SURFACE MAGNETOSTATIC WAVE RESONANCE WITH ORIENTATION

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Abstract. **Surface waves in thin isotropic ferromagnet slab magnetized in its plane are considered in the magnetostatic limit. Magnetostatic waves are considered as a type of spin waves. The special attention is given to the propagation directions of the waves, which are not orthogonal and parallel to the magnetization of the film. Two features of these waves have not been discussed earlier: the wave resonance with orientation and the existence of a new type of the surface magnetostatic waves. The position of the resonance peak depends not only on the frequency, but also on the direction of wave propagation, and we use the term "wave resonance with orientation". The wave resonance with orientation predicted theoretically on the basis of the dispersion relation of Damon-Eshbach [5]. The existence of a new type of surface waves (waves of second type) also follows from [5]. These waves propagate almost parallel to the magnetization of the film. The frequency of waves of second type can greatly exceed the frequency of Damon-Eshbach. The frequency of Damon-Eshbach limits previously studied surface waves (of the first type). Resonance with the orientation of the waves of the second type is also considered. Estimates and graphics are provided for the parameters of YIG films [7].**

Keywords: **magnetostatic waves, spin waves, isotropic ferromagnet, magnetic films, spectra of electromagnetic waves.**

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Contents

1. INTRODUCTION

In [1, 2] the need to develop devices with using of magnetostatic waves (MSW) and ferromagnetic films is associated with the trends in modern information processing systems: to use more broadband signals with a higher center frequency, with the ability to select materials with low attenuation MSW, with prospects for the development of devices with using of magnonic and photonic crystals. Magnetostatic waves can be considered as a simple type of spin waves, when the inhomogeneous exchange interaction can be neglected. The observed complexity rise of the spectra of the MSW with the complication of the structure of the films [3, 4], apparently, indicates the need for a more detailed theoretical investigations of the fundamental properties of the MSW. Research conditions limiting the range of MSW with high frequencies may be important. Well studied and usually used MSW propagating either to parallel or to orthogonal the magnetization of the film [5-8]. Forward volume MSW is tipical in films magnetized perpendicular to their surface. Two other types MSW used in tangentially magnetized films. Backward volume MSW propagate in the direction of the magnetization of the film. Surface MSW (SMSW) has frequencies higher than the backword valume waves. SMSW is generally assumed orthogonal to the magnetization of the film [8]. Dispersion relations are valid for an arbitrary direction of propagation of the wave is obtained for surface magnetostatic waves in

[5]. The existence of the special directions of propagation of waves in a tangentially magnetized films is noted in a number of experimental and numerical studies of surface magnetostatic waves [6, 9, 10]. These directions do not match the magnetization or orthogonal directions however, are the predominant directions. Nonparallel and nonorthogonal to the magnetization SMSW have features that interesting for generalizations design principles of the devices with using of the MSW. The explanation of the observed effects in form a narrow wave rays emanating from the edges of the emitter, explained in [6, 9, 10] using the decomposition of the waves on the Fourier-harmonics and numerical accounting of the dispersion relation, equivalent to the results of [5]. The results of this research must give understanding of more delicate experiments on surface waves.

