1$ ; b) the presence of a random force with intensity  $S = 0.05$  and damping  $2\gamma = g/2 = 0.05$ . The initial values  $D_{r=0}$  and  $\langle D_{r=0} \rangle$  correspond to the value  $D_{r=0} = \langle D_{r=0} \rangle = 10^{-80}$ .

#### 4. CONCLUSION

Examined above problem of LENR implementation based on the use of coherent correlated states allows us to explain, justify and numerically consider all known LENR paradoxes without applying new radical hypotheses, basing only on the powerful foundations of modern quantum theory and nuclear physics.

It is important to note that a variety of LENR effects for light, medium and heavy isotopes, observed in completely different media and systems (crystals, amorphous bodies, liquids, gases, various living systems, etc.) under different causes (hydrogenation of metals during electrolysis and thermal exposure, glow discharge, shock waves, electric discharge, natural metabolic processes with concomitant biological phenomena, etc.) are described by a single universal mechanism! Previously, different authors believed that each of the group of effects is characterized by its unique mechanism, not applicable to another group.

Another undoubtedly positive aspect of the method of coherent correlated states in application to LENR processes is the ability to predict the expected effects, as well as the possibility of preliminary assessment of the potential suitability and efficiency of new designed or newly used devices, systems and objects.

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