The SMSW dispersion relations for thin isotropic ferromagnetic slab are derived in [5] based on Walker's equation [7]. Thin isotropic ferromagnet slab magnetized in its plane is considered in the magnetostatic limit. Magnetic damping is important but not considered. The surfaces of the slab plane are "free", i.e. the slab is surrounded by a vacuum. In [5] it is assumed that film is magnetized to saturation along the z axis. The plane of film is $x = 0$. Thickness of film is s. The components of Polder magnetic susceptibility tensor ($\hat{\chi}$) determine properties of magnetostatic waves $\kappa = \chi_{11} = \chi_{22}$ and $j\nu = \chi_{12} = -\chi_{21}$ (j is imaginary unit). We denote by ($k_x^{(i)}$, k_y, k_z) components of the wave vector of magnetostatic waves in the film. The index i $\in \{1, 2\}$corresponds to waves that are formed within a free film as a result of reflections from the surfaces of the film. The film is called "free" film, if both its surface adjacent to the vacuum. MSW is a limiting case of wave, when in sufficiently thin films wave properties cease to depend on the dielectric permittivity of the materials. Therefore, the results for the "free" films remain true when the film is covered with a thick and good dielectric. (The results obtained for MSW in free films that are obviously false for metallized films). In free films $k_x^{(1)}$ and $k_x^{(2)}$ differ only in sign. It is assumed that the components of the wave vector can be not only real numbers, but also complex. The imaginary part of the wave vector corresponds to an exponential change of amplitude of a wave at the increase of the corresponding coordinates. If $k_x^{(i)}$ is real then wave is called the volume MSW. If $k_x^{(i)}$ is imaginary then wave is called the surface MSW (SMSW). Parameter $\eta = k_z/k_y$ is the cotangent of the angle between the magnetization of the film coinciding with the z-axis, and the direction of propagation of magnetostatic waves in the plane x = const. In this case, the Walker's equation is written in the form [5]:

$$(\kappa+1)((k_x^{(i)})^2 + k_y^2) + k_z^2 = 0, \tag{1}$$

It leads to ratios

$$k_z = \eta k_y;\; k_x^{(i)} = \pm j k_y \sqrt{\frac{1+\kappa+\eta^2}{1+\kappa}}. \tag{2}$$

Therefore, in the following formulas it is possible to exclude k_z, $k_x^{(i)}$ and to leave only one component of the wave vector k_y. The dispersion relation is derived from the conditions for the fields at the boundaries of the film. Appropriate variation of the dispersion relation for a free film with thickness of s can be found in [5, formula (19)].

$$(1+\eta^2) + 2(1+\eta^2)^{1/2}\left(-\frac{1+\kappa+\eta^2}{1+\kappa}\right)^{1/2}(1+\kappa)ctg\left[k_y s\left(-\frac{1+\kappa+\eta^2}{1+\kappa}\right)^{1/2}\right] +$$
$$+(1+\kappa)^2\left(\frac{1+\kappa+\eta^2}{1+\kappa}\right) - \nu^2 = 0. \tag{3}$$

Functions tangent and cotangent are comfortable in the formula (3) on the stage of the derivation of the equation, but complicate the analysis. These functions go to infinity and there their sign is changed. In addition, in the formula (3) there are some other features that would make it difficult analysis by using as numerical and analytical methods. In this article these difficulties, to the extent possible, eliminated.

The purpose of this article is to obtain the conditions of formation of the predominant directions of propagation of the SMSW in tangential magnetized free films of isotropic ferromagnet neither parallel nor orthogonal to the direction of magnetization of the film. The concept of group velocity is not used.

2. THE CONVERSION OF THE DISPERSION EQUATION

Get rid of tangents and cotangens in the formula (3). It is necessary to reduce variable η to $u = \frac{1}{\sqrt{\eta^2+1}}$, The variable u is the sine of the angle between the magnetization of the film and the direction of propagation of the SMSW. In this paper we deal with only surface waves (imaginary $k_x^{(i)}$), so we can use the substitution:

$$ctg(jx) = \frac{a}{b} \Leftrightarrow \exp(-2x) = \frac{a+jb}{a-jb},$$

where j is the imaginary unit.

After transformation formula (3) is reduced to the form:

$$k_y = \frac{u}{2s}\left(\frac{1+\kappa}{1+\kappa u^2}\right)^{1/2} \ln\left(\frac{(uv)^2 - \left(1-\left((1+\kappa u^2)(1+\kappa)\right)^{1/2}\right)^2}{(uv)^2 - \left(1+\left((1+\kappa u^2)(1+\kappa)\right)^{1/2}\right)^2}\right). \quad (4)$$

The amplitude of the MSW is proportional to the wave vector [5], therefore, the formula (4) allows to make qualitative conclusions not only about the length of the MSW, but also about the amplitude of the waves. (The components of the wave vector are proportional to k_y in accordance with (2)) From the formula (4) it follows that if the direction of wave propagation coincides with the direction of magnetization of the film ($u = 0$), then SMSW is not exist. This fact allows us to understand the attention of researchers mainly to SMSW at $u = 1$. This estimate does not preclude the existence of surface magnetostatic waves at the corners, a small but not zero.

Formula (4) can be regarded as a method of calculating the wave vector only with restrictions. Sometimes the formula (4) is useless as there are uncertainties at the most interesting frequencies. At frequency $w_\perp$, the value κ is equal to (-1). In $\left(\frac{1+\kappa}{1+\kappa u^2}\right)^{1/2}$ there is an uncertainty factor for $u = 1$ and $w = w_\perp$, however, the expression under the logarithm in (4) takes a value of 1 so the wave vector is zero. MSW not exist at a frequency $w_\perp$. At frequencies more than $w_\perp$, factor $\left(\frac{1+\kappa}{1+\kappa u^2}\right)^{1/2}$ is real therefore the MSW are SMSW. When the expression (1 + κu) vanishes, uncertainties in the formula (4) complicate the analysis and can lead to incorrect estimates. These uncertainties create special difficulties at analytical and numerical analysis.

In [7, 8] properties of an isotropic ferromagnet is determined using the characteristic frequencies $w_M = 4\pi\gamma M_0$ and $w_H = \gamma H_0$. γ is the gyromagnetic ratio, M_0 is the saturation magnetization, H_0 is the internal magnetic field corresponding to the saturation magnetization. The components of the tensor Polder are expressed through characteristic frequencies [6-8]:

$$\kappa = \chi_{11} = \chi_{22} = \frac{w_M w_H}{w_H^2 - w^2};$$

$$v = \frac{\chi_{12}}{j} = -\frac{\chi_{21}}{j} = \frac{w w_M}{w_H^2 - w^2}.$$

Necessary condition for the existence SMSW is 1 + κ > 0, it is transformed to

$$w > w_\perp = \sqrt{w_H\left(w_H + w_M\right)}.$$

In accordance with [11] we introduce the notation

$$W_\perp = w^2 - w_H^2 - w_M w_H,$$

$$W_u = w^2 - w_H^2 - w_M w_H u^2,$$

$$W_H = w^2 - w_H^2.$$

The value $W_\perp$ turns to zero at the frequency $w_\perp = \sqrt{w_H\left(w_H + w_M\right)}$. W_u vanishes when $w_u = \sqrt{w_H\left(w_H + w_M u^2\right)}$. If $w = w_H$ then $W_H = 0$. (4) reduces to

$$k_y = \frac{uW_\perp^{1/2}}{2sW_u^{1/2}} \ln\left(\frac{\left(ww_M u\right)^2 - \left(W_H - W_\perp^{1/2}W_u^{1/2}\right)^2}{\left(ww_M u\right)^2 - \left(W_H + W_\perp^{1/2}W_u^{1/2}\right)^2}\right). \quad (5)$$

In (5) there is uncertainty on the frequency w_u. The denominator of (5) and the logarithm is equal to 0. Let us introduce the function:

$$m(z) = \frac{\ln\left(1+z\right)}{z},$$

The formula for calculating the components of the wave vector is

$$k_y = \frac{uW_\perp^{1/2}R}{2s} m\left(RW_u^{1/2}\right), \quad (6)$$

where

$$R = \frac{4W_H W_\perp^{1/2}}{\left(ww_M u\right)^2 - \left(W_H + W_\perp^{1/2}W_u^{1/2}\right)^2}. \quad (7)$$

The function $m(z)$ has no singularities at zero since $\lim_{x\to 0} m(z) = 1$. $zm(z) = ln(1 + z)$, so $\lim_{x\to\infty} zm(z) = \infty$.

When R goes to infinity, then k_y and the amplitude of the wave goes to infinity. I.e., take place the wave resonance. If denominator in the formula (7) goes to zero we obtain the resonance condition.

$$\left(ww_M u\right)^2 - \left(W_H + W_\perp^{1/2}W_u^{1/2}\right)^2 = 0. \quad (8)$$

Solving equation (8), we obtain 4 solutions for predominant waves

$$u_{1,2} = \frac{w \pm W_\perp^{1/2}}{w_M + w_H} = \frac{w \pm \sqrt{w^2 - w_\perp^2}}{w_M + w_H};$$

$$u_{3,4} = -\frac{w \pm \sqrt{w^2 - w_\perp^2}}{w_M + w_H}. \quad (9)$$

Solutions $u_{3,4}$ and $u_{1,2}$ are symmetrical about the origin. Solutions $u_{1,2}$ form two branches.

Predominant wave branch with the sign "+" is more familiar. The angle between the wave vector of a wave and the magnetization of the film is close to right angle. This branch can be called as "orthogonal" SMSW, or "the first type SMSW".

There are no SMSW at $w = w_{\perp}$. At this frequency $W_{\perp}^{1/2} = 0$, hence $R = 0$ and $k_y = 0$. However, from (9) at a frequency of $w = w_{\perp}$ we derive $u_{\perp} = \frac{w_{\perp}}{w_M + w_H}$. We can say that $u_{\perp}$ makes sense thresholds, which separates different branches SMSW.

In (9) with the sign "-", the branch corresponds to the waves, not previously known to the author. These waves are possible in the region of small angles (if the angles of 10-30 degrees can be called small).

Frequencies of "orthogonal" SMSW are limited to top the Damon-Eshbach's frequency $w_{DE} = w_H + w_M/2$ [5-9]. We substitute $w = w_{DE} = w_H + w_M/2$ in the formula (9) and get:

$$u_{1,2} = \frac{w_H + w_M/2 \pm \sqrt{(w_H + w_M/2)^2 - w_H(w_M + w_H)}}{w_M + w_H} =$$

$$= \frac{w_H + w_M/2 \pm w_M/2}{w_M + w_H}; \quad u_1 = 1; \quad u_2 = \frac{w_H}{w_M + w_H}.$$

Predominant direction $u_1 = 1$, as expected, corresponds to the "orthogonal" SMSW. With increasing frequency above w_{DE} from the formulas of this branch, we get the values of u, contradicting the condition $1 > u = sin(\varphi)$. Such decisions should be deleted. The second predominant direction $u_2 = \frac{w_H}{w_M + w_H}$, occurring at the Damon-Eshbach's frequency, corresponds to the "parallel" SMSW or "the second type SMSW". The absolute value of u_2 is less than unity. It corresponds to a corner and continues to exist with increasing frequency above the Damon-Eshbach's frequency. For frequencies much larger than $w_{\perp}$, we get the asymptotic evaluation:

$$u_2 = \frac{w - \sqrt{w^2 - w_{\perp}^2}}{w_M + w_H} \approx \frac{w_H}{2w}.$$

In more detail the conditions for the existence of "parallel" SMSW discussed in section 3.

3. GRAPHS OF THE DISPERSION RELATION FOR YIG

The most simple model of Yttrium Iron Garnet (YIG) is the model of an isotropic ferromagnet, neglecting anisotropy, exchange fields and magnetic damping [7]. (The term "isotropy" characterizes the state of a ferromagnet before its magnetization. After magnetization film up to saturation, film becomes anisotropic.) Equation (3) and formula (6), (7) have been derived with this simple model of the isotropic ferromagnet. The following fact, that has been discussed in [7], is useful for development of MSW models. For more accurate models, you can use the formulas MSW-approximation (this article is of the formula (6)-(7)), if to consider amendments at w_H. The rationale of this technique is also considered in [12]. To account for the magnetic damping, it is enough to do the replacement $w_H \rightarrow w_H + j\alpha w$. Value w_H becomes a complex number that depends on the frequency w. Value α is the coefficient of magnetic damping in the equation of motion of the magnetization in the ferromagnet, written in the form of Hilbert [7]. More complex but the same way is possible to take into account the inhomogeneous exchange field. Numerical experiments show that, the magnetic damping cannot be neglected, but a more careful specification of α without changing its sign does not change graphs for YIG noticeable way. Accounting for exchange field complicates the solution of the problem, but also does not lead to a noticeable change in graphic for isotropic YIG. In the present work for YIG, magnetic damping is taken into account, and the inhomogeneous exchange field is ignored.

As parameters we select values $\gamma = 1.76086 \cdot 10^7 c^{-1} T^{-1}$, $H_0 = 1250\ Oe$, $M_0 = 139\ Gs$, $\alpha = 5 \cdot 10^{-5}$. For these parameters, the following values of the characteristic frequencies in *GHz* have been derived:

$$\frac{w_H}{2\pi} = 3.50, \quad \frac{w}{2\pi} = 4.90, \quad \frac{w_{DE}}{2\pi} = 5.42, \quad \frac{w}{2\pi} = 5.95.$$

$2\pi/cm$ is unit for wave vector, $3 \cdot 10^{-4}\ cm$ as a thickness of slab.

In [7, page 175] for these parameters, there are graphs of the frequency on the wave vector, when the wave vector and the magnetization are orthogonal.

Fig. 1 shows a plot of the wave vector on the direction for frequencies at 5% below the Damon-Eshbach's frequency (near the middle of the frequency range between $w_{\perp}$ and w_{DE}). The wave vector is shown in the graph only one component k_y. This is sufficient, since according to (2), the remaining components of the wave vector are proportional to k_y. The direction of wave propagation is represented by the value u (varying from (-1) to (+1)); u is equal to the sine of the angle between the magnetization of the film and the direction of wave propagation (directions lie in the plane of the film). In the graphs

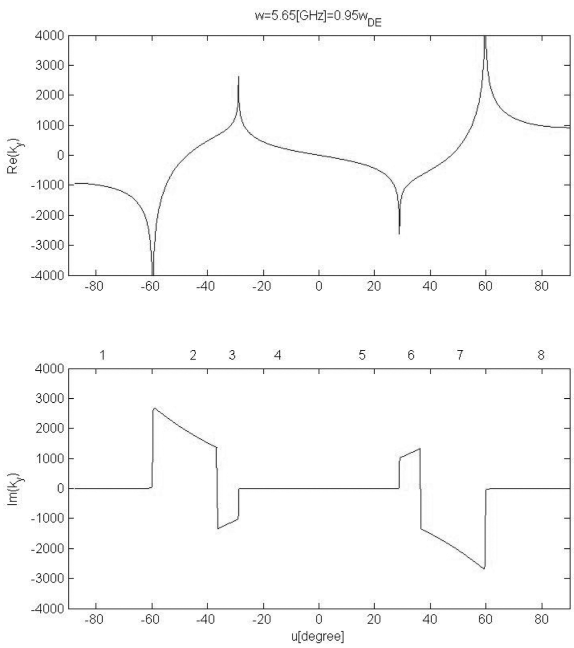

Fig. 1. *For YIG, the dependence of the wave vector on the wave direction is presented when frequency is at 5% below the Damon-Eshbach's frequency.*

the value of the parameter *u* are translated into degrees of the corresponding angle. For one value of a sine corresponds not one, but two angle that are symmetric with respect to the right angle. I.e. graphs can symmetrically continuing, reflecting from vertical boundaries.

As follows from (5), graphics should be centrosymmetric with point reflection at zero. This is a nontrivial property of waves of free films as a manifestation of nonreciprocity wave in magnetic media and as a result of the vector nature of the magnetization. If **k** is the wave vector of a surface waves, the wave propagating in the opposite direction **-k** can have very different properties. Vector **-k** is the decision where the amplitude should increase with distance from the surface of the media and hence must be removed from the results. In graph, for a section of surface waves, there are corresponding symmetrical sections with unrealizable waves that should be excluded from the solution.

In the Fig. 1, there are 8 regions with different behaviour. They are labelled on the top of Im(k_y).

Region 8 corresponds to the more familiar "orthogonal" SMSW. At region 8, the undamped (within the MSW approximation) surface waves occur near the direction orthogonal to the magnetization of the film (90 degrees). The real part of the wave vector increases with the deviation of u from orthogonal directions. The amplitude SMSW is proportional to the wave vector, so we can talk about resonant increase of the wave amplitude in the direction of the wave, i.e., about the "wave resonance with orientation". The resonance peak differs from that of a Gaussian curve, characteristic of usual resonance. When the maximum of k_y is reached then the damping of waves turns on. The direction corresponding to the maximum of k_y , we call the predominant (resonance) direction. In fig. 1 predominant direction is separates the region 8 and 7. Region 8 "orthogonal" SMSW corresponds to the region 1, where waves are unrealizable wave. They should be excluded from the solutions of the problem.

A large dumping of the wave amplitude takes place on region 7. The length of the damping and the wavelength has the same order. Region 2 is symmetric to region 7.

Region 4 corresponds to the "parallel" branch SMSW. Conditions for the existence of these waves differ from the conditions of existence "orthogonal" SMSW. To change the sign of the parameter *u* need to get a wave propagating in the opposite direction, or to magnetize the ferromagnetic material in the opposite direction, or place the emitter on the opposite end of the film. The peak of "wave resonance with orientation" takes place on the border of the region 4 and region 3. At region 3 there are large dumping of the waves. On the border region 4 and region 5 the parameter *u* and the wave vector vanish.

At region 5 and 6 the waves are unrealizable and should be excluded from the solution.

Note that to obtain Fig. 1 with the correct signs damping of the waves must be taken into account magnetic damping. If we change the sign of the coefficient of magnetic damping then the picture changed dramatically in regions where a large increase or damping of the waves. Moreover, the change in the value of the coefficient of magnetic damping, without changing its sign does not change significantly the graph. Magnetic damping cannot be neglected when describing the wave resonance with orientation.

Fig. 2 shows a graph of the frequency twice the Damon-Eshbach's frequency. The branch of "orthogonal" SMSW disappeared, but the branch of "parallel" SMSW exist. The region of surface waves

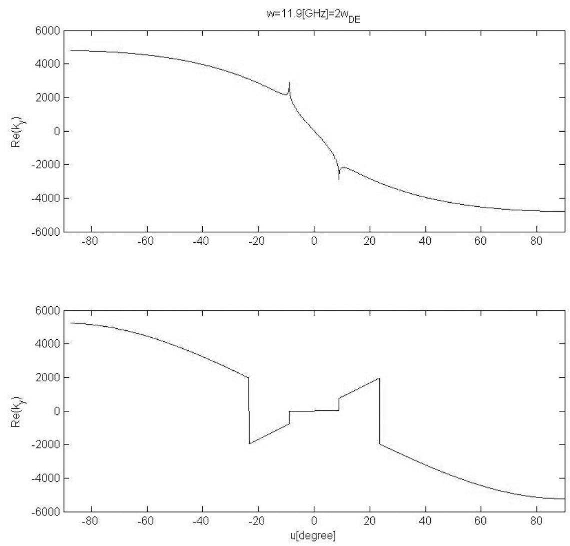

Fig. 2. *For YIG, the dependence of the wave vector on the direction is presented, when wave frequency is twice the Damon-Eshbach's frequency*

narrows if the frequency increases. It is inversely proportional to frequency.

4. PREDOMINANT DIRECTIONS OF SMSW IN YIG

Fig. 3 allows us to estimate the angles between the magnetization and the predominant direction of wave propagation. We plot the ratio (9), where instead of the parameter u used the angle between the wave vector and the magnetization of the film, expressed in degrees. With plotting the chart, we take into account that for any value of a sine not one but two angle take place. They are symmetric with respect to the right angle. The frequency is given in *GHz*.

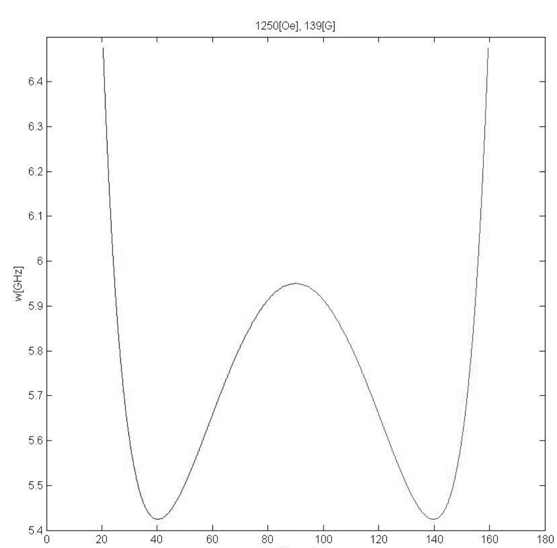

Fig. 3. Directions of the wave resonance with orientation (predominant modes) for YIG

A 90-degree angle in Fig. 3 corresponds to the Damon-Eshbach's frequency. At lower frequencies there are symmetric predominant directions. The minima in Fig. 3 corresponds to the frequency $w_\perp$ and direction $u_\perp$. At this frequency cannot be SMSW. $u_\perp$ is the ultimate value that separates the branch "orthogonal" SMSW from "parallel" SMSW. The frequency of "parallel" PMSW may greatly exceed the Damon-Eshbach's frequency.

5. CONCLUSION

In magnetic film an wave resonance with orientation can be considered as selecting of predominant harmonics. This phenomenon is very similar, but different from the caustic. Caustics are interpreted in terms of geometric optics, but with interesting wave justifications and generalizations [13]. In [6, 9, 10] the formation SMSW by means emitter of finite size is explained as the result of the interaction of spatial modes.

Understanding the details of the analysis of the dispersion relation allows to clarify this interpretation. The middle part of the flat emitter gives the main contribution to the amplitude of the harmonics propagating orthogonal to the emitter, while the ends of the flat emitter give harmonic propagating in all directions. Resonant amplification of selected two harmonics leads to formation of two narrow SMSW beams from each of the ends of the emitter when the frequency is slightly lower Damon-Eshbach's frequency. Although any resonance phenomenon can be mathematically described in the style of the catastrophe theory [13], it seems promising to move away from a beautiful picture of the wave fields, and focus on the more simple effects and experiments. "whiskers" waves can be interpreted as resonance peaks with unusual properties. After research of features of this wave resonance can put more specific targets for investigation of wave fields.

Developed in the present work, ideas, formulas and estimations can be useful when searching other, maybe less spectacular, but more constructive solutions for the development of new SMSW devices. Probably if to go closer to the predominant directions we can to try to reduce the power consumption of the devices on SMSW and to propose new methods of signal processing. "Parallel" SMSW predicted by the theory, have a frequency range exceeding the Damon-Eshbach's frequency. From the point of view of the opinions expressed in [1, 2], (mentioned in the beginning of the article) the study of such waves and their properties should be promising. For this branch SMSW the wave resonance with orientation takes place also. At high frequency, the predominant direction comes near to direction of film magnetization inversely proportionally to frequency.

The orientation of the resonance and the existence of branch of the "parallel" SMSW derived theoretically and experimental confirmations are required. If experiment would not confirm theoretical prediction, what conclusions may follow? All transformations of equation (3) presented in this article do not contribute to the model SMSW any simplifications or generalizations, they only result in equation (3) is more convenient for the analysis of mind and computer. So the question would be arise: why is the model SMSW proposed in a classic paper [5] would sometimes not give the correct results? Refined definition of the problem will be not less interesting than the direct experimental confirmation of the theoretical predictions.

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